The School Mathematics Study Group: Lessons in Mathematics Education

Madeline E. Polhill

University of Richmond Summer Research Fellowship

Dr. Della Dumbaugh

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I pledge that I have neither received nor given unauthorized assistance during the completion of this work.

Table of Contents

Introduction: Why is learning about the SMSG important?

> The world of today demands more mathematical knowledge on the part of more people than the world of yesterday, and the world of tomorrow will demand even more. It is therefore important that mathematics be taught in a vital and imaginative way which will make students aware that it is a living, growing subject which plays an increasingly important part in the contemporary world (Begle, "The School Mathematics Study Group" 616).

In his 1958 article in *The Mathematics Teacher*, School Mathematics Study Group (SMSG) director Edward G. Begle outlined the impetus for this organization's work in the "new math" movement of mid-century America. These words from 62 years ago could just as well have been spoken 62 seconds ago, as Begle's call for students prepared to think mathematically in an ever-advancing technological world proves more timely than ever. Accordingly, a thoughtful examination of the work of the SMSG proves particularly relevant in our modern world.

The School Mathematics Study Group (1958–1972) declared its mission in 1961 to "foster research and development in the teaching of school mathematics" (Malkevitch 8; Wooton 128). The SMSG was one of several "new math" projects "that had an aim of reforming, repairing, or enhancing mathematics education on the K–12 level" (Bossé 173; Phillips, *The New Math: A Political History* 2). Though the "new math" movement has been regarded in public memory as a "failure," this paper offers an alternative, more productive consideration of this movement and, more specifically, the School Mathematics Study Group (Malkevitch 8). As the recipient of over $10 million in purely federal funding, the SMSG holds a prominent place among the "new math" projects as the "official" representation of the movement and thus serves as a fitting focus for this study (Goodman; Hayden 123–25, 137; Phillips, *The New Math: A Political History* 2, 15; Phillips, "In Accordance" 543).

This paper argues that, despite its place in history, the School Mathematics Study Group offers a valuable case study for mathematics educators seeking to venture into the future better informed about both the successes and failures of previous projects. Understanding this project requires recognizing that the School Mathematics Study Group was wholly a product of the forces—personal, educational, mathematical, and political—that shaped it. Admittedly, some of the SMSG's shortcomings resulted from its members' lack of understanding of the changes needed in mathematics education. Still, the majority of the SMSG's public vilification resulted through no fault of its own, but rather from shifts and unrest in the very forces that shaped the organization. With the benefit of nearly half a century of hindsight, this study aims to draw from the work of the School Mathematics Study Group with an eye towards identifying meaningful lessons for mathematics educators today.

Part I: From what context did the SMSG emerge?

The emergence of the School Mathematics Study Group (SMSG) represented the convergence of a variety of influential forces. In particular, the SMSG appeared as individual people, as well as educational, mathematical, and political movements, demonstrated interest in mathematics education reform as the 1950s drew to a close.

Edward Begle: Primed for a creative career in mathematics education

As the director of the School Mathematics Study Group for the duration of its existence, Edward Griffith Begle held significant influence over the organization (Kilpatrick). Consequently, the forces that shaped Begle's adventures in mathematics education prove enlightening in terms of his own role in leading the SMSG. In particular, Begle's connection to Princeton University, where he earned his PhD in 1940 for work in topology under Solomon Lefschetz, offers insight into the origins of Begle's interest in the learning, teaching, and study of mathematics (Kilpatrick). Begle's subsequent professional and personal experiences led him straight to his position with the SMSG.

Although Begle could not have foreseen his interest in mathematics education, few programs could better have encouraged him towards his innovative career than the mathematics department at Princeton University. In particular, Begle's connection to his advisor Solomon Lefschetz offers one indication as to Begle's passionate work in mathematics education. During his tenure at Princeton, Lefschetz played a key role in creating an outstanding mathematics program at Princeton and himself held specific opinions about how professors should teach introductory classes (Lefschetz 346–47; Peterson 154). Lefschetz also maintained a reputation for working so closely with the graduate program and with his many graduate students that former colleague Albert W. Tucker remembered him as a "father figure" to his advisees (Aspray, "The Emergence of Princeton" 359; Aspray, "Albert Tucker: The People at Princeton" 10). Lefschetz, in other words, demonstrated significant interest in the educational experiences of the next generation of mathematics scholars. Begle's eventual role as a forerunner in mathematics education work ultimately, then, seems a natural extension of what he observed as a graduate student at Princeton.

The unique environment created by Fine Hall, the Princeton mathematics department's home beginning in 1931, allowed for significant networking that prepared Begle for his later work (Aspray, "The Emergence of Princeton" 346). Fine Hall featured common areas where students and professors could discuss mathematics (Aspray, "The Emergence of Princeton" 354–55). This model stood in stark contrast to that of many other prominent universities, where mathematics professors held offices scattered across campus (Aspray, "The Emergence of Princeton" 355). When the Institute for Advanced Study also took up residence in Fine Hall from 1933–1939, members of the IAS could readily exchange ideas and assist graduate students with their dissertations on many occasions (Aspray, "The Emergence of Princeton" 355, 358). Fine Hall's mathematics program, therefore, supported collaboration among its members like nearly no other contemporary institution (Aspray, "The Emergence of Princeton" 355).

These connections surely inspired Begle in his later work on the outskirts of traditional mathematics research. In a 1984 interview with Albert W. Tucker, a professor at Princeton in the 1930s and later chair of the mathematics department there, interviewer William Aspray wondered if Princeton's graduate tradition characterized by collaborative work impacted the future pursuits of its PhD students: "Do you think the way that education went on at Princeton, where one wasn't closely tied to an advisor but had a whole smorgasbord of mathematics to experience, somehow translated itself into mathematicians who were more willing to go outside the narrow confines of traditional discipline-boundaries?" (Aspray, "Albert Tucker: Overview of Mathematics" 3; Leitch 318). Tucker replied, "Oh yes. Among the Princeton [PhDs] there were unusually many who went into things that were on the edge of traditional mathematics—or altogether outside of traditional mathematics" (Aspray, "Albert Tucker: Overview of Mathematics" 3). In other words, Tucker suggests that since graduate students at Princeton constantly sought out ideas from a variety of scholars, these students later felt encouraged to explore the broad variety of mathematics-related fields and challenge the standard definition of a career in mathematics.[1] Thus, Begle's studies at Princeton prepared him well to undertake work on the fringes of mathematics in mathematics education.

As a professor at Yale University later in his career, Begle's academic and personal experiences pushed him to leave pure mathematics permanently and placed him in the ideal position to serve as the SMSG's director. While at Yale, Begle became motivated to reform mathematics education at the college level (Kilpatrick). Former colleague Charles E. Rickart recalled spending significant time speaking with Begle about the best ways to teach introductory mathematics classes to college students, noting the clear truth that Begle's "destiny lay in another direction" than pure mathematics research (Zelinka 629). During this time, Begle published his 1954 textbook, *Introductory Calculus, with Analytic Geometry*, which proved innovative in its focus on the student's perspective rather than the professor's (Kilpatrick; Zelinka 629). From a professional standpoint, Begle also felt increasingly drawn to administration, believing he himself "wasn't good enough or pure enough" to maintain a role as a research mathematician (Phillips, "In Accordance" 550; Phillips *The New Math: A Political History* 42). Experiences in his personal life further influenced the direction of his career: He realized the importance of improving mathematics instruction at the pre-collegiate level when he attempted to help his daughter with her mathematics homework but discovered her textbooks to be "so revolting that" he "had to do something" (Goodman; Hayden 120). With the academic, professional, and personal motivations to serve in an administrative role and create change in mathematics education at the secondary level, Edward Begle found himself prepared to accept the call to lead

[1] A few other examples include Henry Wallman, who went on to complete electronic work for the war effort; Paco Lagerstrom, who took up aeronautical engineering; John Tukey, who completed a PhD in topology but proceeded to study statistics and data; and Marvin Minsky, who went on to study electrical engineering and artificial intelligence (Aspray, "Albert Tucker: Overview of Mathematics" 3–4).

the new School Mathematics Study Group. When Yale University indicated an interest in hosting the SMSG, Begle proved the ideal candidate (Wooton 13).

