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Visual Presentation of MIRR and MNPV Calculations

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Project cash flows and modified cash flows are presented in an illustrative graphic within Excel for the live or virtual classroom. Further, the graphic computes and displays the relevant modified internal rate of return (MIRR) and modified net present value (MNPV), with associated formulas. The presentation allows for a discussion of the reinvestment assumption attributed to the internal rate of return (IRR) and the net present value (NPV) calculations.

INTRODUCTION

By adjusting the Excel timeline technique of Alexander, Arnold, and Farizo (2020), a visual presentation of modified time value of money techniques is generated, specifically the modified internal rate of return (MIRR) and modified net present value (MNPV, McClure and Girma, 2004). The calculation of the MIRR and MNPV are related to what is called the "reinvestment assumption" within the internal rate of return calculation (IRR) and technically, the calculation of the net present value (NPV) (see Arnold and Nixon, 2006 and 2013 and Rich and Rose, 2014 concerning IRR). However, it is possible to consider MIRR and MNPV as valid metrics without viewing the measures as a "correction" for the reinvestment assumption. Just the same, as will be demonstrated, it is very easy to illustrate the reinvestment assumption with the MIRR and MNPV calculations.

Arnold and Nixon (2013) explain the reinvestment assumption using a bond as an example. Suppose a par value 10-year $1,000 bond with 6% annual coupons is held to maturity. Because the bond sells for par, the yield to maturity is the same as the coupon rate of 6%. Calculating the holding period return (HPR) for the bond demonstrates that if the bond cash flows are not reinvested, the HPR is less than the yield to maturity.

$$HPR = \left[\frac{\$1,000\ +(10\ \times\ 6\%\ \times\ \$1000)}{\$1,000}\right]^{1/10} - 1 = 4.812\% \tag{1}$$

Because the bond yield to maturity is the equivalent of the IRR of the bond, the IRR calculation is considered to have a reinvestment assumption. In other words, the IRR calculation assumes that cash flows are reinvested through time at the IRR. In the bond example, let the cash flows be reinvested at 6% annually.

$$HPR = \left[\frac{\$1,000.00\ +\$790.85}{\$1,000.00}\right]^{1/10} - 1 = 6.000\% \tag{2}$$

The HPR now equals the IRR because cash flows are reinvested at the IRR. Consequently, it can be interpreted that the IRR calculation implicitly assumes reinvestment of intermediate cash flows at the IRR (i.e. the reinvestment assumption).

The reinvestment assumption exists for the NPV calculation as it does with the IRR calculation. Using the bond example again, assume the appropriate risk adjusted rate for discounting the bond cash flows is 6% annually (i.e. the sum of the discounted cash flows is $1,000). However, let the bond price be $800 which implies an NPV of $200 (= $1,000 - $800). Allow the $800 investment to earn 6% annually (i.e. be invested at the appropriate discount rate). The appreciated value of the investment at the maturity of the bond is $1,432.68 (= $800 × [1 + 6%]^{10}). Compare this number to the accumulation of the cash flows received from the bond, $1,600 (= $1,000 + 10 × 6% × $1,000). The value created by the bond over ten years is $167.32 (= $1,600 - $1,432.68), which is below the appreciated value of the NPV of $358.17 (= $200 × [1 + 6%]^{10}).

Similar to the IRR scenario, if the bond cash flows are invested at 6% annually (i.e. the appropriate discount rate) and grow to $1,790.85, the value created at the maturity of the bond is $358.17 (= $1,790.85 - $1,432.68). This value is equivalent to the appreciated value of the NPV and implies a reinvestment of cash flows at the appropriate discount rate.

Rich and Rose (2014) provide a discussion as to the legitimacy of the reinvestment assumption concerning the IRR calculation. The discussion could also be applied to the NPV calculation. While debating the validity of the reinvestment assumption within the IRR and NPV calculations is not the goal of this presentation, the graphical technique introduced below illustrates these concepts and can offer additional context to these discussions.

SECTION 1: MIRR-MNPV Graphic in Excel

In Exhibit 1, a three-year version of the MIRR-MNPV graphic is presented with the associated Excel programming. A ten-year version of the graphic is available for download at:

