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Degradation Models

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Introduction

Reliability testing typically generates product lifetime data, but for some tests, **covariate** information about the wear and tear on the product during the **life test** can provide additional insight into the product's lifetime distribution. This usage, or **degradation**, can be the physical parameters of the product (e.g., corrosion thickness on a metal plate) or merely indicated through product performance (e.g., the luminosity of a light emitting diode). The measurements made across the product's lifetime are *degradation data*, and *degradation analysis* is the statistical tool for providing inference about the lifetime distribution from the degradation data.

Degradation testing and analysis is tied in with accelerated life testing (ALT) because both methods have evolved in recent years to suit reliability tests for which product lifetimes are expected to last far beyond the allotted test time (*see* **Accelerated Life Models**). ALT is meant to expedite product failure during test intervals by stressing the product beyond its normal use. This helps to bring in more information if the link between the accelerated test environment and the regular use environment is known. The same is true of degradation analysis. If the link between the measure of degradation and lifetime is clearly known, the degradation data provide valuable information about product reliability. Accelerated degradation testing (ADT) combines these two approaches by testing products in harsh environments and measuring the evidence of product degradation during the ALT.

Degradation analysis is especially useful for tests in which soft failures occur; that is, the lifetime of the test item is said to end after the measured performance decreases to a predetermined threshold value that designates a nonfunctioning state or an incipient failure. For a test of material strength, as an example, the degradation measurement might be the increasing size of the largest observed crack in the material, or the amount of corrosion measured on the surface. A failure event can be designated long before the material actually breaks. As another example, electronic components function reliably if their resistance, capacitance and voltage stay within design limits, and failure can be said to occur when one or more of these parameters degrades beyond a specified limit. Ohring [1] presents a comprehensive array of physical models for electronic devices. Similarly, light emitting devices may be considered to fail only after the luminosity degrades below a fixed measured limit.

Figures 1 and 2 illustrate two different examples of degradation data. Figure 1, from Bogdanoff and Kozin [2] and featured in Meeker and Escobar [3], shows the measured crack size for alloy specimens that were fatigued by rapid cycling. Twelve of the 21 test items failed because their crack sizes exceeded the fixed threshold of 1.6 inches. Nine other test items did not fail, and if the degradation model is sound, more information will be gained from the degradation measurements in these nine observations compared to the respective lifetime measurements, which are right censored.

Figure 2 shows the measured degradation path for light emission of seven vacuum fluorescent displays (VFDs). The VFDs were tested for 200 h in an accelerated failure environment, so this is an example of an ADT. See Bae and Kvam [4] for details. In this case, the degradation path of each test item reaches the failure threshold (defined as the time the luminosity decreases by 50%) where a soft failure is defined.

Degradation Modeling

Models for degradation are generally either data-driven or derived from physical principles via stochastic processes. Although the data-driven model is more commonly applied to analyze degradation data, viewing degradation through stochastic processes helps researchers theoretically characterize the failure process.

Data-Driven Model

The measured degradation path for the ith tested device ($i = 1, \ldots, n$) will consist of a vector of m_i measurements made at time points $t_{i1}, \ldots, t_{im_i}$. The VFDs in Figure 2, for example, were each measured at the same five time points (0, 24, 72, 100, and 200 h). The measured degradation at t can be modeled as the actual unknown degradation $\eta(t)$

