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Visual Timelines in Excel to Illustrate TVM Calculations

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Visual Timelines in Excel to Illustrate TVM Calculations

Time value of money calculations are illustrated through developing a timeline with cash flow graphics in Excel. The cash flow graphics can be used in the live or virtual classroom and as a resource for students outside of the classroom. Further, the graphic is readily adjustable to different scenarios making it useful for multiple time value of money topics.

INTRODUCTION

Widely adopted corporate finance and investment texts make frequent use of timelines to demonstrate time value of money (TVM) calculations and applications.[1] Additionally, instructors use timelines extensively in the classroom, and have done so for decades. Yet, students have a reluctance to use timelines initially or abandon using timelines altogether, despite the great amount of intuition one can garner when using a timeline. In this presentation, Excel programing with a timeline graphic are used to illustrate the timeline and the associated TVM calculations. A simple graphic is generated by merging and adding borders to cells to establish the discrete time periods. An instructor can display the Excel timeline graphic on an overhead screen or through Zoom for lecture and post the file as a resource for students to use on their own outside the classroom.

In the next section, the programing for working with a single cash flow is presented. In the following section, a similar graphic utilizing multiple cash flows is presented. Although the multiple cash flows graphic technically subsumes the single cash flow graphic, the former is necessary to generate intuition and should not be overlooked. The third section concludes the article.

SECTION 1: Single Cash Flow Graphic in Excel

In Exhibit 1, a three-year time line with the associated Excel programming is displayed. A ten-year version of the graphic is available for download at:

https://scholarship.richmond.edu/finance-faculty-publications/X/

[1] See, for example, Chapter 14 of Bodie, Kane, and Marcus (2018), Chapter 4 of Ross, Westerfield, Jaffe, and Jordan (2016) , Chapter 5 of Ross, Westerfield, and Jordan (2016), and Chapter 2 of Welch (2017).

The programming prevents the user from entering multiple cash flows and from entering multiple points in time for determining the time value of money calculation.

Exhibit 1: Single Cash Flow Graphic in Excel

	A	B	C	D	E	F	G	H	I
1	Periodic interest rate:	10.00%							
2									
3									
4	Enter a single cash flow above associated moment in time:					$121.00			
5									
6									
7		**0**		**1**		**2**		**3**	
8	Type in "X" to indicate the point in time at which the time value of money is calculated:	X							
9									
10	Answer:	$100.00							
11									
12	Calculation:	$121 ÷ (1 + 10%)^2 = $100.000							
13									
14	FV = cash flow × (1 + interest rate)^N								
15	PV = cash flow ÷ (1 + interest rate)^N								
16									
17									
18		0		0		1		0	
19		1		0		0		0	
20		2	0	÷	×				
21		2	÷	121					
22									
23									
24									

Merge cells: B4 and C4, D4 and E4, F4 and G4, H4 and I4
Provide associated cell borders in rows 5 and 6 to mimic a time line.
Merge cells: B7 and C7, D7 and E7, F7 and G7, H7 and I7
Merge cells: B8 and C8, D8 and E8, F8 and G8, H8 and I8
Merge cells: B10 and C10
Cell B10: = D21 / (1 + B2)^(B20 – C20)
Merge cells: B12 to E12
Cell B12: = CONCAT(TEXT(D21,"$ #,###.00")," ", C21," ","(1 + ",B1*100,"%) ^ ",B21," = ",TEXT(B10, "$ #,###.00"))

These formulas can be "hidden" or "cut and pasted" to another worksheet to hide from view:
Merge cells: B18 and C18, D18 and E18, F18 and G18, H18 and I18
Merge cells: B19 and C19, D19 and E19, F19 and G19, H19 and I19
Cell B18: =IF(B4 = 0, 0, 1)
Cell D18: =IF(D4 = 0, 0, 1)
Cell F18: =IF(F4 = 0, 0, 1)
Cell H18: =IF(H4 = 0, 0, 1)

Cell B19: =IF(B8 = "X", 1, 0)
Cell D19: =IF(D8 = "X", 1, 0)
Cell F19: =IF(F8 = "X", 1, 0)
Cell H19: =IF(H8 = "X", 1, 0)

Cell B20: =IF(SUM(B18:I18)=1,SUMPRODUCT(B7:I7,B18:I18),"ERROR")
Cell C20: =IF(SUM(B19:I19)=1,SUMPRODUCT(B7:I7,B19:I19),"ERROR")
Cell B21: = ABS(B20 – C20)
Cell C21: =IF((B20 – C20) > 0, D20, E20)
Cell D21: = SUM(B4:I4)