https://scholarship.richmond.edu/finance-faculty-publications/X/

Exhibit 1: MIRR-MNPV Graphic in Excel

	A	B	C	D	E	F	G	H	I
1	Re-investment Rate (usually set at the Weighted Average Cost of Capital, WACC):	10.00%							
2	Project Discount Rate:	15.00%							
3									
4	Enter cash flows above associated moment in time:	-$10,000.00		-$7,000.00		$6,000.00		$26,000.00	
5									
6									
7				0		1		2		3
8									
9	STEP 1: Present Value all cash outflows using re-investment rate	-$10,000.00		-$7,000.00					
10									
11									
12				0		1		2		3
13									
14			PV(Outflows):		-$16,363.64				
15									
16			= -$10,000.00		+ -$6,363.64				
17									
18			= -$10,000.00 / (1 + 10%) ^ 0		+ -$7,000.00 / (1 + 10%) ^ 1				
19									
20	STEP 2: Future Value all cash inflows using re-investment rate					$6,000.00		$26,000.00	
21									
22									
23				0		1		2		3
24									
25			FV(Inflows):		$32,600.00				
26									
27			= $0.00		+ $0.00		+ $6,600.00		+ $26,000.00
28									
29			= $0.00 * (1 + 10%) ^ 3		+ $0.00 * (1 + 10%) ^ 2		+ $6,000.00 * (1 + 10%) ^1		+ $26,000.00 * (1 + 10%) ^ 0
30									
31	STEP 3: Display modified cash flows	-$16,363.64						$32,600.00	
32									
33									
34				0		1		2		3
35									
36	STEP 4: Compute Modified Internal Rate of Return (MIRR) as the holding period return of the STEP 3 modified cash flows:	MIRR:		25.83%		= [$ 32,600.00 / $ 16,363.64] ^ (1 / 3) -1			
37									

#						
38	Compute Modified Net Present Value (MNPV) based on the project discount rate and the STEP 3 modified cash flows:	MNPV:	$5,071.39	= $ 32,600.00 / (1 + 15%) ^ 3 – $ 16,363.64 / (1 + 15%) ^ 0		
39						
40	Internal rate of Return (IRR) based on initial cash flows:	IRR:	30.00%			
41						
42	Net Present Value:	NPV:	$5,545.33			
43						
44						
45						
46		0	1		0	0
47		0	1		2	3
48		1				
49		3				
50						

Merge cells: B4 and C4, D4 and E4, F4 and G4, H4 and I4
Provide associated cell borders in rows 5 and 6 to mimic a time line.
Merge cells: B7 and C7, D7 and E7, F7 and G7, H7 and I7

Merge cells: B9 and C9, D9 and E9, F9 and G9, H9 and I9
Provide associated cell borders in rows 10 and 11 to mimic a time line.
Merge cells: B12 and C12, D12 and E12, F12 and G12, H12 and I12
Cell B9: =IF(B4<0,B4,0)
Cell D9: =IF(D4<0,D4,0)
Cell F9: =IF(F4<0,F4,0)
Cell H9: =IF(H4<0,H4,0)

Merge cells: B14 and C14 and D14 and E14
Cell D14: =NPV(B1,B9:I9)*(1+B1)

Merge cells: B16 and C16, D16 and E16, F16 and G16, H16 and I16
Cell B16: =IF(B12>B48,” “,CONCAT(“= “,TEXT(B9/(1 + B1)^B12,”$ #,##0.00”)))
Cell D16: =IF(D12>B48,” “,CONCAT(“= “,TEXT(D9/(1 + B1)^D12,”$ #,##0.00”)))
Cell F16: =IF(F12>B48,” “,CONCAT(“= “,TEXT(F9/(1 + B1)^F12,”$ #,##0.00”)))
Cell H16: =IF(H12>B48,” “,CONCAT(“= “,TEXT(H9/(1 + B1)^H12,”$ #,##0.00”)))

Merge cells: B18 and C18, D18 and E18, F18 and G18, H18 and I18
Cell B18: =IF(B12>B48,” “,CONCAT(“= “,TEXT(B9,”$ #,##0.00”),” / (1 + “,B1*100,”%) ^ “,B12))
Cell D18: =IF(D12>B48,” “,CONCAT(“+ “,TEXT(D9,”$ #,##0.00”),” / (1 + “,B1*100,”%) ^ “,D12))
Cell F18: =IF(F12>B48,” “,CONCAT(“+ “,TEXT(F9,”$ #,##0.00”),” / (1 + “,B1*100,”%) ^ “,F12))
Cell H18: =IF(H12>B48,” “,CONCAT(“+ “,TEXT(H9,”$ #,##0.00”),” / (1 + “,B1*100,”%) ^ “,H12))

Merge cells: B20 and C20, D20 and E20, F20 and G20, H20 and I20
Provide associated cell borders in rows 21 and 22 to mimic a time line.
Merge cells: B23 and C23, D23 and E23, F23 and G23, H23 and I23
Cell B20: =IF(B4>0,B4,0)
Cell D20: =IF(D4>0,D4,0)
Cell F20: =IF(F4>0,F4,0)
Cell H20: =IF(H4>0,H4,0)
Merge cells: B25 and C25, D25 and E25
Cell D25: =NPV(B1,B20:I20)*(1+B1)^(B49+1)