Educational movements: Change needed in mathematics

By the late 1950s, mathematics education was in desperate need of reform. Mathematics classes often placed an emphasis on computations rather than on conceptual understanding (Wooton 3–4). Oftentimes, students learned only mathematical procedures and failed to understand underlying mathematical truths: According to SMSG member William Wooton, "The only requisites for successful achievement in high school mathematics were, in many cases, a good memory and a willingness to follow directions" (4). At a 1959 teacher orientation conference, educator Martha Hildebrandt suggested, "The present textbooks are highly mechanized…The teacher does the first exercise. Very little attention is paid to any explanation, because the pupil expects this first exercise to be done by the teacher and then the rest is mere imitation which can be done with the mind following other pursuits. Often the pupil knows no more when he has completed the last problem than he did after completing a few problems of the list. No learning is going on because lists of exercises are usually all alike" (School Mathematics Study Group, *Report* 69). In other words, many schoolchildren were learning to replicate calculations but not why the underlying mathematical concepts worked. With many students receiving a less-than-stimulating mathematics education, America's mathematics classrooms needed a change.[2]

[2] The progressive movement, and in particular the "life adjustment movement," which had been designed to provide practical training for growing numbers of high school students who would not attend college, had received considerable, although perhaps excessive, criticism for its "anti-intellectual" bent (Mondale and Patton 68–69; Phillips *The New Math: A Political History* 60–61; Phillips, "The New Math and Midcentury American Politics" 459). Though many politicians pointed to the progressive movement as one of the factors necessitating stronger

Prior to the emergence of the SMSG, educators and mathematicians alike had begun calling for change. As early as the 1820s, mathematics educator Warren Colburn had, in fact, recognized that completing mindless drills would not help students understand mathematics or gain skills to apply their knowledge to solve other problems (Hayden 48–49). But beginning in the mid-1950s, the National Council of Teachers of Mathematics and the Commission on Mathematics of the College Entrance Examination Board began to call for serious mathematics education reform (Phillips, *The New Math: A Political History* 98–99). The Commission on Mathematics, in particular, recognized a growing need for improved mathematics education in high schools and advocated for the introduction of a modern mathematical perspective and the inclusion of new topics in mathematics classrooms (Hayden 112). Several future members of the SMSG, like Princeton's Samuel Wilks and Edward Begle himself, held affiliations with the College Entrance Examination Board and its Commission on Mathematics (Begle, "The School Mathematics Study Group" 617; Hayden 110–11, 115, 120; Mosteller 12–13). The SMSG, in fact, used the Commission's report as a "reasonable starting point" for its work (Phillips, *The New Math: A Political History* 49). The SMSG, therefore, had roots in existing mathematics education reform work and was wholly a product of its educational context.

Mathematical community: New understandings

The School Mathematics Study Group also emerged as some members of the global mathematical community began to reformulate their understanding of the very nature of mathematics. The activities of Nicolas Bourbaki, an infamous group composed primarily of

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mathematics programs, the progressive education movement was actually much less of a strong, united front than its critics seemed to suggest (Phillips, "The New Math and Midcentury American Politics" 456, 459). Still, its role in precipitating change in mathematics education deserves recognition.

French mathematicians who worked to reformulate mathematics in a pure, abstract, and unified style, reveal most clearly the nature of these changes and influences (Beaulieu 219–20; Phillips, *The New Math: A Political History* 50–51). In its work, Bourbaki emphasized "precise terminology" and "structure" (Phillips, *The New Math: A Political History* 51). The group's understanding of mathematics found its way across the Atlantic Ocean into the SMSG's mathematics education work. Not only did the Commission on Mathematics (from which the SMSG sought many of its objectives) draw on the ideas of Bourbaki in developing a plan for the reformulation of mathematics education, but many individuals associated with the SMSG itself found inspiration from Bourbaki (Phillips, *The New Math: A Political History* 49–53). These simultaneously Bourbaki- and SMSG-affiliated individuals included Andrew Gleason, Robert Rourke, Marshall Stone, Albert Tucker, and Edward Begle himself (Phillips, *The New Math: A Political History* 52–53; Phillips, "In Accordance" 546, 549). According to historian Christopher Phillips, "Perhaps recognizing the divisiveness and hyperbole of Bourbaki's rhetoric, Begle never explicitly proselytized for Bourbaki's vision, but he was nonetheless a believer in their model of mathematics and echoed them when he defined mathematics as a 'set of interrelated, abstract, symbolic systems'" (*The New Math: A Political History* 52). Especially since Begle and his closest colleagues held so much influence within the SMSG and since "[b]orrowing the language of Bourbaki meant taking sides in the mathematical world," the SMSG's activities and textbooks took a mathematically-partisan position from the organization's outset (Phillips, *The New Math: A Political History* 50, 52; Phillips, "In Accordance" 556). No mathematical aberration, the SMSG was very much a product of its mathematical context.

Political climate: Training to think mathematically

Though the "new math" movement maintains a strong association with the Cold War, political calls for mathematics reform hearken back to World War II and its aftermath (Roberts and Walmsley 468–69). In particular, the war effort's reliance on mathematics underscored the importance of rethinking America's mathematics classrooms (Hayden 80, 85). Moreover, the United States Navy's difficulty recruiting servicemen with a strong mathematics background highlighted the desperate need for mathematics education reform ("The Letter of Admiral Nimitz" 213–14). Heightened calls for change would not, however, come until the second half of the next decade.

Until the Cold War—in particular the launch of Sputnik 1—focused the American public on the critical nature of learning mathematics to preserve the free world, the United States federal government maintained minimal involvement in educational projects (Hayden 116–17; Phillips, *The New Math: A Political History* 24–25). Widespread belief in an imminent Soviet threat quickly shifted political perspectives. By the mid-1950s, teacher training initiatives received increased National Science Foundation (NSF) funding, and the NSF had begun considering "a large-scale mathematics curriculum project" (Phillips, *The New Math: A Political History* 23, 38). The October 4, 1957, launch of Sputnik served as "an instigator of funding" by raising concerns about the nation's ability to compete with the Soviet Union and catapulting the importance of learning mathematics and improving mathematics education into the public eye (Hayden 117; Roberts and Walmsley 468; Wooton 9). Amidst fears about national security, federal education initiatives centered around fighting the "Cold War of the classrooms" with "scientific manpower" received bipartisan support, and NSF funding for what became the SMSG passed easily (Phillips, *The New Math: A Political History* 22, 35, 40–41). Within a year after Sputnik's launch, the NSF's annual funding rose from $50 million to $136 million, with a large portion of the money intended for education projects (Phillips, *The New Math: A Political History* 40; Phillips, "The New Math and Midcentury American Politics" 458). Readily-available federal funding ultimately helped the SMSG attempt to meet the federal government's demands for a reevaluation of mathematics education.

Underlying this federal interest in focusing on mathematics education reform was a belief that students trained to think mathematically would become "intelligent citizens" who could win the ideological warfare of the Cold War (Phillips, "The New Math and Midcentury American Politics" 460). According to Phillips,

> Anti-Communists had conflicting ideas about how to counteract Soviet influence, but there was broad agreement that the promotion of disciplined intelligence was critical not only for checking the demagoguery of Joseph McCarthy's followers but also for guarding against any actual Communist threat. Fears of anti-intellectualism and American stagnation in the competition with the Soviet Union drove many to think deeply about the relationships among education, 'Western' democratic values, social order, and complex domestic and international security challenges ("In Accordance" 541).

In other words, many Americans believed that winning the Cold War would require ensuring students learned to think "correctly" (Phillips, "The New Math and Midcentury American Politics" 470). Specifically, proper intellectual activity meant thinking like a mathematician, or in a "'rational,' 'mathematical,' and 'structural'" way (Phillips, "The New Math and Midcentury American Politics" 455, 466, 470). This type of thinking stood in stark contrast to the supposed "inflexible and rigid" mindset of the Soviets (Phillips, *The New Math: A Political History* 79). Presumably, national security relied on citizens who could think mathematically.

In this climate, the emergence of the School Mathematics Study Group proved inevitable. Coming from a mathematical background that encouraged thinking about mathematics in innovative ways, Edward Begle had developed a significant interest in administration and mathematics education. Groups involved in education had recognized the need for change within mathematics classrooms. Some subsections of the mathematics community had begun reformulating their understanding of mathematics itself. And finally, the federal government and the American public had begun to believe in the importance of training students to think mathematically to ensure the nation's successful emergence out of the Cold War. Once these factors converged, the SMSG took shape almost immediately.

Part II: What was the SMSG?

The SMSG takes shape.

On February 21, 1958, the National Science Foundation held a conference at the University of Chicago that aimed to "survey the problem of supply and demand with respect to research mathematicians" (Hayden 118; Wooton 9–10). Subsequently referred to as the Chicago Conference on Research Potential and Training, the group recognized the need for large-scale reforms in mathematics education within America's public schools in order to promote strength in research mathematics (Hayden 118; Wooton 9–10). The conference called on the American Mathematical Society to work with the Mathematical Association of America and the National Council of Teachers of Mathematics to create a committee dedicated to improving the mathematics curriculum (Wooton 10).

One week later, an NSF-sponsored conference at MIT led by Mina Rees developed goals for the new committee (Hayden 118; Wooton 10–11). Specifically, the so-called Cambridge Conference tasked this committee with holding a "four- or five-week writing session" during the following summer to create "model" syllabi and prepare for the creation of monographs for secondary mathematics students (Wooton 11).

