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# Stress-Lifetime Joint Distribution Model for Performance Degradation Failure

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**Abstract**—The high energy density self-healing metallized film pulse capacitor has been applied to all kinds of laser facilities for their power conditioning systems under several stress levels, such as 23kV, 30kV and 35kV, whose reliability performance and maintenance costs are affected by the reliability of capacitors. Due to the costs and time restriction, how to assess the reliability of highly reliable capacitors under a certain stress level as soon as possible becomes a challenge. Accelerated degradation test provides a way to predict its lifetime and reliability effectively. A model called stress-lifetime joint distribution model and an analysis method based on accelerated degradation data of metallized film pulse capacitors are described in this paper. Also described is a method for estimating the distribution of time to failure. The estimators of the unknown parameters in the model are given respectively. Both the failure probability density function (pdf) and the cumulative distribution function (cdf) can be presented by the model. Based on these estimators and the pdf/cdf, the reliability model of metallized film pulse capacitors is obtained. According to the reliability model, the probability of capacitors under stress 23kV that survive to 20000 shots is presented. Then the Kolmogorov-Smirnov test is performed to validate the model. The result shows that the reliability of capacitors under a certain stress level can be assessed as soon as possible by using the model, once the estimators of the unknown parameters in the model are obtained.

**Keywords**—stress-lifetime joint distribution model; accelerated degradation data; metallize film pulse capacitor; reliability

## I. INTRODUCTION

A high power pulsed energy system, which consists of many energy-storage capacitor units, is the key component of many testing facility for complicated science projects. At present, thousands of capacitors with high specific storage in parallel under several stress levels, such as 23kV, 30kV and 35kV, supply energy to the facilities of inertial confinement fusion (ICF) [1]. The reliability of these capacitors directly or indirectly affects the reliability performance and maintenance costs of the entire system. Due to the costs and time restriction, how to assess the reliability of highly reliable capacitors under a certain stress level as soon as possible becomes a challenge.

The electrodes of self-healing metallized film pulse capacitor are vapor-deposited metallization directly on the dielectric substrate. Typically the metal is aluminum, or an alloy, and the thickness of the layer is 20-100 nms. When a dielectric breakdown occurs, the high current density in the metallized electrode around the breakdown site will

immediately vaporize the metallization. The breakdown of the dielectric results in the loss of a small amount of electrode. The only permanent change in the capacitor is the loss of a small amount of capacitance [2, 3]. Expecting self-healing, the deterioration of metal spraying also results in the loss of capacitance [4, 9].

The “self-healing” process of metallized film pulse capacitor is modeled as a counting process. We can consider the process as a homogeneous or non-homogeneous Poisson process  $\{N(t); t \geq 0\}$  with intensity function  $\lambda(t)$ . Within the fixed stress level, we can assume that the capacitance loss of one self-healing is  $y_i$  ( $i = 1, 2, \dots$ ), and  $\{y_i; i = 1, 2, \dots\}$  are independent and identically distributed as normal random variables, that is  $y_i \sim N(\mu_y, \sigma_y^2)$ .

In  $[0, t]$ , the capacitance degradation of capacitor is  $x(t)$ , and

$$x(t) = \sum_{i=1}^{N(t)} y_i. \quad (1)$$

If  $m(t) = \int_0^t \lambda(\tau) d\tau$ , the mean function of degradation is

$$E[x(t)] = m(t) \cdot E[y_i] = \mu_y m(t). \quad (2)$$

The greater the voltage, the faster the capacitance decreases. When the capacitance loss is within specified limit, the performance of capacitor is steady. If the capacitance loss goes beyond the specified limit, the performance of capacitor becomes deteriorative. The degradation of dielectric and the rate of capacitance loss start to increase rapidly [9, 10]. The end of life for metallized electrode capacitor is defined as the maximum percentage loss of the capacitance, typically 5% [5, 6]. For metallized film pulse capacitor, the lifetime is defined as the number of repetitive charge/discharge cycles on the condition that the loss of capacitance is within 5% of the original capacitance.