Merge cells: B27 and C27, D27 and E27, F27 and G27, H27 and I27
Cell B27: =IF(B23>B49,” “,CONCAT(“= “,TEXT(B20*(1 + B1)^(B49 – B23),”$ #,##0.00”)))
Cell D27: =IF(D23>B49,” “,CONCAT(“= “,TEXT(D20*(1 + B1)^(B49 – D23),”$ #,##0.00”)))
Cell F27: =IF(F23>B49,” “,CONCAT(“= “,TEXT(F20*(1 + B1)^(B49 – F23),”$ #,##0.00”)))
Cell H27: =IF(H23>B49,” “,CONCAT(“= “,TEXT(H20*(1 + B1)^(B49 – H23),”$ #,##0.00”)))

Merge cells: B29 and C29, D29 and E29, F29 and G29, H29 and I29
Cell B29: =IF(B23>B49,” “,CONCAT(“= “,TEXT(B20,”$ #,##0.00”),” * (1 + “,B1*100,”%) ^ “,B49-B23))
Cell D29: =IF(D23>B49,” “,CONCAT(“+ “,TEXT(D20,”$ #,##0.00”),” * (1 + “,B1*100,”%) ^ “,B49-D23))
Cell F29: =IF(F23>B49,” “,CONCAT(“+ “,TEXT(F20,”$ #,##0.00”),” * (1 + “,B1*100,”%) ^ “,B49-F23))
Cell H29: =IF(H23>B49,” “,CONCAT(“+ “,TEXT(H20,”$ #,##0.00”),” * (1 + “,B1*100,”%) ^ “,B49-H23))

Merge cells: B31 and C31, D31 and E31, F31 and G31, H31 and I31
Provide associated cell borders in rows 32 and 33 to mimic a time line.
Merge cells: B34 and C34, D34 and E34, F34 and G34, H34 and I34
Cell B31: =D14

The project illustrated has two investments, $10,000 initially and $7,000 a year later. The project then produces positive cash flows of $6,000 and $26,000 in Years 2 and 3 respectively. To generate modified cash flows, a re-investment rate (usually set at the weighted average cost of capital and set to 10% in Exhibit 1) is applied to the inflows and outflows of the project. The outflows are discounted and summed to produce a single outflow of $16,363.64 at Year 0 (STEP 1 in Exhibit 1). The inflows are appreciated at the re-investment rate to the end of the project (Year 3 in Exhibit 1) to produce a terminal inflow of $32,600 (STEP 2 in Exhibit 1).

The modified cash flows are displayed on the timeline in STEP 3 of Exhibit 1. These cash flows may then be used for the MIRR and MNPV calculations, presented in STEP 4 of Exhibit 1. The MIRR is the HPR of the modified cash flows.

$$HPR = \left[\frac{\$32{,}600}{\$16{,}363.64}\right]^{1/3} - 1 = 25.83\% = MIRR \qquad (3)$$

The MNPV is the NPV of the modified cash flows using the project discount rate.

$$MNPV = \frac{\$32{,}600}{(1+15\%)^3} - \frac{\$16{,}363.64}{(1+15\%)^0} = \$5{,}071.39 \qquad (4)$$

The IRR and NPV are calculated within the graphic and are shown to be 30% and $5,545.33 respectively.

In the classroom (live or virtual), the graphic allows the student to see the steps necessary for computing the modified cash flows and the results of the MIRR, MNPV, IRR, and NPV calculations (note: the first two calculations are also illustrated within the graphic). The instructor can then lead a discussion of the differences between, and the potential benefits of, each technique. The graphic can also be used as a resource for the student to use on their own or as an illustration of the MIRR and MNPV techniques to be programmed on another platform in a fintech class.

A demonstration of the reinvestment assumption is easily displayed by setting the re-investment rate to the IRR and showing how the IRR and MIRR become equal (Exhibit 2).

Exhibit 2: MIRR = IRR when the IRR is the Re-investment Rate

	A	B	C	D	E	F	G	H	I
1	Re-investment Rate (usually set at the Weighted Average Cost of Capital, WACC):	30.00%							
2	Project Discount Rate:	15.00%							
3									
4	Enter cash flows above associated moment in time:	-$10,000.00		-$7,000.00		$6,000.00		$26,000.00	
5									
6									
7			0		1		2		3