On April 3, American Mathematical Society President Richard Brauer of Harvard University appointed eight mathematicians to this committee (Wooton 12). In late April, the Committee of Eight, which included Samuel Wilks and Edward Begle himself, convened at Hunter College to determine the structure of the organization that would become the School Mathematics Study Group (Wooton 13–14, 147–48). With the appointment of Edward Begle to the position of director; the establishment of the SMSG's headquarters at Yale; the May 7, 1958, grant of $100,000 to the SMSG from the NSF; and the appointment of twenty-six individuals to the SMSG's Advisory Committee, the group could begin to prepare for its first writing session (Wooton 13–15, 44–45).[3]

By this first writing session in June, Begle needed to find mathematicians and educators to participate, make logistical preparations for the event, and organize the group's headquarters at Yale (Wooton 15, 44–45). With the help of the Advisory Committee, Begle took advantage of his extensive list of contacts to convince forty-five enthusiastic mathematicians, educators, and education officials to drop their plans to participate in the SMSG's first writing session (Wooton 15–16). With these arrangements complete, the SMSG's official work could commence.

[3] Yale housed the university's headquarters until Begle accepted a position at Stanford University in 1961, when the SMSG moved with him (Wooton 44–45, 114–15).

The SMSG begins working.

The Yale University writing session during the summer of 1958 represented the first of many intense writing sessions at various locations across the nation at which the SMSG developed its textbooks (Phillips, *The New Math: A Political History* 44; Wooton 18, 58). Though mathematicians ultimately oversaw the functions of the SMSG, Begle worked diligently to ensure that the passionate members of the SMSG's writing teams hailed from a variety of backgrounds: mathematicians working in different subject areas, teachers representing different types of high schools, and professionals with experience at the RAND Corporation, Bell Telephone Laboratories, and the American Association for the Advancement of Science, for example (Phillips, "The New Math and Midcentury American Politics" 459; Wooton vi, 15–16, 59). SMSG leaders hoped mathematicians would ensure the soundness of the textbooks' mathematical content and educators would ensure the pedagogical quality of the material (Wooton 23). From 1958 to 1966, around 400 colleagues from thirty-seven states worked on the SMSG's writing teams (Phillips, *The New Math: A Political History* 99).

Remembered by some as a "sponsored crash program" developed amidst the anxieties of the Cold War, the SMSG produced its materials almost impossibly quickly (as cited in Walmsley 44). The SMSG's "huge writing teams" and "huge budgets" contributed to this rapid development (Hayden 143). SMSG member William Wooton reported that the 1959 writing sessions in Ann Arbor, Michigan, and Boulder, Colorado, required individual teams to write textbooks and teacher commentaries within two months, a small fraction of the usual multiple-year process (75, 149–50). Another SMSG member, Edwin Moise, mentioned that "[o]ne wag has suggested that [the group's] whole style of operation is based on a misinterpretation of a ‘word problem’ in algebra: ‘If one man can do a job in three years, how many men does it take to do the same job in half an hour?’” (88, 90). Though perhaps absurdly exaggerated, this sly commentary emphasizes the rapid nature of the group’s work.

These hasty writing sessions successfully developed a variety of curricular materials during the SMSG’s existence. The SMSG focused initially on developing materials for students in grades 7–12 (Begle, “The School Mathematics Study Group” 617). In 1961, the National Science Foundation extended its funding to the SMSG for the additional creation of elementary school mathematics materials (Phillips, *The New Math: A Political History* 80–81). The SMSG sent its textbooks to various schools that helped test the materials, with the ultimate goal that these textbooks would serve as models for commercial textbook publishers (Phillips, *The New Math: A Political History* 102; Wooton vi, 46). The SMSG also created supplemental materials, including monographs for strong students and guides for teachers (Wooton 50–51, 53). By the time the SMSG folded, four million copies of the organization’s almost thirty publications had reached nearly every state in the nation (Phillips, *The New Math: A Political History* 3, 98; Phillips, “In Accordance” 543).

The SMSG hoped to demonstrate a particular understanding of mathematics in its texts. Specifically, the group wanted students to understand why mathematics topics worked on a conceptual level (Phillips, “The New Math and Midcentury American Politics” 464; Phillips, “In Accordance” 551). As Phillips explains, “[s]tudents had to obtain the correct answer via the correct process” (“The New Math and Midcentury American Politics” 463). New topics like sets and modular arithmetic found their way into textbooks to help students grow in their ability to think in a mathematical fashion, namely “logically and structurally” (Phillips, “The New Math and Midcentury American Politics" 461). The SMSG's writers placed greater emphasis on the process of mathematical thinking rather than on knowing particular ideas out of the hope that students would subsequently develop more sophisticated mental abilities (Phillips, "The New Math and Midcentury American Politics" 463).

Despite the SMSG's intent to revolutionize how American schoolchildren thought about mathematics, and despite the vast number of textbooks the SMSG distributed, the organization experienced an inelegant fall from popularity beginning in the early 1970s. All was not lost, however. Fortunately, the nature of the demise of the SMSG and the "new math" movement more broadly, as well as the actual work of the SMSG during its fourteen years of existence, offer significant, timeless insight into the nature of mathematics curriculum work.

Part III: What can today's mathematics educators learn from the SMSG?

"Anyone who knows anything about the new math knows that it failed" (Phillips, *The New Math: A Political History* 121). But did the "new math" really fail? Or did public perception create and perpetuate the notion of failure? On closer look, many critics of the SMSG and the "new math" based their views on a limited understanding of the actual content of these projects. For example, Morris Kline, the most well-known critic of the "new math" movement, demonstrated in his publications "vagueness, distortion of facts, undocumented statements, overgeneralization, and a general lack of knowledge of what [was] going on in the schools" (as cited in Hayden 165; Kline 22). Focusing on the specific work of a singular project—the School Mathematics Study Group—will offer a more productive opportunity to learn from this period of mathematics education reform. Furthermore, this project aims to consider the SMSG's work not for the purpose of making a judgment about the quality or success thereof, but rather to offer contemporary scholars with an interest in mathematics education informed guidance for the future. In particular, the structure of the SMSG, a mathematician-dominated project hastily set up in response to national security concerns, inadvertently caused the project to falter. This paper identifies several overarching lessons from this unfortunate demise.

Teacher preparation

The first lesson concerns the SMSG's inability to prepare teachers adequately to introduce its textbooks into classrooms, despite the vital role teachers play in the learning process. By 1969, Begle had recognized that mathematics education reform choices depended on "the effectiveness of the teachers who will be called on to implement the changes" ("The Role of Research" 235). But especially because of the pioneering nature of the SMSG's work and because of the SMSG's affiliation with research mathematicians rather than with mathematics educators, the SMSG's reforms caught teachers unprepared. This realization may help explain the ways in which the SMSG fell short of its lofty goals.

Many of the problems with teachers' inability to implement these reforms stemmed from their own educational experiences. In particular, weaker students constituted a large portion of the teaching profession. After all, much more lucrative professions existed for mathematics majors, and low-status elementary education programs often scared stronger students away (Wooton 5; Kolata 855). Teachers could hardly inspire their students to mathematical greatness if they demonstrated uncertainty in the subject themselves.

In addition to many teachers lacking academic strength, most teacher preparatory programs provided a weak foundation in mathematics and mathematics teaching. At the time the SMSG began its work, one estimate suggested that more than a quarter of secondary mathematics teachers had never taken a calculus class (Wooton 52). These mathematics teachers, then, possessed only slightly more mathematical understanding beyond that of their students. As of 1960, 29 states required no mathematics courses for elementary teacher certification (Wooton 99).[4] Compounded with the fact that most states required at most one year of mathematics at the high school level (and the fact that this year may well have merely reviewed old concepts), many elementary teachers had only learned about as much mathematics as they would need to teach their students (Hayden 173–75). Ultimately, the picture painted of the mathematical preparation of America's teachers of mathematics, both at the secondary level and also at the elementary level, is not a pretty one. Curricular materials covering new types of mathematics and new ways of thinking about mathematics were doomed from the start when placed in the hands of educators who lacked mathematical understanding themselves (Hayden 191; Phillips, *The New Math: A Political History* 106).

Especially in light of the need for well-prepared teachers to carry out the project's reforms, the School Mathematics Study Group recognized the significance of teacher preparation (Wooton 136–37). Therefore, the SMSG produced several guides so current and future mathematics teachers could better learn relevant mathematical concepts: These texts included *Studies in Mathematics*, *Some Basic Mathematical Concepts*, *Structure of Elementary Algebra*,

[4] Phillips offers slightly different estimates for elementary school teachers. He suggests that these teachers had taken two mathematics courses and one mathematics education course in college (*The New Math: A Political History* 106). The point stands, however, that elementary school teachers could hardly consider themselves mathematics experts.