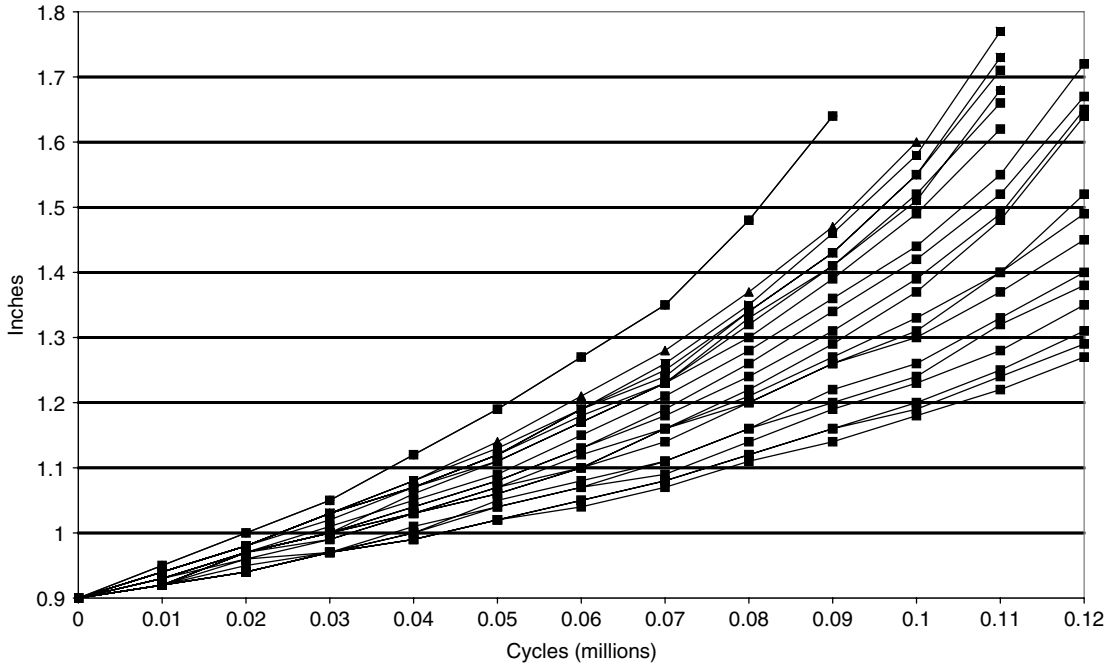

Figure 1 Alloy fatigue crack size (in inches), observations, measured in millions of cycles

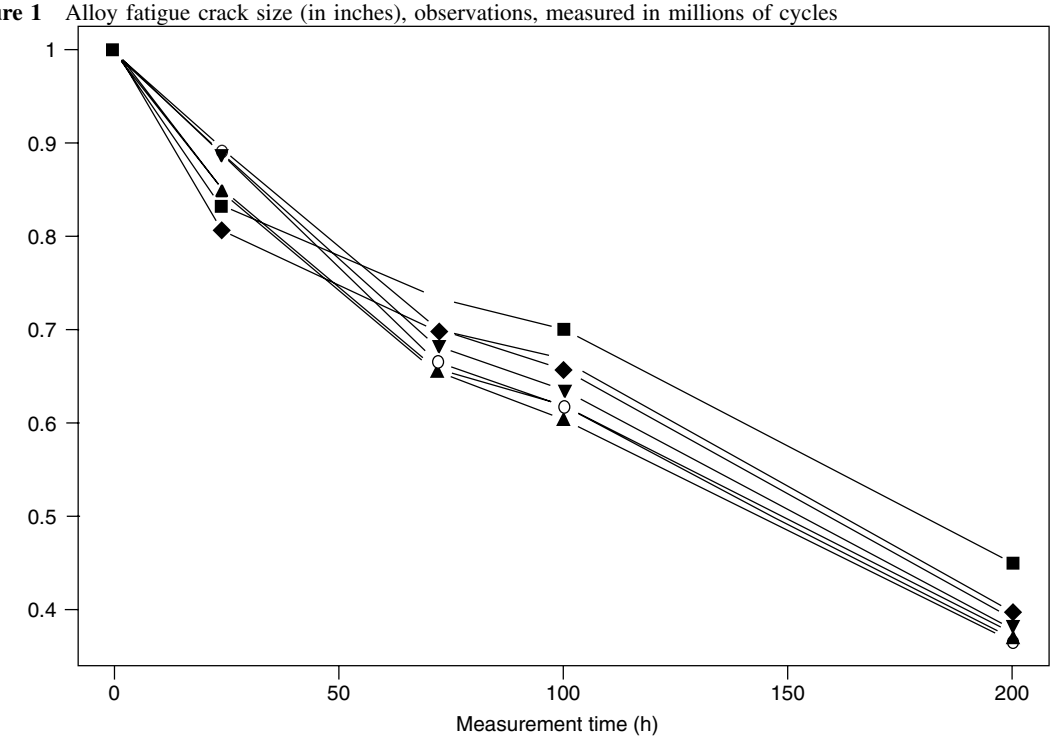

Figure 2 Luminosity degradation in seven VFD devices

plus a measurement error term ϵ. At the m_i time points, the degradation measurements of device i are

$$y_{ij} = \eta(t_{ij}) + \epsilon_{ij},\ 1 \le i \le n,\ 1 \le j \le m_i \quad (1)$$