A ten-period version of this file can be downloaded from: https://scholarship.richmond.edu/finance-faculty-publications/X/

Additional features if desired:

Highlight cells B4 to I4, go to “Data” menu on Excel dashboard, go to “Data Validation” pulldown menu and an input box displays with three sub-menus:
In “Settings” menu: select “Custom” and enter =COUNT($B$4:$I$4) <= 1 as the Formula
In “Input Message” menu: activate “Show input message when cell is selected” box (this is usually the default), and enter “Enter a single cash flow above associated moment in time” into “Input message” text box
In “Error Alert” menu: activate “Show error alert after invalid data is entered” box (this is usually the default), set “Style” as “Stop” (this is usually the default), and enter “Only enter one cash flow on the timeline.” into the “Error message” text box

Highlight cells B8 to I8, go to “Data” menu on Excel dashboard, go to “Data Validation” pulldown menu and an input box displays with three sub-menus:
In “Settings” menu: select “Custom” and enter =COUNTA($B$8:$I$8) <= 1 as the Formula
In “Input Message” menu: activate “Show input message when cell is selected” box (this is usually the default) and enter “Type in “X” to indicate the point in time at which the time value of money is calculated” into “Input message” text box
In “Error Alert” menu: activate “Show error alert after invalid data is entered” box (this is usually the default), set “Style” as “Stop” (this is usually the default), and enter “Only enter “X” at a single point in time.” into the “Error message” text box

The single cash flow graphic displays the timeline and the associated TVM calculation. The instructor should show the student how to determine the present and future values based on when a cash flow appears in time relative to the point in time the cash flow is to be evaluated (i.e. the position of “X” in the graphic). Further, the instructor should demonstrate the effect of changing the discount rate on the present and future values. The student should then become comfortable with reproducing these computations on their own.

For example, increasing the discount rate to 12% in Exhibit 1 makes the $121.00 cash flow received in Year 2 worth $96.46 today (see Exhibit 2).

Exhibit 2: The Effect of Increasing the Discount Rate

	A	B	C	D	E	F	G	H	I
1	Periodic interest rate:	12.00%							
2									
3									
4	Enter a single cash flow above associated moment in time:					$121.00			
5									
6									
7		0		1		2		3	
8	Type in “X” to indicate the point in time at which the time value of money is calculated:	X							
9									
10	Answer:	$96.46							
11									
12	Calculation:	$121 ÷ (1 + 12%)^2 = $96.46							
13									
14	FV = cash flow × (1 + interest rate)^N								
15	PV = cash flow ÷ (1 + interest rate)^N								

Have the student note that the present value equation has a larger denominator as the discount rate increases and results in a lower present value for the cash flow.

An additional feature of the calculator is that it does not require that the present or future value be calculated relative to time 0. That is, one can find the future value of some future cash flow at a point in time beyond when the future cash flow is received. For example, if “X” is moved to Year 3, the $121.00 in Year 2 earns one year of interest and will be worth $135.52 in Year 3 (see Exhibit 3).

Exhibit 3: Illustrating Future Value

	A	B	C	D	E	F	G	H	I
1	Periodic interest rate:	12.00%							
2									
3									
4	Enter a single cash flow above associated moment in time:					$121.00			
5									
6									
7		**0**		**1**		**2**		**3**	
8	Type in “X” to indicate the point in time at which the time value of money is calculated:							X	
9									
10	Answer:	$135.52							
11									
12	Calculation:	$121 × (1 + 12%)^1 = $135.52							
13									
14	FV = cash flow × (1 + interest rate)^N								
15	PV = cash flow ÷ (1 + interest rate)^N								

What is important is that the student begins to understand that TVM calculations are about finding the value of cash flows *relative to a desired point in time*. Students tend to think present value is always a calculation for determining the value of future cash flows at Year 0 and that future value is only applied at the point in time when the final cash flow is received. While a number of TVM calculations including security pricing and saving for a retirement goal are computed in this manner, there are many TVM calculations that do not follow this format. A student’s failure to understand this intuition early in the process of learning time value of money concepts may hinder understanding of more advanced computations. The easiest way to build this intuition is by considering a single cash flow and then changing the relative moments in time between the cash flow and when the cash flow is to be valued.

SECTION 2: Multiple Cash Flows Graphic in Excel

When considering multiple cash flows on a timeline, a number of new educational possibilities emerge (see Exhibit 4 for the programming).