8						
9	STEP 1: Present Value all cash outflows using re-investment rate	-$10,000.00	-$7,000.00			
10						
11						
12		0	1	2	3	
13						
14		PV(Outflows):	-$15,384.62			
15						
16		= -$10,000.00	+ -$5,384.62			
17						
18		= -$10,000.00 / (1 + 30%) ^ 0	+ -$7,000.00 / (1 + 30%) ^ 1			
19						
20	STEP 2: Future Value all cash inflows using re-investment rate			$6,000.00	$26,000.00	
21						
22						
23		0	1	2	3	
24						
25		FV(Inflows):	$33,800.00			
26						
27		= $0.00	+ $0.00	+ $7,800.00	+ $26,000.00	
28						
29		= $0.00 * (1 + 30%) ^ 3	+ $0.00 * (1 + 30%) ^ 2	+ $6,000.00 * (1 + 30%) ^1	+ $26,000.00 * (1 + 30%) ^ 0	
30						
31	STEP 3: Display modified cash flows	-$15,384.62			$33,800.00	
32						
33						
34		0	1	2	3	
35						
36	STEP 4: Compute Modified Internal Rate of Return (MIRR) as the holding period return of the STEP 3 modified cash flows:	MIRR:	30.00%	= [$ 33,800.00 / $ 15,384.62] ^ (1 / 3) -1		
37						
38	Compute Modified Net Present Value (MNPV) based on the project discount rate and the STEP 3 modified cash flows:	MNPV:	$6,839.43	= $ 33,800.00 / (1 + 15%) ^ 3 – $ 15,384.62 / (1 + 15%) ^ 0		
39						
40	Internal rate of Return (IRR) based on initial cash flows:	IRR:	30.00%			
41						
42	Net Present Value:	NPV:	$5,545.33			

One can also demonstrate the reinvestment assumption within the NPV calculation by setting the re-investment rate to be the same as the discount rate, thus making the MNPV = NPV (Exhibit 3).

Exhibit 3: MNPV = NPV when the Project Discount Rate is the Re-investment Rate

	A	B	C	D	E	F	G	H	I
1	Re-investment Rate (usually set at the Weighted Average Cost of Capital, WACC):	15.00%							
2	Project Discount Rate:	15.00%							
3									
4	Enter cash flows above associated moment in time:	-$10,000.00		-$7,000.00		$6,000.00		$26,000.00	
5									

		0	1	2	3		
6							
7		**0**	**1**	**2**	**3**		
8							
9	STEP 1: Present Value all cash outflows using re-investment rate	-$10,000.00	-$7,000.00				
10							
11							
12		**0**	**1**	**2**	**3**		
13							
14		PV(Outflows):	-$16,086.96				
15							
16		= -$10,000.00	+ -$6,086.96				
17							
18		= -$10,000.00 / (1 + 15%) ^ 0	+ -$7,000.00 / (1 + 15%) ^ 1				
19							
20	STEP 2: Future Value all cash inflows using re-investment rate			$6,000.00	$26,000.00		
21							
22							
23		**0**	**1**	**2**	**3**		
24							
25		FV(Inflows):	$32,900.00				
26							
27		= $0.00	+ $0.00	+ $6,900.00	+ $26,000.00		
28							
29		= $0.00 * (1 + 15%) ^ 3	+ $0.00 * (1 + 15%) ^ 2	+ $6,000.00 * (1 + 15%) ^1	+ $26,000.00 * (1 + 15%) ^ 0		
30							
31	STEP 3: Display modified cash flows	-$16,086.96			$32,900.00		
32							
33							
34		**0**	**1**	**2**	**3**		
35							
36	STEP 4: Compute Modified Internal Rate of Return (MIRR) as the holding period return of the STEP 3 modified cash flows:	MIRR:	26.93%	= [$ 32,900.00 / $ 16,086.96] ^ (1 / 3) -1			
37							
38	Compute Modified Net Present Value (MNPV) based on the project discount rate and the STEP 3 modified cash flows:	MNPV:	$5,545.33	= $ 32,900.00 / (1 + 15%) ^ 3 – $ 16,086.96 / (1 + 15%) ^ 0			
39							
40	Internal rate of Return (IRR) based on initial cash flows:	IRR:	30.00%				
41							
42	Net Present Value:	NPV:	$5,545.33				

As stated previously, the purpose of this presentation is to provide a graphic to illustrate the modified cash flows and the calculation of the MIRR and the MNPV for a classroom setting. However, because the reinvestment assumption can be readily illustrated using the graphic, a discussion about the validity of the reinvestment assumption is certainly possible if desired. Ultimately, the MIRR and MNPV calculations provide additional techniques for project analysis that merit inclusion in the finance classroom aside from illustrating the reinvestment assumption.

SECTION 2: Conclusion

MIRR and MNPV provide metrics of project assessment that include the investment of project cash flows as they are generated over time. This gives the decisionmaker additional insights regarding the given project that are not available from other metrics.

The graphic allows the student to see the process of modifying cash flows based on a re-investment rate, along with the calculation of the MIRR and MNPV based on these modified cash flows. The graphic is suitable for classroom presentation (live or virtual) or as a resource for the student outside of the classroom.

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