The form of η can be chosen to have a strict form or it can be more arbitrary. For example, the degradation of many electronic components is known to be of a log-linear form. That is, if we replace y with $\log(y)$, η can be a linear function of time such as $\eta(t) = \beta_1 + \beta_2 t$. On the other hand, the form for degradation can be unknown and nonparametric regression techniques are required to analyze the degradation data. Shiau and Lin [5] use this approach, but compared to a well chosen parametric model, their method can be quite inefficient. However, they show that the nonparametric technique can be useful for selecting a suitable parametric form of η.

If a specific form is described for η, we will have an unknown set of parameter values $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_k)$ that must be estimated in order to fully characterize the degradation. In any realistic application of degradation analysis, the test units will degrade in a similar way but on distinct paths. To model this *unit-to-unit variability*, a distribution is assigned to $\boldsymbol{\beta}$, allowing n distinctive paths to describe the degradation of the n test units. Meeker and Escobar [3] present a convincing argument for why the **normal distribution** adequately characterizes this randomness (i.e., $\boldsymbol{\beta} \sim \mathcal{N}(\bar{\beta}, \Sigma)$). Along with $\boldsymbol{\beta}$, let λ represent unknown parameters that are common across test units (thus no random effects are necessary) so the degradation path is expressed by

$$y_{ij} = \eta(t_{ij}; \lambda, \boldsymbol{\beta}_i) + \epsilon_{ij},\ 1 \le i \le n,\ 1 \le j \le m_i \quad (2)$$

Degradation Processes

In many experiments, degradation is a continuous progression of wear and decay, so it makes intuitive sense to model the degradation path with a stochastic process. If we let a stochastic process $W(t)$ to describe the degradation level of an item at time t, the mean degradation $\mu(t) = E[W(t)]$ is typically increasing and sometimes known through physical principles. For example, the wear on an automobile tire might be measured in terms of usage ($t =$ odometer reading) and if the tire wear is constant, $\mu(t) = \lambda t$ for some unknown value λ.

The degradation characteristics of several electronic components can be described through stochastic processes. For example, Mitsuo [6] determined the mean degradation for light emitting diodes as $\mu(t) = \lambda t^k$ for some $\lambda > 0$ and $k \in \Re$. Aven and Jensen [7] show how different processes imply different lifetime distributions. For example, in the case where the degradation follows a Wiener process, the time when the degradation level first reaches a fixed failure threshold has an inverse Gaussian distribution.

Stochastic processes are also helpful to infer lifetime distributions from **damage models**, which are a special case of degradation models, for example, see Kahle and Wendt [8]. For example, suppose $M(t)$ is a Poisson shock process with rate λ. Let P_k be the probability that the system survives k shocks, so that $1 = P_0 \le P_1 \le P2 \le \ldots$. Then, the system survival can be expressed as

$$\sum_{k=0}^{\infty} P_k \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

If $P_{i+j} \le P_i P_j$, then compared to a new device, the probability of surviving k additional shocks is smaller when the device has already absorbed some shocks. Based on this premise, it is possible to show that the lifetime distribution is *new better than used*, that is, for any $x, y > 0$, $P(X \ge x)P(X \ge y) \ge P(X \ge x + y)$ (*see* **Stochastic Orders and Aging**).

Relating Degradation to Lifetime

If the **physics of failure** is known through the model of an item's degradation over time, it's lifetime distribution can be inferred from this model as well (*see* **Degradation and Failure**). For materials, specimens exposed to constant stress cycles in a given stress range, lifetime is measured in number-of-cycles until failure (N). The Whöler curve (or $S - N$ curve) relates stress level (S) to N as $NS^b = k$, with known material parameters b and k. The $S - N$ equation is expressed in log form as $Y = \log N = \log k - b \log S$. If N is log-normally distributed, Y is normally distributed and regular regression models can be applied for predicting cycles-to-failure. The log-normal distribution is considered for modeling the failure time distribution when the corresponding degradation process based on rates that combine multiplicatively, and it is convenient for modeling fatigue

crack growth in metals and composites. Sobczyk and Spencer [9] features numerous settings and examples.

