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AN EXPERIMENTAL INVESTIGATION OF A NEW PRIORITY INDEX FOR JOBS WITH DUE DATES

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ABSTRACT

This paper describes an extension of the critical ratio procedure that is used to assign priorities to jobs that are waiting to be processed on a set of machines. The extended procedure is compared to the conventional procedure in a flow shop setting using a set of test problems. Several measures of schedule performance that are related to job due dates are considered.

INTRODUCTION

Converting the relationship between lead time remaining (LTR) for a job and the time until its due date (TTD) to a single number provides a means of establishing job priorities. One method is to calculate the critical ratio (CR) for each job, defined as CR = TTD/LTR, and assign the highest priority to the job with the smallest CR. If a CR is larger than or equal to one, there is adequate time to complete the job on or before the due date. If the CR is less than one, then the job will be late unless some component of flow time can be reduced. The numerator of the CR is defined as TTD = Due Date - Date Now and so a CR less than zero is associated with a job that is past due. Since the due dates and processing times are generally different for each job, the CRs change over time at different rates. This difference in the rate of change of the CRs is not considered when priorities are assigned in the conventional way.

In this paper we present a procedure for scheduling competing jobs on a set of machines by implicitly considering the rate of change of the critical ratios. This is done by examining the CRs that would result from potential scheduling decisions. These so- called contingent critical ratios (CCR) are evaluated and potential decisions that result in unfavorable schedules at a future stage are avoided. The rudimentary idea of considering these contingent critical ratios was introduced by Gooding and Rudisill [4]. Here we make a more thorough comparison of the conventional technique and the CCR procedure. A set of test problems is used to compare the two techniques under the criteria of average job lateness, maximum job lateness and percentage of jobs completed on time.

THE SETTING

The conventional CR technique and the CCR technique are examined in a flow shop setting. Each of n jobs $J_1, J_2, \ldots, J_n$ are to be processed on m machines $M_1, M_2, \ldots, M_m$ in that order. The processing time for J_i on M_k is denoted t_{ik} and is assumed to include sequence independent set up times for whatever operations J_i requires on M_k. Job preemption is not allowed and it is assumed that due dates and processing times are known in advance. Detailed descriptions of the flow shop problem can be found in [1] and [3]. Also an excellent recent review of the literature on production scheduling including the flow shop problem is available in [5].

Only permutation schedules are considered in the comparison. Permutation schedules are those where the sequence of jobs is identical on each machine and so a schedule of this type can be completely characterized by a single permutation of the job indices. It is well- known that permutation schedules are optimal for all two machine problems with any regular measure of performance and for three machine problems where the objective is to minimize the maximum flow time. In many other studies authors have restricted their attention to schedules of this type [2, 6].

It should be noted, however, that both the conventional CR technique and the CCR technique would presumably perform better if this assumption were not made. This improvement would come at the price of additional complexity required to dynamically apply the technique at progressive stages of the production process. This unrestricted comparison is worthy of investigation, but the purpose here is to make an initial comparison of the CR and CCR techniques.

THE CONTINGENT CRITICAL RATIO TECHNIQUE

Given an n x m flow shop problem, application of the conventional CR technique would yield a vector of n critical ratios. The analogous application of the CCR technique yields an n x n matrix M with undefined elements on the main diagonal. The matrix is formed by computing, for each job i competing for M_1, the CRs that would result if job i is processed first. In other words, the matrix M of contingent CRs is given by $M = [M_{ij}]$ with M_{ij} equal to the CR for job j when job i finishes work on the machine.

The following maximin decision rule is then applied.

1) For each row i, choose job k such that $M_{ik} = \min_j M_{ij}$. Let $M'_i = M_{ik}$. Thus job k is the most critical entry in row i.
2) Assign the highest priority to job L with $M'_L = \max_i M'_i$.

Application of this rule assigns the highest priority to the job that will yield the most favorable "worst-case" CR for the remaining jobs. The second highest job priority is assigned by applying the maximin decision rule to the (n - 1) x (n - 1) matrix formed by considering the contingent CRs for the remaining jobs after the highest priority job has been processed on the machine. The procedure is continued until a complete permutation schedule is determined.

EXAMPLE

Job	t_{i1}	Lead Time Remaining (Includes t_{i1})	Time Until Due Date
1	2	14	27
2	5	13	18
3	9	24	25
4	7	12	17

The first row of the M matrix is constructed by subtracting the processing time for J_1, $t_{11} = 2$, from the TTD of the other jobs and computing the CRs that would result if J_1 was processed first. The CCRs for J_2, J_3 and J_4 are 1.23, .96 and 1.25, respectively. The other rows of the matrix are constructed in a similar fashion. This procedure yields the following 4 x 4 matrix (with undefined diagonal elements).