Geometry, and *Euclidean Geometry Based on Rule and Protractor Axioms* (Hayden 154–55). Regardless of the production of these texts, however, the SMSG held little influence over the training of teachers. The organization aimed to design a new mathematics curriculum and perhaps to undertake "in-service" teacher preparation initiatives, but reforming teacher education programs was simply not on the group's agenda or even within the group's purview (Wooton 52). Providing a few texts would never counteract a system not adequately training its teachers in mathematics and mathematics teaching in the first place.

Even if teachers had a reasonable background in mathematics, SMSG textbooks introduced mathematical concepts that many teachers had never seen before. Writing about Baton Rouge junior high school teachers implementing SMSG materials, Houston Karnes suggested that "[t]he college work of these teachers, in mathematics, was of the traditional type, and therefore did not include certain vocabulary, ideas and concepts these teachers would be using in the experimental material" (468). Many teachers' training courses encouraged drilling computational skills rather than emphasizing the conceptual approach reformers now promoted (Bossé 191). These differences presented teachers with the distinct challenge of radically altering their understanding of mathematics in their teaching style (Bossé 191).

Cognizant of the difficulties teachers would face in preparing to use the new materials, the SMSG made plans to support teachers in implementing its textbooks (Weaver, "The School Mathematics Study Group Project on Elementary School Mathematics, Grades K–3" 516; Wooton 88). The SMSG developed supplemental materials to help familiarize elementary and secondary teachers with the content and structure of the new textbooks and to introduce methods of teaching the new curriculum (Begle, "SMSG: The First Decade" 242–43; Wagner 455, 457–58; Wooton 61–62, 118–19). The National Science Foundation also sponsored conferences and in-service training programs that helped prepare teachers to use SMSG texts (Phillips, *The New Math: A Political History* 99, 105; Wagner 457–59; Wooton 48). Ideally, these initiatives would support teachers in introducing SMSG texts into their classrooms.

But no matter the attempts the group made to prepare teachers to use the new textbooks, many teachers, particularly those at the elementary school level, simply lacked adequate preparation for the new materials. Because America's school systems employed over eight times as many elementary school teachers as high school teachers (in addition to the fact that elementary school teachers had little mathematics experience), the issue of training elementary school teachers in the "new math" proved especially challenging (Hayden 173; Kolata 855). As a result, about one-eighth as many elementary school teachers as secondary mathematics teachers obtained official training in the "new math" (Kolata 855; Phillips, *The New Math: A Political History* 105; Roberts and Walmsley 470). And only about half of these secondary teachers had received training in the first place (Phillips, *The New Math: A Political History* 105). Many of the teachers expected to implement the new materials simply lacked familiarity with the textbooks before they began using them in their classes, which created a significant obstacle for the reforms' success.[5]

Without sufficient preparation, hopes that teachers could successfully implement modern mathematics materials proved misguided. How could a high school mathematics teacher who had never studied calculus and who had always taught via drill techniques have begun to teach

[5] Perhaps Henry Pollak's closing remarks of a speech at an orientation conference best summarize the issues of teacher preparation: "You may feel overwhelmed by what I have been saying, but then I've had to do a year's work in half an hour" (School Mathematics Study Group, *Report* 14). If these efforts existed at all, minimal teacher training initiatives simply could not make up for teachers' lack of preparation.

group theory, as the group hoped (School Mathematics Study Group, *Intermediate Mathematics* 843–66)? The notion is absurd. Though the SMSG attempted to prepare teachers to use its textbooks, the compounded difficulties of teachers' lack of education and the newness of the curricular materials placed teachers in the impossible position of having to teach new mathematics concepts without adequate preparation to do so. Significantly, these inadequacies resulted in large part from factors beyond the SMSG's control, since the group's primary aim involved developing a new curriculum, not training teachers. Because of the group's place in history and in the educational world, the SMSG's desired reforms simply could not flourish in classrooms.

Testing

In addition to revealing the extensive work needed to prepare teachers to teach mathematics and "new math" materials, the SMSG's work laid bare the need for meticulous research regarding the viability of mathematics reforms. One of the first projects of its kind to develop and study mathematics curricular work, the SMSG ultimately paid the price for its own pioneering role in reform.

The SMSG's activities included considerable involvement in curricular testing. SMSG member William Wooton asserted that "[t]he trial of the material was as important to the work of SMSG as the writing itself" (Wooton 46). The group tested the material during summer writing sessions with summer school classes and writing group members' children, later sending the textbooks out to "cit[ies] or localit[ies]" that served as experimental centers (Phillips, *The New Math: A Political History* 100, 102; Weaver, "The School Mathematics Study Group Project on

Elementary-School Mathematics" 34; Wooton 46–47, 80, 82, 100, 129). Under the direction of a center-wide chairman, teachers affiliated with these centers tested SMSG textbooks in their classrooms and reported back to their chairmen about how well these materials worked within classrooms (Wooton 46). Mathematicians served as consultants to the teachers within these centers (Wooton 46). Other schools acted as testing points, which received less financial and educational support in implementing SMSG materials than centers but could still provide the organization with feedback (Wooton 87–88). In addition to studying the individual effects of their textbooks, Begle and his colleagues started a groundbreaking National Longitudinal Study of Mathematical Ability (Phillips, *The New Math: A Political History* 126). With these efforts, the SMSG demonstrated a pioneering commitment to ensuring mathematics education reforms functioned well in classrooms.

Despite the SMSG's work to test its materials, the merit of the SMSG's actual curricular materials remains elusive, particularly in terms of students' achievement with these materials. Indeed, "For every study or article that showed a decline, there was another indicating that certain new textbooks might actually improve scores, or at least stabilize previously falling scores" (Phillips, *The New Math: A Political History* 126). Extant tests, in other words, provided minimal, if any, information about the validity of SMSG materials.

These results varied widely for several reasons. First of all, researchers had only recently taken interest in studying mathematics education, which meant that few studies existed to "establish a baseline score for comparison" (Phillips, "The New Math and Midcentury American Politics" 472; Phillips, *The New Math: A Political History* 126). The SMSG and other "new math" projects undertook so much original research that "[t]he New Math Movement eventually affected research more than research affected the Movement" (Bossé 176). The SMSG and the other "new math" programs laid much of the groundwork for subsequent research in mathematics education. But though their contributions proved significant for future reform, these organizations could not benefit significantly from these initial steps.

Moreover, the very nature of testing students' mathematics achievement with SMSG materials (or "new math" materials more broadly) compared to that of students using traditional materials was inherently difficult. Since the SMSG had intended from the beginning to improve students' conceptual understanding of mathematics, not to increase computational skills, results frequently depended on whether tests focused on traditional or modern concepts (Begle, "SMSG: The First Decade" 244; Hungerman 32, 34, 36–37; Moise 98–99; Phillips, *The New Math: A Political History* 126–27). The vastly different aims of textbooks made it difficult to determine whether or not the SMSG's materials taught mathematics better than traditional textbooks.

Unfortunately, these issues with testing could not find resolution before critics berated the SMSG for failing in its attempts to teach students mathematics in an improved fashion. Opponents of the SMSG ultimately leveraged inconclusive test results against the group. Though research revealed little solid evidence to support or refute the validity of the SMSG's work, critics cited decreasing SAT mathematics scores from 1962 to 1975, in particular, to advocate for mathematics education to move back towards a focus on computational skills (Phillips, *The New Math: A Political History* 125). Though these SAT results hardly served as an indictment of the "new math" programs, since SAT verbal scores decreased by an even larger degree during this period, these decreasing test scores earned the SMSG and the "new math" movement more broadly considerable negative attention (Kolata 854–55; Phillips, *The New Math: A Political History* 125).[6] These criticisms would ultimately haunt the group's image.

The educational cost of a mathematical vision

The SMSG fell victim to the pioneering nature of its work. The group's practice of developing its textbooks with a (perhaps nonexistent) ideal student in mind also significantly threatened the long-term success of the group's reforms. Thus, the SMSG offers cautionary guidance about the potential for well-intentioned education reforms to isolate real students.

Content over pedagogy

The SMSG's preference for content over pedagogy created the first obstacle in terms of reaching real students (Phillips, *The New Math: A Political History* 16). SMSG leadership did recognize the importance of high-quality mathematics teaching, in many cases ensuring sizeable contributions from schoolteachers to develop the pedagogical quality of textbooks (School Mathematics Study Group, *Report* 38; Phillips, "In Accordance" 543). In theory, Begle recognized the importance of attempting to understand how students learn best: "I believe that research in mathematics education must involve research on the learning of mathematics, the teaching of mathematics, problem solving, etc., as well as the syllabus, and that exclusive concentration on any one aspect of this complex is not fruitful" ("Curriculum Research" 44).[7]

[6] Various explanations for these general decreases in scores include greater numbers of people taking the SAT, as well as discrepancies between the focuses of the tests and students' courses (Phillips, *The New Math: A Political History* 125–26).