An alternative model based on fatigue, introduced by Birnbaum and Saunders [10], defined B_n to be the measurable damage (*see* **Cumulative Damage Models Based on Gamma Processes**) to the test item after n cycles with accumulated amount of damage ζ_i in the ith cycle *via* $B_n = \zeta_1 + \cdots + \zeta_n$, $i = 1, \ldots, n$. If the ζ_is are identically and independently distributed as with mean μ and variance σ^2,

$$P(N \leq n) = P(B_n > B^*) \approx \Phi\left(\frac{B^* - n\mu}{\sigma\sqrt{n}}\right) \quad (3)$$

where Φ is the standard normal cumulative distribution function (CDF). This results because B_n will be approximately normal if n is large enough. The reliability function for the test unit is

$$R(t) \approx \Phi\left(\frac{B^* - n\mu}{\sigma\sqrt{n}}\right) \quad (4)$$

This is called the *Birnbaum–Saunders* distribution, and it follows that

$$W = \frac{\mu\sqrt{N}}{\sigma} - \frac{B^*}{\sigma\sqrt{N}} \quad (5)$$

has a normal distribution, which leads to accessible implementation in lifetime modeling. Bogdanoff and Kozin [2] overview properties of the Birnbaum–Saunders distribution for materials testing.

Statistical Inference

If we assume the degradation path model in equation (2), with item-to-item variability reflected through random coefficients in η, we have an accumulated set of unknown parameters $\theta = (\lambda, \bar{\beta}, \Sigma)$, which makes for a difficult computation of the lifetime distribution. Numerical methods and simulations are typically employed to generate point estimates and confidence statements.

In selecting a degradation model based on longitudinal measurements of degradation, monotonic models are typically chosen under the assumption that degradation is a one-way process. From the degradation model, the lifetime distribution is defined as the time at which the degradation first reaches the failure threshold, designated y^*:

$$F(t) = P(y(t) > y^*) = P(\eta(t; \lambda, \boldsymbol{\beta}) + \epsilon > y^*) \quad (6)$$

Least squares (**Least-Squares Estimation**) or maximum likelihood (**Maximum Likelihood**) can be used to estimate the unknown parameters in the degradation model. To estimate $F(t_0)$, one can simulate N degradation curves from the estimated regression by generating N random coefficients $\theta_1, \ldots, \theta_N$ from the estimated distribution $G(\theta; \hat{\beta})$. Next compute the estimated degradation curve for y_i based on the model with θ_i and $\hat{\lambda}$: $y_i(t) = \eta_i(t; \hat{\lambda}, \theta_i)$. Then $\hat{F}(t_0)$ is the proportion of the N generated curves that have reached the failure threshold y^* by time t_0.

A nonparametric **bootstrap** sampling procedure can be used for measuring the uncertainty in the lifetime distribution estimate. The bootstrap procedure *resamples* the sample degradation curves *with replacement* (i.e., so some curves may not be represented in the sample while others may be represented multiple times). Meeker and Escobar [3] summarize a bootstrap procedure for making **confidence intervals** for the lifetime distribution:

1. Compute estimates of parameters $\bar{\beta}$, λ, Σ.
2. Use simulation (above) to construct $\hat{F}(t_0)$.
3. Generate $N \geq 1000$ bootstrap samples, and for each one, compute estimates $\hat{F}^{(1)}(t_0), \ldots, \hat{F}^{(N)}(t_0)$. This is done as before except now the M simulated degradation paths are constructed with an error term generated from $H(\eta; \hat{\Sigma})$ to reflect variability in any single degradation path.
4. With the collection of bootstrap estimates in equation (7), compute a $1 - \alpha$ level confidence interval for $F(t_0)$ as $(\hat{F}^l(t_0), \hat{F}^u(t_0))$, where the indexes $1 \leq l \leq u \leq N$ are calculated as $N^{-1} = \Phi(2\Phi^{-0.5}(p_0) + \Phi^{-0.5}(0.5\alpha))$ and $uN^{-1} = \Phi(2\Phi^{-0.5}(p_0) + \Phi^{-0.5}(0.5(1-\alpha)))$, and p_0 is the proportion of bootstrap estimates of $F(t_0)$ less than $\hat{F}(t_0)$.