Exhibit 4: Multiple Cash Flows Graphic in Excel

	A	B	C	D	E	F	G	H	I
1	Periodic interest rate:	10.00%							
2									
3									
4	Enter a single cash flow above associated moment in time:			$100.00		$100.00		$100.00	
5									
6									
7		**0**		**1**		**2**		**3**	
8	Type in "X" to indicate the point in time at which the time value of money is calculated:	X							
9									
10	Answer:	$248.69							
11									
12	Calculation (Equation):	Year 0:	= $ 0.00 ÷ (1 + 10%)^0						
13		Year 1:	+ $ 100.00 ÷ (1 + 10%)^1						
14		Year 2:	+ $ 100.00 ÷ (1 + 10%)^2						
15		Year 3:	+ $ 100.00 ÷ (1 + 10%)^3						
16									
17	Calculation (Values):	=	$ 0.00						
18		+	$ 90.91						
19		+	$ 82.64						
20		+	$ 75.13						
21									
22	FV = cash flow × (1 + interest rate)^N								
23	PV = cash flow ÷ (1 + interest rate)^N								
24									
25		1		0		0		0	
26		0		1		2		3	
27		÷		÷		÷		÷	
28		$ 0.00		$ 90.91		$ 82.64		$ 75.13	
29		0	÷	×					
30									
31									
32									

Merge cells: B4 and C4, D4 and E4, F4 and G4, H4 and I4
Provide associated cell borders in rows 5 and 6 to mimic a time line.
Merge cells: B7 and C7, D7 and E7, F7 and G7, H7 and I7
Merge cells: B8 and C8, D8 and E8, F8 and G8, H8 and I8
Merge cells: B10 and C10
Cell B10: = SUM(B28:I28)
Merge cells: C12 to F12
Cell C12: = CONCAT("= ",TEXT(B4,"$ #,###.00")," ", B27," ","(1 + ",B1*100,"%) ^ ",ABS(B26))
Merge cells: C13 to F13
Cell C13: = CONCAT("+ ",TEXT(D4,"$ #,###.00")," ", D27," ","(1 + ",B1*100,"%) ^ ",ABS(D26))

Merge cells: C14 to F14
Cell C14: = CONCAT("+ ",TEXT(F4,"$ #,###.00")," ", F27," ","(1 + ",B1*100,"%) ^ ",ABS(F26))
Merge cells: C15 to F15
Cell C15: = CONCAT("+ ",TEXT(H4,"$ #,###.00")," ", H27," ","(1 + ",B1*100,"%) ^ ",ABS(H26))
Merge cells: C17 to D17
Cell C17: = B28
Merge cells: C18 to D18
Cell C18: = D28
Merge cells: C19 to D19
Cell C19: = F28
Merge cells: C20 to D20
Cell C15: = H28

These formulas can be "hidden" or "cut and pasted" to another worksheet to hide from view:
Merge cells: B25 and C25, D25 and E25, F25 and G25, H25 and I25
Merge cells: B26 and C26, D26 and E26, F26 and G26, H26 and I26
Merge cells: B27 and C27, D27 and E27, F27 and G27, H27 and I27
Merge cells: B28 and C28, D28 and E28, F28 and G28, H28 and I28
Cell B25: =IF(B8 = "X", 1, 0)
Cell D25: =IF(D8 = "X", 1, 0)
Cell F25: =IF(F8 = "X", 1, 0)
Cell H25: =IF(H8 = "X", 1, 0)
Cell B26: = B7 – B29
Cell D26: = D7 – B29
Cell F26: = F7 – B29
Cell H26: = H7 – B29
Cell B27: = IF(B26<0,D29,C29)
Cell D27: = IF(D26<0,D29,C29)
Cell F27: = IF(F26<0,D29,C29)
Cell H27: = IF(H26<0,D29,C29)
Cell B28: = B4/(1 + B1)^B26
Cell D28: = D4/(1 + B1)^D26
Cell F28: = F4/(1 + B1)^F26
Cell H28: = H4/(1 + B1)^H26
Cell B29: =IF(SUM(B25:I25)=1,SUMPRODUCT(B7:I7,B25:I25),"ERROR")

A ten-period version of this file can be downloaded from: https://scholarship.richmond.edu/finance-faculty-publications/X/

Additional feature if desired:

Highlight cells B8 to I8, go to "Data" menu on Excel dashboard, go to "Data Validation" pulldown menu and an input box displays with three sub-menus:
In "Settings" menu: select "Custom" and enter =COUNTA($B$8:$I$8) <= 1 as the Formula
In "Input Message" menu: activate "Show input message when cell is selected" box (this is usually the default) and enter "Type in "X" to indicate the point in time at which the time value of money is calculated" into "Input message" text box
In "Error Alert" menu: activate "Show error alert after invalid data is entered" box (this is usually the default), set "Style" as "Stop" (this is usually the default), and enter "Only enter "X" at a single point in time." into the "Error message" text box

In the current form, the student can see what cash flows are being evaluated at Year 0 (i.e. "X" is set at Year 0) and how the overall value is calculated by summing individual discounted cash flows. In this example, having all the cash flows constant allows a discussion about a present value annuity.