[7] Begle even wondered about the effectiveness of various pedagogical approaches, such as discovery teaching, team teaching, and classroom conversations, but the SMSG itself did not work directly with these innovations ("The Role of Research" 232; "Some Lessons Learned" 210–11).

However, with research mathematicians occupying the leadership and decision-making positions of the SMSG, their reform efforts demonstrated little concern with how students actually learn mathematics (Phillips, "The New Math and Midcentury American Politics" 459).

A quick perusal of an SMSG publication reveals the group's close association with mathematicians. Mathematics textbooks in the K–12 curriculum must straddle a delicate balance between the mathematical needs of the general population and the interests of mathematics scholars in universities (Phillips, *The New Math: A Political History* 48). The SMSG's texts, however, made very clear the organization's allegiance to mathematicians. The SMSG's high school texts featured a variety of exercises requiring students to prove results, as well as lessons that incorporated elaborate proofs (School Mathematics Study Group, *Introduction to Algebra: Part II* 230–32; School Mathematics Study Group, *Report* 10; School Mathematics Study Group, *Intermediate Mathematics* 10, 16–17, 56, 70–71, 107). For example, the eleventh-grade textbook included a section with the detailed proof that a repeating decimal corresponds with a rational number (School Mathematics Study Group, *Intermediate Mathematics* 69–71). Group theory concepts could also be found sprinkled throughout SMSG texts, with a significant coverage of groups and fields at the eleventh-grade level (Phillips, "In Accordance" 553–54; School Mathematics Study Group, *Intermediate Mathematics* 843–66). The eleventh-grade text even included several sections explaining recent developments in mathematics to remind students of the ever-evolving nature of mathematics research (School Mathematics Study Group, *Intermediate Mathematics* 254–55, 727). These publications, then, demonstrated a significant interest in sparking mathematical curiosity and establishing the mathematical foundations for students to pursue advanced mathematics in the future.

Theoretically, the SMSG sought guidance from psychologists in developing its curricular materials, despite Begle's own distrust of psychologists (Phillips, *The New Math: A Political History* 87–90). Looking more closely, however, the SMSG in fact took advantage of disagreements between psychologists over how children learn (Phillips, *The New Math: A Political History* 87).[8] One psychologist, Jerome Bruner, completed research that suggested that "any subject could be taught effectively in some intellectually honest way to any child at any state of development" (Phillips, *The New Math: A Political History* 89). Bruner's work supported the idea that younger students could learn more advanced topics and that mathematical thinking could improve students' intellectual capacity more broadly (Phillips, *The New Math: A Political History* 89). Despite psychologist William Brownell's insistence that the SMSG consider students' previous mathematical experiences rather than focusing on what students could theoretically learn at any "mental stage" and his horror over the "SMSG's lack of actual research into how children really learn," the SMSG surged forward to attempt to teach younger and younger students more advanced mathematics (Phillips, *The New Math: A Political History* 89–90). Phillips recognizes that the SMSG's cherry-picking of certain psychologists' ideas revealed the group's true motives: "If there was little consensus among psychologists as to the underlying development of the students' mind, there seemed to be ample evidence that students would be able to handle the new concepts SMSG wanted to introduce" (*The New Math: A Political History* 89). "That Brownell's warnings went unheeded," Phillips asserted, "confirms the centrality of mathematics itself to SMSG's version of curriculum reform" (*The New Math: A Political History* 90). The SMSG placed the content-related, mathematical interests of the

[8] Certainly, the lack of psychologists interested in education during this period did not help the SMSG reflect students' psychological needs in the group's work, either (Phillips, *The New Math: A Political History* 87).

organization's mathematicians above the true needs of mathematics students, despite its claims to the contrary.

The SMSG's emphasis on content over pedagogy shone through in its work. According to one of Begle's former students, the SMSG thought that "if you get the mathematics right, and get it organized right, then kids will learn" (as cited in Phillips, *The New Math: A Political History* 90). Research convinced Begle that textbooks played a much more significant role in determining students' learning of mathematics than the ways in which teachers taught the material (Begle, "Some Lessons Learned" 209–10; Kolata 855–56; Phillips, *The New Math: A Political History* 98).[9] Though F. L. Elder, an educator who spoke at an SMSG teacher orientation conference, recognized that lectures and textbook reading alone might not prove sufficient in helping students learn mathematics, the SMSG itself was not willing to prioritize or even consider these ideas and instead "pushed for few institutional changes" (School Mathematics Study Group, *Report* 125; Phillips, *The New Math: A Political History* 16). In 1959, educator W. Eugene Ferguson may have captured the critical flaws in the SMSG's approach when he articulated that "[w]e are probably lacking in ideas of how to teach the material, but we felt that the people teaching the material could figure out how to teach it if they understood it themselves" (School Mathematics Study Group, *Report* 79–80).[10] The SMSG seemed to recognize that pedagogical issues existed, but the group's treatment of the development of its textbooks demonstrated the group's avoidance of studying pedagogy itself.

Especially without honestly considering studies on how students learn mathematics, the SMSG's work consistently treated with preference the content mathematicians *hoped* students

[9] Incidentally, "new math" opponent Morris Kline held the opposite opinion, that a teacher could "overcome" a poor curriculum with effective pedagogy (Kline 170).
[10] Of course, many of these teachers did *not* understand the material in the first place.

would learn over the mathematics students *could* learn.[11] These issues with the group's approach only became more pronounced as the group attempted to develop texts for weaker mathematics students.

Academic strength

The SMSG's decidedly mathematically-focused reform choices particularly failed students who struggled with mathematics. Though the SMSG's work supposedly did not explicitly aim to create more students who would study advanced mathematics, Begle himself admitted the SMSG focused initially on stronger students, those preparing to attend college, since these students could ultimately become mathematicians and scientists (Begle, "The School Mathematics Study Group" 618; Phillips, "The New Math and Midcentury American Politics" 461; Wooton 18–19). The "new math" reformers initially focused on affecting "the course offerings of an educational system they believed to be uninspiring, unintellectual, and insufficiently challenging for its most gifted students," despite Begle's claims that the SMSG would not overlook less mathematically-gifted students (Begle, "The School Mathematics Study Group" 618; Dumbaugh and Schwermer 82). This initial focus no doubt centered the group's work.

Though the SMSG claimed a desire to help weaker mathematics students learn "the nature of mathematics and the role of mathematics in our society," its actual actions for these

[11] We should be careful here not to assert that the SMSG should have insisted on teachers using specific pedagogical strategies. After all, the work of Yale's Seymour Sarason revealed that, especially at the elementary level, "teachers resisted curricular innovations because they challenged the teachers' competence and voice in the educational process" (Phillips, *The New Math: A Political History* 128). The point, rather, is that the SMSG likely should have paid greater attention to pedagogical concerns in developing its texts. Ensuring greater input from teachers, particularly in the elementary writing groups, where teachers were grossly underrepresented, might have further enhanced the pedagogical quality of the group's work (Phillips, *The New Math: A Political History* 19, 81).

students reveal its true intentions (Walmsley 44). The SMSG held sessions on more average (non-college-bound) mathematics students to consider the best ways to approach these students' mathematics textbooks (Wooton 90, 113). The SMSG's approach for these students involved presenting the same advanced concepts at slower rates and with a few extra "carefully designed exercises" (Wooton 91–92, 101, 103). With this approach, the SMSG drew on psychologist John Carroll's work suggesting students of any ability could learn the same mathematics with the right amount of work (Begle, "The Role of Research" 237–38). Theoretically, then, the SMSG aimed to teach students of less exemplary mathematical ability. But the fact that the SMSG's approach for the strongest mathematics students framed its subsequent approach for students who struggled in mathematics raises questions about the group's true commitment to individuals who struggled more in mathematics.

In addition to the problematic nature of the SMSG's work regarding weaker students, its obvious interest in the experiences of gifted students helps reveal the group's aims. The SMSG developed a variety of enrichment opportunities for the strongest mathematics students. For example, the SMSG published monographs meant "to disseminate good mathematics at the secondary school level which will supplement the usual high school curriculum; to awaken interest among gifted students; and to present mathematics as a satisfying, meaningful human activity" (Wagner 457; Begle, "SMSG: The First Decade" 242). In its matrix algebra text, the SMSG included additional "Research Exercises" intended to introduce the best students to "real mathematical research" (Wagner 456; Wooton 86). The SMSG even demonstrated interest in serving the most advanced students by connecting these students with nearby mathematicians who could serve as mentors (Begle, "SMSG: The First Decade" 241). The SMSG, then, very clearly demonstrated its investment in supporting the absolute strongest mathematics students in their continued mathematical endeavors.

After the group disbanded, one mathematician affiliated with the SMSG reflected that the "SMSG might have…begun a curriculum that necessitated a tempo beyond the ability of most students['] capabilities" (Bossé 172, 192). A curriculum project initially designed with the strongest few students in mind simply could not be expected to work well for students less naturally inclined towards mathematics. This narrow vision would ultimately not only stymie the success of the group's materials in the classroom, but also prevent the group from fully understanding relevant factors outside the classroom that affected mathematics learning.

