Cosmological Inflation in N-Dimensional Gaussian Random Fields with Algorithmic Data Compression

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Abstract

The leading modern theories of cosmological inflation are increasingly multi-dimensional. The "inflation field" \( \phi \) that has been postulated to drive accelerated expansion in the very early universe has a corresponding potential function \( V \), the details of which, such as the number of dimensions and shape, have yet to be specified. We consider a natural hypothesis that \( V \) ought to be maximally random. We realize this idea by defining the \( V \) as a Gaussian random field in some number \( N \) of dimensions. We repeatedly simulate the evolution of \( \phi \) in a large set of initial conditions on the "landscape" of \( V \). We simulate a "path" by first choosing \( \phi \) at random in \( \phi \)-space while simultaneously computing \( V \) and its derivatives along the path via a constrained Gaussian random process, incorporating the information from prior steps. When \( N \) is large, this method significantly reduces computational load as compared to methods which generate the potential landscape all at once. Even so, computation of the covariance matrix \( V \) of constraints on \( \phi \) can quickly become intractable. Inspired by this problem, we present data compression algorithms to prioritize the necessary information already simulated, then keep an arbitrarily large portion. Information such as the evolution of the scale factor and tensor and scalar perturbations can be extracted from any particular path, then statistical information about these quantities can be gathered from repeated trials. In this way, we present a versatile multi-variable program for exploration into how accurately this emergent model can fit to observation.

Problem

Creating an inflationary model means defining a potential function \( V \) of many variables \( \phi^{(1)}, \phi^{(2)}, \ldots, \phi^{(N)} \). The field \( \phi \) is required to have some key properties:

- **Minimum condition.** The function must have a local minimum at \( V = 0 \).
- **Slow roll condition.** The function must have gradual and gradually changing slopes near the minimum.
- **Energy density.** \( V \) represents an energy density in the early universe, so its values must be large enough to make it dominate at that time. This ensures that \( V \) governed large-scale changes in the size of the universe.

There are many examples of inflationary potentials, but we are interested in creating \( V \) from a random process. We do this point-by-point. The first potential value \( V(\phi) \) is a completely random value, drawn from a Normal distribution with mean zero and variance \( V \), or \( V(1) \). The second value \( V(2) \) is constrained by the first value depending on the difference between \( \phi \), and \( \phi_{(s)} \) in general, \( V(0) = V(1) \), where

\[
\mathbf{W}(\phi) = \begin{pmatrix} V(2) - V(1) \\ V(2) \end{pmatrix}, \quad \mathbf{Z}(\phi) = \begin{pmatrix} V(1) \\ V(2) \end{pmatrix}, \quad V(2) = V(1) + (V(2) - V(1)) = V(1) + \mathbf{W}(\phi) \mathbf{Z}(\phi)^{-1} \mathbf{W}(\phi)^T
\]

\( \mathbf{W}(\phi) \) is the list of all previous potential values and \( F \) is the covariance matrix of constraints on \( V \). Subscripts \( 0 \) and \( N \) abbreviate "old" and "new", respectively. This process of randomly generating \( V \) is used when solving the Klein-Gordon differential equations for the inflation field evolution of a model universe:

\[
\frac{d^2 \phi}{d\tau^2} - 3 \frac{d\phi}{d\tau} \frac{dV}{d\phi} + \frac{1}{V} \frac{dV}{d\tau} = 0
\]

In these equations, \( N \) is the "number of e-folds", a measure of both time and the relative size of the universe and \( 1/H \) are functions of the other quantities. If we know the values of \( V \) at the relevant region, we can solve for \( \phi \) from its initial position to its final position, ideally in the minimum of \( V \). The trajectory (evolution) of any \( \phi \) depends on its underlying potential function, which is randomized to some extent but controlled by parameters \( s, V(0), V(1) \), and \( N \). Today, there exist some observable quantities \( v, \omega, n_s \) that are theoretically linked back to the trajectory, and thus to the choice of potential. Testing the plausibility that the Gaussian random potential represents our physical universe involves simulating many potentials, solving for many trajectories, and extracting observable quantities from each trajectory for statistical analysis.

Program

Restricting \( N = 2 \) for visual purposes, the Gaussian random potentials end up looking something like this.

To use this function, or one like it, to solve for the inflation field evolution in a model universe, imagine releasing a ball at some point around the minimum. The Klein-Gordon equations tell us that the ball will accelerate downhill along the slope of \( V \) until it reaches zero, after which it will roll uphill. The trajectory (evolution) of any \( \phi \) under a random potential, then extract from it observable quantities. Figure 2 describes the process, from input to output, that we designed in Python to do the tasks.

Predictions

Since our model shows promise, there are several ways in which we can computerize our results as plausible:

- Explore parameter space. Varying \( s, V(0), V(1) \) and \( \phi(0) \) and cataloging many simulations will yield the best range of our model.
- Translate to other languages. Other programming languages, like C and FORTRAN, can quickly execute the program described in Figure 2.
- Incorporate other observables. Some models have been tested against other quantities, such as the non-Gaussianity of the CMB, adding tests which could "make or break" the model.

References

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