Another set of cash flows allows for the discussion of the net present value (NPV) of a project. Assume a project costs $400 and generates three annual cash flows of $100, $200, and $300 over the next three years with an associated discount rate of 15%

annually. The timeline in Exhibit 5 illustrates the cash flows and the calculation of the NPV as $35.44.

Exhibit 4: Net Present Value (NPV) Illustrated

	A	B	C	D	E	F	G	H	I
1	Periodic interest rate:	15.00%							
2									
3									
4	Enter a single cash flow above associated moment in time:	($400.00)		$100.00		$200.00		$300.00	
5									
6									
7		0		1		2		3	
8	Type in "X" to indicate the point in time at which the time value of money is calculated:	X							
9									
10	Answer:	$35.44							
11									
12	Calculation (Equation):	Year 0:	= -$ 400.00 ÷ (1 + 15%)^0						
13		Year 1:	+ $ 100.00 ÷ (1 + 15%)^1						
14		Year 2:	+ $ 200.00 ÷ (1 + 15%)^2						
15		Year 3:	+ $ 300.00 ÷ (1 + 15%)^3						
16									
17	Calculation (Values):	=	($400.00)						
18		+	$ 86.96						
19		+	$ 151.23						
20		+	$ 197.25						
21									
22	FV = cash flow × (1 + interest rate)^N								
23	PV = cash flow ÷ (1 + interest rate)^N								

Next, one can introduce the "Goal Seek" feature to change the discount rate to a value that sets the NPV to zero: go to the "Data" menu on the Excel dashboard and then to the "What-If Analysis" pulldown menu to set cell B10 = 0 by changing cell B1.

Exhibit 5: Net Present Value (NPV) Illustrated

	A	B	C	D	E	F	G	H	I
1	Periodic interest rate:	19.44%							
2									
3									
4	Enter a single cash flow above associated moment in time:	($400.00)		$100.00		$200.00		$300.00	
5									
6									
7		0		1		2		3	
8	Type in "X" to indicate the point in time at which the time value of money is calculated:	X							
9									
10	Answer:	$0.00							
11									
12	Calculation (Equation):	Year 0:	= -$ 400.00 ÷ (1 + 19.44%)^0						
13		Year 1:	+ $ 100.00 ÷ (1 + 19.44%)^1						
14		Year 2:	+ $ 200.00 ÷ (1 + 19.44%)^2						

15		Year 3:	+ $ 300.00 ÷ (1 + 19.44%)^3						
16									
17	Calculation (Values):	=	($400.00)						
18		+	$ 83.73						
19		+	$ 140.20						
20		+	$ 176.07						
21									
22	FV = cash flow × (1 + interest rate)^N								
23	PV = cash flow ÷ (1 + interest rate)^N								

After iterating, the NPV becomes zero and the discount rate of 19.44% is revealed to be the internal rate of return (IRR). This illustrates that the IRR is a calculation found through iteration rather a solution found directly by applying a formula. Demonstrating this visually helps the student to understand how the IRR is determined.

By changing the cash flows in the following manner: Year 0 = - $118,525.54, Year 1 = $424,321.44, Year 2 = - $505,155.86, and Year 3 = $200,000.00, a different type of project is illustrated that requires additional investment in Year 2. For example, the project could be the purchase of a mine that needs safety maintenance in Year 2 and is then expected to be sold in Year 3. Assuming an annual discount rate of 8.00%, the project has an NPV = $41.40 making it potentially an acceptable project (see Exhibit 6).

Exhibit 6: Net Present Value (NPV) with Cash Flows that Change Sign

	A	B	C	D	E	F	G	H	I
1	Periodic interest rate:	8.00%							
2									
3									
4	Enter a single cash flow above associated moment in time:	($118,525.54)		$424,321.44		($505,155.86)		$200.000.00	
5									
6									
7		**0**		**1**		**2**		**3**	
8	Type in "X" to indicate the point in time at which the time value of money is calculated:	X							
9									
10	Answer:	$41.40							
11									
12	Calculation (Equation):	Year 0:	= -$ 118,525.54 ÷ (1 + 8%)^0						
13		Year 1:	+ $ 424,321.44 ÷ (1 + 8%)^1						
14		Year 2:	+ -$ 505,155.86 ÷ (1 + 8%)^2						
15		Year 3:	+ $ 200,000.00 ÷ (1 + 8%)^3						
16									
17	Calculation (Values):	=	($118,525.54)						
18		+	$ 392,890.22						
19		+	($ 433.089.73)						
20		+	$ 158,766.45						