Looking beyond the classroom

"[O]ur educational system does not work equally well for all students" (Begle, "Some Lessons Learned" 207). Though Begle had recognized this truth by 1973, the SMSG's activities revealed that the group's narrow vision further blinded the group to students' needs beyond the classroom. In particular, the group's limited consideration of the realities of students' backgrounds and schools' needs reduced the organization's ability to serve the nation's students. The SMSG's understanding of the country's mathematics students "entailed a genderless, raceless, classless version of the student body, a vision not tied to particular schools or individuals. Of course, this was a fiction. If a student was supposed to be trained to reason like a mathematician, it mattered that the mathematician at this time was overwhelmingly likely to be white, middle class, and male" (Phillips, *The New Math: A Political History* 77). The SMSG's (perhaps inadvertent) focus on the most privileged of mathematics students in the United States revealed itself clearly in the group's activities.

In particular, the SMSG operated from the premise that all students attended well-funded, well-equipped schools. This simply was not true. Consequently, the SMSG's work remained inaccessible to many disadvantaged schools. Schools that struggled to meet students' basic needs could hardly afford a fundamental restructuring of their mathematics programs (Phillips, *The New Math: A Political History* 122). They also could not participate in SMSG testing opportunities. For example, the requirement that a mathematician serve as a consultant to testing centers required that these centers be located near a college or university (Wooton 47). This requirement disadvantaged remote rural districts. Moreover, schools that served as testing points for SMSG textbooks had to use internal funding to hire mathematicians as consultants during the curriculum testing process (Wooton 87–88). In the case of testing points, schools that wanted an official means through which to determine the quality of SMSG materials also needed access to money to do so. Apparently never designed with these schools in mind in the first place, the SMSG's work was simply out of reach of poor districts. With the blinders of their socioeconomic privilege, SMSG leaders ultimately expanded inequitable opportunities, albeit likely unintentionally.

In addition to neglecting poorer students and districts, the SMSG demonstrated a strong misunderstanding of the place of culture within mathematics education. By 1969, Begle had developed a belief that cultural factors affect a student's "ability to learn and do mathematics" ("The Role of Research" 241). One SMSG text described these "culturally disadvantaged" students as individuals who were "'mainly…urban slum-dwelling people' whose problem was both low economic status and a lack of participation in 'middle class culture'" (Phillips, *The New Math: A Political History* 92). Moreover, the belief was that mathematics could help these groups think in a "proper" fashion (Phillips, *The New Math: A Political History* 92). As such, the SMSG did undertake a few studies regarding students' cultural backgrounds, including one study looking at children from "slum areas of six cities" and another exploring the attitudes of "black," "Oriental," "Mexican," and white students regarding their mathematics materials (Begle, "SMSG: The First Decade" 241; Begle, "Some Lessons Learned" 212). Notably, both of these studies described students from culturally-diverse backgrounds as "low-achieving" or "less able" (Begle, "SMSG: The First Decade" 240–41; Begle, "Some Lessons Learned" 212). The SMSG in turn developed texts for these "culturally disadvantaged…children," but these textbooks merely covered material more slowly than the SMSG's other texts (Phillips, *The New Math: A Political History* 3, 93). Taken together, these details suggest the SMSG equated an affiliation with a minority culture with a lack of academic ability. By viewing cultural differences as a problem rather than as an opportunity to transform its traditional approach, the SMSG not only erroneously conflated intellectual potential with students' cultures, but the group also repeated the mistake it had made with less mathematically-gifted students: viewing mathematics reforms through the lens of a "model" student, in this case ignoring real cultural factors beyond the classroom.

The fact that the SMSG's elite position blinded the group's work, in turn causing the group to misunderstand students' needs outside the classroom, ultimately contributed to the program's downfall. By the time Begle recognized the effect of a student's environment on her ability to learn and understand mathematics, the damage had already been done ("Curriculum

Research" 44; "Some Lessons Learned" 207). By the end of the 1960s, many education experts had begun to understand that because of the significance of a student's background, meaningful educational change would extend beyond curriculum adjustments in classrooms to involve more extensive "institutional reform" (Phillips, *The New Math: A Political History* 128–29, 132). The SMSG's narrow understanding of mathematics education reform became incompatible with the shifting climate. Amidst these realizations that many students and schools faced significant obstacles, "[o]rganizations like SMSG, which hoped to teach *all* students essentially the *same* sort of mathematics, seemed increasingly naïve" (Phillips, *The New Math: A Political History* 131–32, emphasis original). Regardless even of the SMSG's specific approach to presenting mathematics, faith waned in the appropriateness of focusing on students' individual mathematical thought processes as opposed to on a larger restructuring of educational systems (Phillips, *The New Math: A Political History* 128–29). After all, "[d]ebates about 'modern' math and intellectual discipline seemed ridiculous when some students lacked even basic access to quality schools and teachers" (Phillips, *The New Math: A Political History* 132). The SMSG's approach was simply out of touch with the realities of the United States's educational system.

Too many voices

It is unrealistic to judge the work of the School Mathematics Study Group as an isolated reform project that revolutionized the teaching of mathematics on its own. The organization hardly existed in a vacuum. Consequently, the effectiveness and reception of the SMSG's work must be considered alongside its relationships with other organizations with an interest in mathematics education, namely mathematicians in the academy, the commercial textbook industry, and the political sphere.

Contentions between mathematicians

Since the SMSG's work unequivocally reflected the interests of certain mathematicians, tensions heightened within the mathematical community over the group's portrayal of mathematics (Phillips, "In Accordance" 544). Though the SMSG's textbook publications seemed to suggest common agreement among mathematicians on an understanding of mathematics and the group's association with the AMS elevated its work, mathematicians across the field disagreed about the place of mathematics within the broader scientific world (Phillips, *The New Math: A Political History* 67, 73–74). Generally speaking, these arguments centered around mathematicians like Edward Begle and Marshall Stone, who viewed the subject as one worth studying purely for its structural merit, and mathematicians like Morris Kline, who viewed the subject as one whose value revolved around its use in scientific settings (Phillips, *The New Math: A Political History* 72–74). This debate over "what mathematics was and why it should be learned" caused significant disagreement over the work of the SMSG and other "new math" projects (Phillips, *The New Math: A Political History* 74).

These tensions in the mathematical research community made their way into the pages of the SMSG's textbooks. The SMSG's Algebra I textbook illustrates the group's focus on the underlying structure of mathematics over computational skills, or on the method rather than the solution (Phillips, *The New Math: A Political History* 1). For example, the authors outlined that "[t]here will be many places in this course and in future mathematics where the <u>pattern</u> or <u>form</u>

of a problem is much more important than any single answer" (School Mathematics Study Group, *Introduction to Algebra: Part I* 77, underline original). Many exercises reflected this aim and required students to prove results (School Mathematics Study Group, *Introduction to Algebra: Part II* 230, 232). For example, one exercise asked students to "[p]rove that if $a < 0$ and $b > 0$ then $\frac{1}{a} < \frac{1}{b}$" (School Mathematics Study Group, *Introduction to Algebra: Part II* 230, 232). These elements of the textbook demonstrated the SMSG's focus on the structure of mathematics. With this focus, the SMSG hoped students would grow in their understanding of mathematics as a way of thinking (Phillips, *The New Math: A Political History* 147). Not all mathematicians appreciated this approach, however. Though mathematicians generally agreed that drill was not a helpful way to learn mathematics, many SMSG opponents demonstrated concern over the ways the SMSG diverted attention from the techniques that allowed students to apply mathematics to scientific domains and instead focused on the underlying mathematical structures (Phillips, *The New Math: A Political History* 74; Phillips, "The New Math and Midcentury American Politics" 461).

The SMSG also revealed its preference for mathematics as a way of thinking versus as a tool to study science through its minimal use of applications in its textbooks. The SMSG did seem to recognize the importance of mathematics within a variety of applied domains. Mathematician Paul Rosenbloom emphasized at an SMSG teacher orientation conference that "it is important for the development of mathematics that our students learn how mathematics is applied to almost every field of knowledge, is essential to many vocations, and plays a fundamental role in government, industry, science, and in every part of our culture" (School Mathematics Study Group, *Report* 20). And the SMSG did attempt to incorporate applications into its publications. The group's junior high school texts, as a noteworthy example, included sections about the use of mathematics in natural science, social science, and computing fields (School Mathematics Study Group, *Report* 21–23). The SMSG also worked in association with the Physical Science Study Committee and the Biological Sciences Curriculum Study, the physics and biology equivalents to the SMSG, to incorporate some of these groups' problems into its textbooks (Hayden 159). Even still, the SMSG cared primarily about cultivating students' understanding of the underlying mathematical structures. At the same orientation conference where Rosenbloom spoke, Edwin Dudley emphasized that "[i]t is the awareness of the model, not the many individual applications[,] that should be foremost in the student's mind" (School Mathematics Study Group, *Report* 68). The primary goal was always to learn mathematics for its own structure, with the notion that scientific applications should follow, not precede, the mathematical models (Phillips, *The New Math: A Political History* 68). This lack of direct focus on mathematical applications drew criticism from applied mathematicians, in particular (Phillips, *The New Math: A Political History* 69). At a 1961 panel run by the Society for Industrial and Applied Mathematics, renowned applied mathematician Richard Courant "warned that 'the life blood of our science rises through [mathematics'] roots,' and 'these roots reach down in endless ramification deep into what might be called reality'" (Phillips, *The New Math: A Political History* 67–68). Courant feared that overemphasizing pure mathematics would separate mathematics from the vital ways in which the subject allows modern society to function.