Now we use 5% of initial capacitance  $c_0$  as the failure criterion  $l$ , that is  $l = 5\% \cdot c_0$ . Then the cdf of capacitor failure can be written as:

$$F(t) = P\{x(t) \geq l\}. \quad (3)$$

## II. ACCELERATED DEGRADATION EQUATION

According to the recent studies [7], [8], [9] and [10], the capacitance loss caused by one "self-healing" is proportional to the energy of electric weaknesses absorption through self-healing. And the energy of electric weaknesses absorption can be given by

$$W \propto d^\alpha V^\beta, \quad (4)$$

where  $d$  is the electrode thickness,  $V$  is the voltage, and  $\alpha$  and  $\beta$  are unknown parameters.

From (2) and (4), we can assume that  $\mu_y = aV^b \propto W$  and the variance  $\sigma_y^2$  is unrelated with the stress. Then we can get

$$\mu_s(t) \triangleq E(x(t)) = aV^b m(t). \quad (5)$$

We call (5) the Accelerated Degradation Equation of the metallized film pulse capacitor.

For convenience, we define  $x_V(t) = \sum_{i=0}^{m(t)} y_{i,V}$ . In the case of stress  $V$ ,

$$x_V(t) \sim N(\mu_V(t), \sigma^2(t)),$$

where  $\mu_V(t) = aV^b m(t)$ , and  $\sigma^2(t) = m(t) \cdot \text{Var}[y_{i,V}] = \sigma_y^2 m(t)$ .

Then according to (3), under stress  $V$ , the cdf of capacitor failure can be written as:

$$\begin{aligned} F_V(t; a, b, \lambda(t), \sigma_y) &= P\{x_V(t) \geq l\} \\ &= 1 - \Phi\left(\frac{l - \mu_V(t)}{\sigma(t)}\right), \quad (6) \\ &= \Phi\left(\frac{a}{\sigma_y} V^b (m(t))^{1/2} - \frac{l}{\sigma_y} (m(t))^{-1/2}\right) \end{aligned}$$

where  $m(t) = \int_0^t \lambda(\tau) d\tau$ .

## III. RELIABILITY ASSESSMENT BASED ON ACCELERATED DEGRADATION DATA

We can use a homogeneous Poisson process to describe the process of metallized film pulse capacitor's self-healing, that is  $m(t) = \int_0^t \lambda d\tau = \lambda t$ .

Assume that there are  $q$  kinds of stress level, that is  $V_1 < V_2 < \dots < V_q$ , and under every stress level there are  $m$  samples for the capacitance degradation test. During the test, periodic measurements of the capacitance were made, typically 1000 shots. The repetitive charge/discharge cycles were continued up to  $n \cdot 1000$  shots, then we can get the following capacitance degradation data

$$\{(x_{ijk}, t_k); i=1, 2, \dots, q, j=1, 2, \dots, m, k=1, 2, \dots, n\}. \quad (7)$$

For the case of  $i$  ( $i=1, 2, \dots, q$ ), using the data  $\{(x_{ijk}, t_k); j=1, 2, \dots, m, k=1, 2, \dots, n\}$ , we can get the estimators of capacitance degradation's mean and variance within the measurement periods:

$$\hat{\mu}_i(t_k) = \frac{1}{m} \sum_{j=1}^m x_{ijk}, \quad \hat{\sigma}^2(t_k) = \frac{1}{m-1} \sum_{j=1}^m (x_{ijk} - \hat{\mu}_i(t_k))^2. \quad (8)$$

The mean function and variance function of capacitance degradation are:

$$\mu_V(t) = a\lambda t V^b \triangleq k_1 V^b t, \quad \sigma^2(t) = \lambda t \sigma_y^2 \triangleq k_2 t,$$

which are linear functions of  $t$ , so the cdf of capacitor failure can be written as:

$$F(t; k_1, k_2, b) = \Phi\left(\frac{k_1}{\sqrt{k_2}} V^b t^{1/2} - \frac{l}{\sqrt{k_2}} t^{-1/2}\right). \quad (9)$$

From  $\{(\hat{\mu}_i(t_k), \hat{\sigma}^2(t_k)); k=1, 2, \dots, n\}$ , we can get the estimators of  $k_1 V^b$  and  $k_2$  with the given stress  $V_i$  by linear regression, further we can get the estimators of  $k_1$  and  $b$ . By substituting the estimators of  $k_1$  and  $b$  into (9), the metallized film pulse capacitor's failure model is established. We call (9) the Stress-lifetime Joint Distribution Model.