					M'_i
	-	1.23	.96	1.25	.96
M =	1.57	-	.83	1.0	.83
	1.29	.69	-	.67	.67
	1.43	.85	.75	-	.75

Note that $M'_L = \underset{i}{\text{Max}}\ M'_i = .96$.

Hence J_1 is assigned the highest priority. The next job in the sequence is determined by constructing the following 3 x 3 matrix under the assumption that J_1 will be processed first.

				M'_i
	-	.75	.83	.75
M =	.54	-	.50	.50
	.69	.75	-	.69

Here M'_L = .75 so that J_2 will be processed second. Note that row 1 in the 3 x 3 M matrix corresponds to J_2.

The 2 x 2 M matrix is constructed in the same way as above.

			M'_i
M =	-	.08	.08
	.46	-	.46

Here M'_L = .46 so J_4 is third and J_3 will be processed last. The sequence has been determined to be $J_1 - J_2 - J_4 - J_3$. With this schedule J_1 would be 13 days early and J_2 would be 3 days early. J_3 and J_4 would be 13 and 2 days late, respectively.

The conventional CR technique would yield a schedule of $J_3 - J_2 - J_4 - J_1$ with finish times of 1 day early, 9 days late, 4 days late and 8 days late, respectively. Average job lateness is 3.75 days for the CCR technique and 5.25 days for the conventional CR technique.

The purpose of this example is only to demonstrate the CCR technique. Conclusions about the effectiveness of the technique are reserved for the following section.

RESULTS AND CONCLUSIONS

A set of 200 test problems was used to compare the two techniques in this flow shop setting. The number of jobs, n, ranged from 2 to 10 in increments of 2 and the number of machines, m, ranged from 3 to 9 in increments of 2. There were 10 problems generated for every n, m pair. For each problem, the processing times were generated as randomly chosen integers between 2 and 49 inclusive. The due dates for each job within a problem were generated according to the formula Due Date = 8(n+m) + 72u where u is the value of a standard uniform random variable.

For each of the 200 test problems, two schedules were generated, one using the CR technique and one using the CCR technique. The same schedule was generated by both techniques in 46 cases. Of the 46 problems for which identical schedules were generated, 34 were problems with only two jobs. When n ≥ 4 (there are 160 of these problems), only 12 had identical schedules generated by the two techniques.

Five measures of schedule performance were considered and three of these criteria, number of late jobs, average tardiness and maximum tardiness, are functions of due dates. Average flow time and makespan are not functions of due dates but were also considered. Results are presented in Table 1 for ten 8 job, 9 machine problems, and a summary of the results of the 154 problems that did not yield identical schedules is presented in Table 2.

TABLE 1

RESULTS FOR TEN 8 JOB, 9 MACHINE PROBLEMS

Problem Number	Rule	NLATE	AVGTARD	MAXTARD	MSPAN	AVERAGE FLOWTIME
1	CR	6	85	185	480	313
1	CCR	6	66	176	464	327
2	CR	6	70	176	495	273
2	CCR	6	84	224	518	323
3	CR	7	101	186	465	278
3	CCR	5	54	136	431	231
4	CR	6	78	199	495	264
4	CCR	5	52	136	449	263
5	CR	8	111	234	533	337
5	CCR	8	81	157	466	324
6	CR	7	94	193	497	333
6	CCR	7	95	192	509	336
7	CR	7	126	264	576	315
7	CCR	7	110	247	576	336
8	CR	7	113	212	530	328
8	CCR	7	96	211	529	335
9	CR	6	58	149	477	281
9	CCR	5	40	118	460	275
10	CR	6	81	187	489	284
10	CCR	6	71	221	525	313

TABLE 2

SUMMARY OF RESULTS FOR 154 PROBLEMS

Criterion:	NLATE	AVGTARD	MAXTARD	MSPAN	AVERAGE FLOWTIME
CCR superior	72	121	109	103	70
CR superior	8	28	38	35	84
Tie	74	5	7	16	0

The information in Table 2 suggests that the CCR technique performs better than the conventional CR technique for all criteria except for average flow time where the rules perform about equally well. The results concerning makespan and average tardiness are pleasantly surprising since we did not expect better performance in these areas.

One other result leads us to conclude that the CCR technique can be expected to perform better in most cases than the CR technique with regard to criteria that depend on due dates. A technique was defined to be uniformly dominant for a particular problem it if generated a schedule with strictly better values of number of late jobs, average tardiness and maximum tardiness (all the criteria that depend on due dates). Of the 154 problems with different schedules for the two techniques, the CCR technique generated 59 uniformly dominant schedules (38%) while the CR technique generated only 5 uniformly dominant schedules (3%).

One would expect some improvement when the CCR technique is used due to the additional information provided by the slightly increased computational requirements. At least for the set of test problems considered in this study, the improvement available by using the CCR technique seems to justify the small increase in computational complexity required to implement the technique.

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