The SMSG's prominence and unofficial "official" status prompted "critiques of SMSG's textbooks [that] were particularly contentious" (Phillips, *The New Math: A Political History* 48, 67). Mathematicians across the field recognized the ability of SMSG textbooks to present a certain understanding of the nature of mathematics to broad subsections of the American population, and some members of the mathematical community who disagreed with the SMSG's approach felt called to vocalize their concerns that schoolchildren were not learning mathematics in what they believed to be the proper way. With the portrayal of their area of study and American schoolchildren's understanding of mathematics on the line, many researchers in mathematics fields successfully problematized the SMSG's work from the beginning (Phillips, *The New Math: A Political History* 67). Later on, some of these same critics would help contribute to the project's public downfall (Phillips, *The New Math: A Political History* 124–25).

Commercial textbook industry

Though less obvious, the commercial textbook industry inadvertently played a major role in shaping the SMSG's lasting public image. From the beginning, the SMSG developed its publications such that "no one had exclusive rights to the material of the SMSG books, and no royalties would have to be paid" with the intention that these texts would provide examples for commercial textbook writers (Phillips, *The New Math: A Political History* 102, 104). The ways the commercial textbook industry functioned in relation to the SMSG's work, however, had permanent negative effects on the organization's legacy.

In particular, the distinct relationships between the SMSG and the commercial textbook industry over secondary texts versus elementary texts highlight this industry's harmful impact on the group. According to Phillips,

> One paradox of the new math was that its most successful incarnation—the fundamental alteration of the secondary-school mathematics curriculum—was also invisible. Most memories of the books and indeed most contemporary discussion of the reforms involved the elementary schools and their changing textbooks. From the point of view of SMSG, this was doubly unfortunate in that

> the organization did not ever receive lasting credit for the monumental effort in the secondary schools but did face any criticism for the elementary school reforms made largely outside of its purview (*The New Math: A Political History* 119–20).

Exploring the particular experiences of the SMSG's work at the secondary level and at the elementary level will help illuminate this point.

The case of the secondary schools, the SMSG's first target group, involved relatively well-developed texts whose commercial counterparts never provided the group with significant lasting popularity. The group's own secondary school publications received largely positive reviews, which resulted in significant interest in developing similar commercial texts (Hayden 143; Phillips, *The New Math: A Political History* 104–05). Despite the fact that many SMSG writers went on to write commercial textbooks, these commercial versions of the SMSG's high school texts proved disappointing to Begle and his SMSG colleagues (Phillips, *The New Math: A Political History* 103–04). In fact, *Modern Algebra* by Mary Dolciani and *Geometry* by Edwin Moise and Floyd Downs were "the only two books ever considered to be suitable replacements for the SMSG texts" (Phillips, *The New Math: A Political History* 104).[12] Even if the SMSG's publications were well-developed themselves, commercial writers would not necessarily stay true to the SMSG's vision.

But regardless of the fate of the secondary texts, the elementary texts had a much greater effect on the SMSG's lasting image. The SMSG itself held little influence over the entrance of "new math" textbooks into elementary schools, particularly because the SMSG started developing elementary texts after commercial texts at the elementary level

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[12] Perhaps not surprisingly, Dolciani, Moise, and Downs all initially served on the SMSG (Phillips, *The New Math: A Political History* 103–04). Dolciani, in particular, played a key role as a commercial writer for Houghton Mifflin (Phillips, *The New Math: A Political History* 103–04).

had gained popularity (Phillips, *The New Math: A Political History* 96–97). Put simply, "[t]he experience of the new math in the elementary schools was not one dictated by SMSG" (Phillips, *The New Math: A Political History* 96–97). The group's actual elementary texts did not receive the popularity its secondary texts did, and most commercial elementary texts had no connections with the group (Phillips, *The New Math: A Political History* 108). The SMSG did undertake some advisory work with state education boards over elementary mathematics standards, but the group ultimately did not significantly impact elementary school curricula (Phillips, *The New Math: A Political History* 108).

At the same time, commercial elementary school texts spread quickly—perhaps too quickly—across the nation. Since the SMSG held few relationships with elementary school educators, since elementary school teachers lacked a "formal organization" to support their efforts, and since schools usually bought an entire series of elementary school texts at once, the SMSG had greater difficulty introducing its individual texts into classrooms (Phillips, *The New Math: A Political History* 105, 108). Therefore, district administrators, state education offices, and ultimately the textbook industry held more influence at this level of education (Phillips, *The New Math: A Political History* 105, 108). Large states like Texas and California with statewide curriculum mandates set the trend for widespread obsession with using "new math" texts (Phillips, *The New Math: A Political History* 110–13). Many of these commercial texts, however, merely introduced "new math" ideas like sets without significant focus on "the goals or needs of the schools" (Phillips, *The New Math: A Political History* 112–13). Moreover, the rapid spread of these textbooks without adequate teacher training initiatives left many teachers unprepared to implement these new textbooks in their classrooms (Phillips, *The New Math: A Political History* 111). The elementary "new math" texts, in other words, spread widely, but not necessarily wisely.

Ultimately, the fact that the elementary texts were both haphazardly developed and also broadly distributed permanently sunk the SMSG's image, despite the organization's minimal involvement with these texts. Though the SMSG had little to do with these elementary texts, public recognition of the "new math" movement and the work of the SMSG became linked to these very publications (Phillips, *The New Math: A Political History* 120). Especially given the problems associated with the elementary school texts, this legacy based on a misunderstanding placed the SMSG at a distinct disadvantage.

National climate

Ultimately, widespread public opinion proved most influential both in creating the SMSG and later in condemning the organization's efforts. During the first half of the SMSG's history, "new math" programs received positive reception (Hayden 143–44, 158). Politically, the "new math" projects had received bipartisan support from liberals who valued "the government's active involvement in improving education" and from conservatives who appreciated that mathematicians and scientists, not educators, spearheaded the reform efforts (Phillips, *The New Math: A Political History* 118). Even when the SMSG disbanded, the group's work was widely considered to have enhanced mathematics education in the United States (Phillips, "The New Math and Midcentury American Politics" 471). This positive reception, however, would not last.

Examining the ways the SMSG's image diminished as the national political climate also changed will help illuminate the nature of the SMSG's loss of popularity.

As the 1960s progressed, reasons for federal interest in mathematics education reform shifted from national security concerns to domestic desires to transform the nation through education (Phillips, *The New Math: A Political History* 118, 120, 133). In particular, President Lyndon B. Johnson's 1965 signing of the Elementary and Secondary Education Act reflected his interest in creating a Great Society through social reform (Phillips, *The New Math: A Political History* 118, 120, 133). At this time, the SMSG began to undertake work on the elementary school curriculum, particularly out of a belief that "the challenges of the contemporary world might be addressed, in part, by changing how first graders learned to add" (Phillips, *The New Math: A Political History* 19, 77, 96–97, 120). Mathematics education reform thus refocused its attention to the elementary level to help reshape American society by helping students learn to reason properly. Notably, the mid-1960s represented both the zenith of the "new math" reforms and also "the high point of public faith in the ability of federal initiatives—particularly in education—to cure the nation's ills" (Phillips, "The New Math and Midcentury American Politics" 473). Widespread support of the SMSG and other "new math" programs during this period reflected a belief that a better nation might be created through government support of education.

But by the early 1970s, the SMSG began to lose this approval. As the "forces" that brought the SMSG into existence collapsed, the fate of the SMSG hung in the balance (Hayden 241–42). Formally, federal financial challenges officially ended the School Mathematics Study Group in the summer of 1972 (Phillips, *The New Math: A Political History* 123). After all, exorbitant spending on the Vietnam War had caused the National Science Foundation to lose much of its funding (Bossé 195; Phillips, *The New Math: A Political History* 123, 134). For example, in 1969 the SMSG had received only $641,000 of the $1.2 million the group had desired (Phillips, *The New Math: A Political History* 123). One of the most devastating financial effects was the loss of funding for the NSF's teacher training programs, which effectively discouraged many teachers from incorporating the "new math" materials into their classrooms (Hayden 244–45). Broadly speaking, without financial support for the SMSG's work and related efforts, these reforms simply could not flourish.