21									
22	FV = cash flow × (1 + interest rate)^N								
23	PV = cash flow ÷ (1 + interest rate)^N								

Increasing the annual discount rate to 15% generates an NPV = -$17.53. This is not necessarily surprising given the intuition that a higher discount rate leads to lower valuations relative to Year 0. However, increasing the annual discount rate to 25% generates a positive NPV of $31.86, which is very counter intuitive. In this case, the higher discount rate is affecting the value of the investment being made in Year 2 more so than the inflows in Years 1 and 3.

At this point, the instructor can introduce the student to the concept of multiple project IRRs (in this case: 10%, 18%, and 30%; an algorithm for generating cash flows based on multiple IRRs is available in the appendix), and that such a situation occurs whenever the cash flows change sign through time, from positive to negative or vice versa. Further, the instructor can graph the NPV based on different discount rates that incorporate the multiple IRRs to demonstrate how the NPV has positive and negative regions based on the discount rate.

SECTION 3: Conclusion

The use of a timeline to illustrate TVM concepts is not a new practice. Textbooks and instructors have done so for decades. Yet, students often abandon this powerful tool early when developing TVM skills, perhaps by shifting to quickly to using calculators or paying too much attention to written formulas without understanding the intuition behind them. Using Excel to create a timeline within a cash flow graphic enhances the benefit of using timelines by displaying cash flows through time as well as the associated TVM calculations. The graphic is readily adaptable to different scenarios to make the file useful for multiple in-person or virtual lectures. It is also suitable as a standalone resource for students to use outside the classroom. Although examples are provided for NPV and IRR, a number of other possibilities exist that can be readily illustrated using the cash flow graphic.

REFERENCES:

Bodie, Zvi, Alex Kane, and Alan Marcus. 2018. Investments, 11th Edition, McGraw-Hill, New York, N.Y.

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Ross, Stephen, Randolph Westerfield, and Bradford Jordan. 2016. Fundamentals of Corporate Finance , 11th Edition, McGraw-Hill, New York, N.Y.

Welch, Ivo. 2017. Corporate Finance, 4th Edition.

APPENDIX: Multiple IRR Cash Flow Algorithm

The algorithm is based on the creation of a polynomial of degree "N" where "N" equals the number of years in the future the cash flows are to span (Note: the constant generated in the polynomial is the value of the cash flow for Period 0). The number of IRRs to be generated is less than or equal to "N." If one desires to have "N" cash flows, but with the number of IRRs being less than "N", use duplicate IRRs within the polynomial until the number of different IRRs is the desired amount.

Let "Z" be a constant, "X" is a variable, and IRR_i is the "i-th" defined IRR. The associated polynomial is:

$$Z \times \prod_{i=1}^{N} \left(X - {}^{1}\!/_{(1 + IRR_i)} \right) \quad \text{(A.1)}$$

The associated cash flows are the coefficients for X^i with the constant (i.e. the value assigned to X^0) being the cash flow for Period 0.

Using the example from Section 2, the three IRRs are 10%, 18%, and 30% with Z = \$200,000. To simplify the calculation, let A = -1 / (1 + 10%), B = -1 / (1 + 18%), and C = -1 / (1 + 30%). Equation (A.1) becomes:

$$= \$200{,}000X^3 + \$200{,}000(A + B + C)X^2 + \$200{,}000(C[A + B] + AB)X^1 + \$200{,}000(ABC) \quad \text{(A.2)}$$

The corresponding cash flows based on the polynomial coefficients become:

Period 0: $\$200{,}000(ABC)$

Period 1: $\$200{,}000(C[A + B] + AB)$

Period 2: $\$200{,}000(A + B + C)$

Period 3: \$200,000

Corollary:

Assuming more than one IRR, set one of the IRRs to zero, the payback period will be "N" and the discounted payback period with an IRR (not equal to zero) as a discount rate will also equal "N." This is unusual because the payback period is expected to be less than the discounted payback period.

Using the example above, set the IRR of 10% to 0 (equivalent to A = -1).

Period 0: -$130,378.10

Period 1: $453,725.78

Period 2: -$523,337.68

Period 3: $200,000.00

Notice, the number of periods to recover all costs is 3 (i.e. the payback period).

If you discount the cash flows at 18% or 30%, the number of periods it takes the discounted cash flows to recover all costs is also 3 (i.e. the discounted payback period).