## IV. MODEL VALIDATION

After obtaining the estimators of  $k_1$ ,  $k_2$  and  $b$ , we can assess the reliability of capacitors under a certain stress  $V_0$  by using the stress-lifetime joint distribution model. But we need to further validate the model with a few capacitance degradation data under stress  $V_0$ . Therefore we perform the Kolmogorov-Smirnov test to check whether the capacitance degradation data under stress  $V_0$  come from the normal distribution with  $\mu_0(t) = \hat{k}_1 V_0^b t$  and  $\sigma^2(t) = \hat{k}_2 t$ .

Assume that there are  $m$  samples for the capacitance degradation test under stress  $V_0$ . According to the testing method proposed in the previous section, within the measurement periods, we can get the following capacitance degradation data

$$\{(x_{0jk}, t_k); j=1, 2, \dots, m, k=1, 2, \dots, n\}. \quad (10)$$

Two hypotheses are proposed for the test:

$$H_0 : F(x) = F_0(x), \quad H_1 : F(x) \neq F_0(x),$$

where  $F(x)$  is the empirical cdf of the capacitance degradation data under stress  $V_0$ ,  $F_0(x)$  is the normal cdf with  $\mu_0(t) = \hat{k}_1 V_0^b t$  and  $\sigma^2(t) = \hat{k}_2 t$ .

In the test, the test statistic is [11]

$$\begin{aligned} D_m &= \sup_{x_{0jk}} |F_0(x_{0jk}) - F(x_{0jk})| \\ &= \max \left\{ \left| F_0(x_{0jk}) - \frac{j-1}{m} \right|, \left| F_0(x_{0jk}) - \frac{j}{m} \right| \right\}, \quad (11) \end{aligned}$$

where  $j=1, 2, \dots, m$ ,  $k=1, 2, \dots, n$ .

Then, we compare  $D_m$  with  $d_\alpha$ , which is the cutoff value for determining if  $D_m$  is significant. If  $D_m < d_\alpha$ , the test fails to reject the null hypothesis  $H_0$ , namely the capacitance

degradation data under stress  $V_0$  come from the normal distribution with  $\mu_0(t) = \hat{k}_1 V_0^b t$  and  $\sigma^2(t) = \hat{k}_2 t$ .

## V. DATA ANALYSIS

We will analyze the testing data of one kind of metallized film pulse capacitor which can be worked under several stress levels in the ICF facility. This kind of capacitors is tested under three stress levels (voltage),  $V_0 = 23\text{kV}$ ,  $V_1 = 30\text{kV}$  and  $V_2 = 35\text{kV}$ . The testing data under stress  $V_0$  are used to validate the reliability of capacitors under stress  $V_0$  which is assessed by the stress-lifetime joint distribution model. The failure criterion of capacitor is  $l = 2.8\mu\text{F}$ . Under stress  $V_0$  there are 8 capacitors tested for performance degradation, and under stresses  $V_1$  and  $V_2$  there are 4 capacitors tested respectively. The testing data under stresses  $V_1$  and  $V_2$  are shown in Fig. 1.

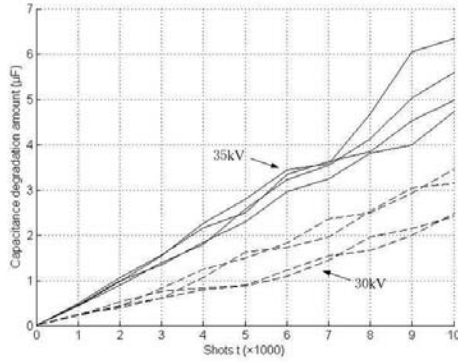


Figure 1. Accelerated degradation data of metallized film pulse capacitors

Using the accelerated degradation data of the capacitors and the fact that  $k_1 V_1^b = r_1$  and  $k_1 V_2^b = r_2$ , we obtain the estimators of  $r_1$ ,  $r_2$  and  $k_2$ :