Example

The VFDs are tested (see Figure 2) at higher filament voltage level than normal usage condition

(Ef = 4.55 V). It is known that the display luminosity for VFDs decreases exponentially over most of the usage period when the degradation path can be expressed as

$$\Lambda(t) = \beta_0 \exp(-\beta_1 t) \tag{7}$$

where $\Lambda(t)$ denotes the luminosity at time t, β_0 is the initial luminosity, and β_1 is the rate of degradation. The light device is considered to fail at the time its luminosity decreases below 50% of its initial measurement. That is, failure is defined as the first time relative luminosity

$$y(t) = \frac{\Lambda(t)}{\Lambda(0)} = \exp(-\beta_1 t) \tag{8}$$

falls below 0.5. Using least-squares regression with the log transformation $\log y(t) = -\beta_1 t$, and by ignoring variation between individuals, the fitted (no-intercept) model is

$$\log y(t) = -0.0048t \tag{9}$$

with $R^2 = 0.9893$. However, the model fails to reflect individual variation of degradation rate (see Table 1).

To reflect individual variation of degradation, a linear random-coefficients model can be applied by assuming that β_1 is random. The random effects model can be written as

$$\log y(t_{ij}) = -(\beta + u_i)t_{ij} + \epsilon_{ij} \tag{10}$$

where t_{ij} is the covariate for the jth measurement time on the ith individual, $\epsilon_{ij} \sim \mathcal{N}(0, \sigma^2)$, and $u_i \sim \mathcal{N}(0, \sigma_u^2)$. Using the SAS NLMIXED procedure, the fitted linear random-coefficients model is computed numerically as

$$\log y(t_{ij}) = -0.004751 t_{ij} \tag{11}$$

Table 1 Estimated degradation rates for VFDs in Figure 2

Sample	Degradation rate
1	4.6×10^{-3}
2	5.1×10^{-3}
3	4.6×10^{-3}
4	5.1×10^{-3}
5	4.8×10^{-3}
6	4.0×10^{-3}
7	5.0×10^{-3}

with $\hat{\sigma}_u^2 = 0.001574$ and $\hat{\sigma}^2 = 1.032 \times 10^{-7}$. Based on the estimated parameters, the fitted lines are given in Figure 3.

Comments

Degradation measurements have great potential to improve lifetime data analysis, but they also introduce new problems to the statistical inference. Lifetime models have been researched and refined for many manufactured products that are put on test. On the other hand, parametric degradation models tend to be based on simple physical properties of the test item and its environment (e.g., the Paris crack law, Arrhenious rule, Power law) which often lead to obscure lifetime models. Meeker and Escobar [3] show that most valid degradation models will not yield lifetime distributions with closed-form solutions. Given the improving computational tools available to researchers, this should be no deterrent to using degradation analysis.

In a setting where the lifetime distribution is known, but the degradation distribution is unknown, degradation information does not necessarily complement the available lifetime data. For example, the lifetime data may be distributed as Weibull, but conventional degradation models will contradict the Weibull assumption (actually, the rarely used *reciprocal Weibull* distribution for degradation with a fixed failure threshold leads to Weibull lifetimes). Bae *et al.* [11] discuss this problem in terms of a simple additive degradation model.

Finally, the monotone relationship between degradation and usage (or time) does not necessarily hold for all applications. Bae and Kvam [12] analyze light display data for a VFD that shows nonmonotonic degradation during its **burn-in** period (*see* **Burn-In and Maintenance Policies**), when impurities in the vacuum are being burned off. As this happens, luminosity actually increases slightly before beginning a long and steady decrease over its usage period. In this case, a more complicated mixture model that captures the burn-in effect proves to be more efficient.