Trust in the federal government to act in citizens' best interests also eroded during the early 1970s, which further destroyed prospects for the SMSG's publications to experience an enduring positive reception. This period in American history witnessed the failures of many of President Johnson's Great Society programs, discontent regarding the country's involvement in the Vietnam War, and distrust of elected officials amidst rampant corruption at the federal level (Phillips, *The New Math: A Political History* 129–30). Without belief in the ability of the federal government to act wisely, federally-supported education programs like the School Mathematics Study Group lost their support. Moreover, the very fact that the SMSG's work had been so rooted in a belief in the ability of mathematical thinking to transform the nation contributed to the program's downfall. After all, "as domestic disillusionment grew, so theories based on the supremacy of the 'modern' American ideal came to seem far less persuasive" (Phillips, *The New Math: A Political History* 130). Crumbling faith in these nationalistic ideals ultimately meant that belief in the ability of the SMSG's textbooks to help America's schoolchildren think properly lost traction.

Mounting distrust of academics to serve the needs of Americans also destroyed support for the SMSG's work (Phillips, *The New Math: A Political History* 129). For one, "[c]olleges came to be associated with protests and radicalism, and with being increasingly isolated from 'mainstream' America" (Phillips, *The New Math: A Political History* 129). Americans turned away from academics as trusted experts who provided reliable guidance and truth, especially as these individuals were now deemed out of touch with American society as a whole (Phillips, *The New Math: A Political History* 129). In particular, distrust of scientists grew due to their roles in supporting military activities and other controversial projects (like nuclear power and the development of harmful chemicals) (Phillips, *The New Math: A Political History* 130–31). Ultimately, then, a curriculum project led by mathematicians could not expect widespread public support.

This changing national climate ultimately caused significant criticisms of the "new math." Phillips claims that "[a]s faith in modernism, federal interventions, and the transformative power of education faded by the early 1970s, so too would the new math's reputation diminish" (*The New Math: A Political History* 120). In particular, opponents of the "new math" projects began to appear in prominent newspapers across the nation during the early 1970s (Phillips, *The New Math: A Political History* 124). Morris Kline's 1973 *Why Johnny Can't Add* offered critics enhanced support with a mathematician ally (Phillips, *The New Math: A Political History* 124–25). These criticisms ultimately succeeded in altering trends in mathematics education.

The reaction to the "new math" reflected specific ideological beliefs about the nation and its citizens' modes of thinking. Though some "new math" opponents pointed to declining test scores or a lack of teacher training as a reason to re-emphasize computational skills in the classroom, the ultimate underlying goal involved a belief in "the need to cultivate traditional habits of thought in a world that seemed to have lost its moral grounding" (Phillips, *The New Math: A Political History* 127, 143). Because they recognized that the style of mathematics curriculum students saw in the classroom could significantly impact "the mind, the family, the society, and the state," "new math" opponents wanted to reframe how mathematics would shape students' minds (Phillips, *The New Math: A Political History* 4).

These assumptions about the place of mathematics in shaping American society resulted in a drill-focused backlash called "back to basics" (Phillips, *The New Math: A Political History* 137). In particular, states like California, New Hampshire, and New York began to re-emphasize computation in their mathematics classrooms in 1973 (Phillips, *The New Math: A Political History* 124). A series of NSF teacher interviews during the mid-1970s revealed that teachers were placing a strong emphasis on the role of "intellectual training, discipline, and social order" in learning mathematics (Phillips, *The New Math: A Political History* 137–38). Despite the lack of usefulness of rote memorization of mathematics facts, an interest in promoting discipline in students led to an emphasis on drills in mathematics classrooms (Phillips, *The New Math: A Political History* 136, 142). According to one mathematics teacher, learning mathematics would ideally encourage in students a "work ethic: responsibility, diligence, persistence, thoroughness, neatness" (as cited in Phillips, *The New Math: A Political History* 142). Another mathematics educator claimed, "What I tell my classes is this: the only practical value you'll get out of studying mathematics is to learn to do as you're told" (as cited in Phillips, "The New Math and Midcentury American Politics" 474). These educators saw the value of learning mathematics in the subject's ability to develop disciplined citizens above all else.

The ultimate demise of the School Mathematics Study Group, then, had almost nothing to do with the group's actual work, but rather everything to do with changing political opinion about the role of learning mathematics within American society. Just as the SMSG did not emerge in a vacuum, so too the group did not fade away in a vacuum. Christopher Phillips's concluding remarks in *The New Math: A Political History* serve as a fitting conclusion to a section exploring the myriad influences on the mathematics classroom:

> The history of the new math should…prompt those who write, use, or evaluate textbooks to remember that the design of the math curriculum is never just a matter of deciding which topics to cover or which pedagogical techniques raise test scores. The constellation of factors that made the new math unique—direct involvement of federal authorities and monies in textbook development, leading roles played by academic mathematicians, Cold War concerns about authoritarian personalities, midcentury claims about the underlying nature of mathematical knowledge—may be period specific. Yet the math classroom will remain a political venue as long as learning math counts as learning to think. Debates about the American math curriculum are debates about the nature of the American subject (149).

Assumptions that mathematics education reform involves only the work of students and teachers in classrooms underestimate the high stakes for other parties: for mathematicians who hope to portray their field in a specific manner, for textbook writers whose profits depend on the popularity of their texts, and for the broader American public that cares deeply about shaping how the next generation of Americans learns to think.

Conclusion

In 1969, Begle reflected on the SMSG's work in mathematics education:

> [W]e cannot stop now. Further improvements are essential. Our children will live in an even more complicated and more quantified world than that of today. They need a better mathematics program than they now are getting. We still have many difficult problems to solve before we can make further improvements. In fact, I believe that so far we have attacked only the easier problems of mathematics education ("The Role of Research" 239–40).

Begle's reflection in the SMSG's later years captures the ongoing need for enhanced excellence and continued growth in mathematics education. As Begle understood, an ever-advancing world will indeed necessitate continued work to ensure *all* students learn mathematics well.

Despite the work the SMSG left undone and the group's largely negative legacy, the SMSG's work has enjoyed significant lasting positive effects. In particular, the work of the SMSG and the "new math" movement more broadly revolutionized the world of mathematics education. Reformers involved with the SMSG and other "new math" projects retained their interest in mathematics education and helped generate lasting interest in studying and improving mathematics instruction in the classroom (Phillips, *The New Math: A Political History* 146). The SMSG's efforts paved the way for future improvements in mathematics education, no matter the success of the group's actual textbooks within classrooms.

The work of the SMSG also offers valuable lessons to mathematics educators today. In particular, the ways in which the group's very structure ultimately caused the project to crumble provide helpful reminders so that, as SMSG member William Wooton asserted, we might not "go back," but rather "go forward more wisely" (1). Especially as one of the first projects to study mathematics reform in the classroom, the SMSG needed to start from scratch with much of its work, including attempts at teacher training and testing of reforms. Because the group only received NSF funding for fourteen years, the project never had "enough time to be tried," as one professor affiliated with the SMSG lamented (Bossé 194). This concern reveals the importance for today's mathematics educators to take great care in taking thorough, gradual steps to create lasting reforms. Especially given the ways in which the focus of mathematics education reforms tends to oscillate between "computation skills" and "conceptual understanding," we might look to the successes and failures of groups like the SMSG for guidance in proceeding with caution to attempt to escape this haphazard binary altogether (Phillips, *The New Math: A Political History* 146). Begle's 1966 recognition that "longer-range planning and experimentation should be started before present materials become frozen into a newly orthodox pattern that will require another upheaval a few years hence" speaks clearly to this point ("Mathematics Curriculum" 632). Instead of rapidly shifting gears when one hastily-developed reform idea fails, we might instead approach future reforms with the utmost intentionality.

Moreover, the manner in which the SMSG's heavily elite, mathematical vision blinded the group to the pedagogical and broader needs of real students and schools offers another lesson for today's reformers. This shortcoming serves as a reminder of the ways in which a rigid, narrow outlook for change can cause even the best-intentioned initiatives to harm the nation's schoolchildren. Mathematics educators should instead approach their efforts with humility and flexibility to recognize that initial plans might require reimagining.

And finally, the noise that hindered the SMSG's efforts in the long term offers a word of caution for eager reformers of the present. In particular, individuals involved in mathematics education must remember the investments so many subsets of the American population hold in the ways in which students learn mathematics. Though surely impossible to satisfy everyone, keeping in mind various parties' stake in mathematics education might help create reforms that meet with approval instead of crippling criticism. Heeding this guidance might help mathematics education reformers of the future create lasting change in ensuring all of the nation's students receive a high-quality mathematics education.

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