$$\hat{r}_1 = 0.2689, \hat{r}_2 = 0.5260, \hat{k}_2 = 0.0312.$$

Because

$$\frac{r_2}{r_1} = \left(\frac{V_2}{V_1}\right)^b,$$

therefore we have:

$$\hat{b} = \frac{\ln(\hat{r}_2/\hat{r}_1)}{\ln(V_2/V_1)} = 4.3525,$$

$$\hat{k}_1 = \frac{\hat{r}_1}{V_1^{\hat{b}}} = 1.0010 \times 10^{-7}.$$

Substitute these estimators into (9), and we obtain the failure cdf of this kind of capacitors under stress  $V_0$ :

$$\begin{aligned} F(t) &= \Phi\left(\frac{\hat{k}_1}{\sqrt{\hat{k}_2}} V_0^{\hat{b}} t^{1/2} - \frac{l}{\sqrt{\hat{k}_2}} t^{-1/2}\right) \\ &= \Phi(0.4786 \times t^{1/2} - 15.8411 \times t^{-1/2}) \end{aligned} \quad (12)$$

Fig. 2 shows the curve of the failure cdf of this kind of capacitors under stress  $V_0$ .

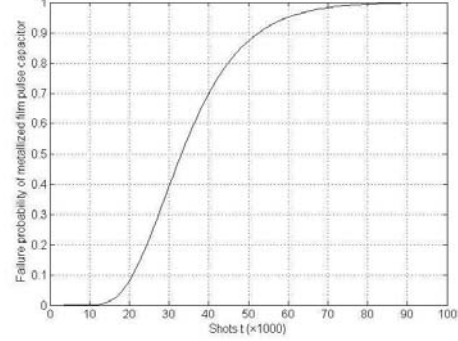


Figure 2. The curve of the failure cdf of capacitors under stress  $V_0$

According to (12), we can calculate the probability of the capacitors that survive to 20000 shots:

$$R(20) = 1 - F(20) = 0.9195.$$

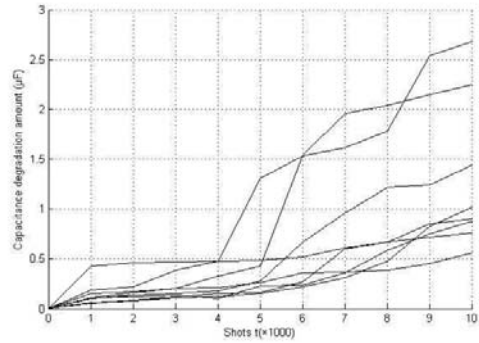


Figure 3. Testing data of metallized film pulse capacitors under stress  $V_0$

Using the testing data under stress  $V_0$  which are shown in Fig. 3 and (11), we obtain the test statistics within the measurement periods. The test statistics are shown in Table I.

TABLE I. THE TEST STATISTICS WITHIN THE MEASUREMENT PERIODS

$t(\times 1000)$	1	2	3	4	5
$D_m$	0.4224	0.3606	0.3197	0.3443	0.3076
$t(\times 1000)$	6	7	8	9	10
$D_m$	0.2533	0.2732	0.2832	0.3440	0.3139

When the significance level  $\alpha = 0.05$ , the cutoff value  $d_{0.05} = 0.45427$ . Obviously, all of the test statistics are less than  $d_{0.05}$ . So the testing data under stress  $V_0$  come from the

normal distribution with  $\mu_0(t) = \hat{k}_1 V_0^b t$  and  $\sigma^2(t) = \hat{k}_2 t$ . Consequently, the model is reasonable, and it can be used to assess the reliability of capacitors under a certain stress level.

## VI. SUMMARIZE AND CONCLUSION

While dealing with performance degradation failure, it is important to analyze the data from accelerated degradation test. This paper describes the basic ideas on accelerated degradation test, and gives the stress-lifetime joint distribution model based on accelerated degradation test. Once the estimators of the unknown parameters in the model are obtained, the reliability of metallized film pulse capacitors under a certain stress level can be assessed as soon as possible by using the model. At last, only a few degradation data under the stress level are needed to validate the result. In the future, we will further compare the stress-lifetime joint distribution model with the conventional reliability model.

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