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The Backward Shift on the Hardy Space

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Preface

Shift operators on Hilbert spaces of analytic functions play an important role in the study of bounded linear operators on Hilbert spaces since they often serve as "models" for various classes of linear operators. For example, "parts" of direct sums of the backward shift operator on the classical Hardy space H^2 model certain types of contraction operators and potentially have connections to understanding the invariant subspaces of a general linear operator.

In this book, we do not want to give a general treatment of the backward shift on H^2 and its connections to problems in operator theory. This has been done quite thoroughly by Nikolskiĭ in his book **[65]**. Instead, we wish to work in the Banach (and F-space) setting of H^p $(0 < p < \infty)$ where we will focus primarily on characterizing the backward shift invariant subspaces of H^p. When $p \in (1, \infty)$, this characterization problem was solved by R. Douglas, H. S. Shapiro, and A. Shields in a well known paper **[29]** which employed the concept of a 'pseudocontinuation' developed earlier by Shapiro **[84]**. When $p \in (0,1)$, the characterization problem is more difficult, due to some topological differences between the two settings $p \in [1, \infty)$ and $p \in (0,1)$, and was solved in a paper of A. B. Aleksandrov **[3]** which was never translated from its original Russian and hence is not readily available in the West. The Aleksandrov paper is also quite complicated and makes use of the distribution theory and Coifman's atomic decomposition for the Hardy spaces of the upper half plane, a topic we feel is not always at the fingertips of those schooled, as we were, in classical function theory and operator theory. It is for these reasons that we gather up these results, along with the necessary background material, and put them all under one roof.

In developing the necessary background results, we do not wish to reproduce the material in the books of Duren **[31]** or Garnett **[39]** (for a general treatment of Hardy spaces) or Stein **[95]** (for a detailed treatment of harmonic analysis and real variable H^p theory). Instead, we will only review this material and refer the interested reader to the appropriate places in these texts for the proofs. The reader is expected to have a reasonable background in functional analysis and function theory (including the basics of H^p theory), but might want to have Rudin's functional analysis book **[78]**, Duren's H^p book **[31]**, and Stein's harmonic analysis book **[95]** at the ready while reading this book. We will try to develop the more specialized topics as we need them.

The authors wish to thank several people who helped us along the way. First, we thank A. B. Aleksandrov, who, through many e-mails, helped us understand the more difficult parts of his papers. Secondly, we thank Alec Matheson and Don Sarason, who read a draft of this book and provided us with useful suggestions and corrections. Thirdly, we thank Olga Troyanskaya, who translated the Aleksandrov paper **[3]** from the original Russian. Finally, the second author wishes to thank

the mathematics department of the University of North Carolina, Chapel Hill, for the comfortable setting for the semester in which he finally got to work with the first author face to face (and not over the Internet) where they assembled the final version of this book.

JAC AND WTR