Compared to ordinary life testing or even ALT, degradation analysis procedures tend to be computationally cumbersome. However, for important applications, the increase in statistical efficiency can be dramatic. In the past, these computations have

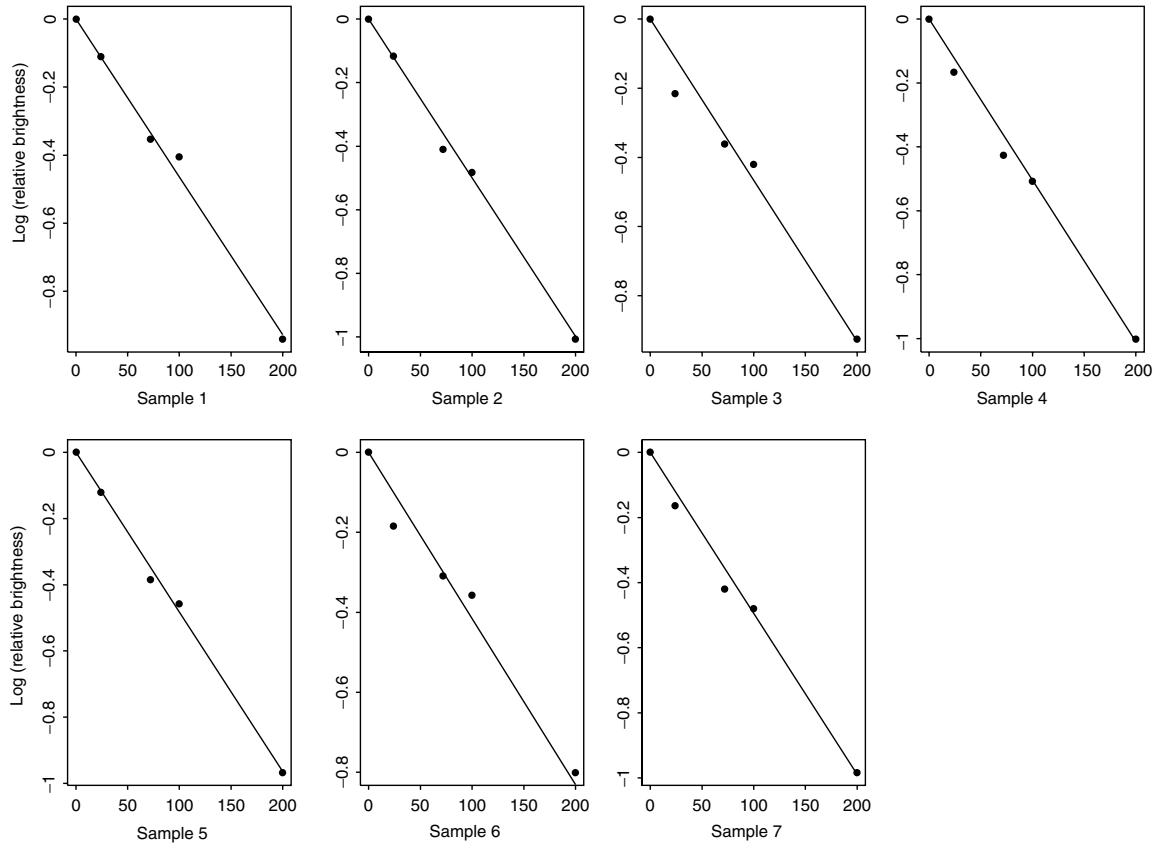

Figure 3 Fitted degradation model

impeded degradation analysis from being a feature of reliability problem solving. Such analyses are easier to implement now, and the reliability analyst need not be coerced into using an overly simplistic model – for instance, a linear model that does not allow for random coefficients. Chen and Zheng [13] construct an **imputation** algorithm to generate a closed-form solution, although its performance is suspect for medium or small samples.

Robinson and Crowder [14] introduce a Bayesian approach and showed if **prior distributions** are chosen from a reasonable class, they have negligible effect on the lifetime **estimation**. Bayesian reliability is summarized in **Bayesian Reliability Analysis**. In most cases the Bayesian procedure is even more computational than the one introduced here, but **Markov chain** Monte Carlo (e.g., **Hierarchical Markov Chain Monte Carlo (MCMC) for Bayesian System Reliability**) methods have straightforward implementation.

Acknowledgments

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Related Articles

Accelerated Life Models; **Bayesian Reliability Analysis**; **Burn-In and Maintenance Policies**; **Cumulative Damage Models Based on Gamma Processes**; **Design for Reliability**; **Degradation Processes**; **Degradation and Failure**; **Mean Square Error**; **Stochastic Orders and Aging**; **Stochastic Deterioration**; **Warranty: Usage and Wear Process for**.

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