

11-2004

## Using the WACC to Value Real Options

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### Recommended Citation

Arnold, Tom and Crack, Timothy Falcon, "Using the WACC to Value Real Options" (2004). *Finance Faculty Publications*. 7.  
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## **Using the WACC to Value Real Options**

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June 03, 2004

***Forthcoming Financial Analysts Journal***

We thank the editor and an anonymous referee. Any errors are our own.

## **Abstract**

### **Using the WACC to Value Real Options**

We present a real option valuation using the weighted average cost of capital (WACC). This is an alternative to risk-neutral real option valuation. Using the WACC involves a marginal increase in mathematical complexity, but it is easy to implement in a spreadsheet, and it is easy to present to management. Our analysis reveals, however, that because the real option valuation is immune to choices of admissible discount rates (as per Arnold and Crack 2003a), the critical issue is correct estimation of volatility, not choice of discount rate. We also point out that the natural and conservative tendency to overestimate risk is anything but conservative in a real option valuation.

*“As we go forward in time through the tree (binomial tree for pricing options), the outcome changes at each node. Because of this, however, the discount rate should also change. In real options, the correct discount rate is determined through the ‘risk neutral’ valuation. When I discuss real options internally, often the initial response is that it’s nothing new. Explaining why it is – because it gets the discount rate right – is not simple, yet it is clearly very important in determining the correct valuation. This problem is exacerbated because most people are given a discount rate by the Treasury group for all calculations, and in general do not question its appropriateness.”*

John Stonier, Airbus Industrie

from Chapter 2 of Copeland and Antikarov (2001)

### **Introduction**

Our quote from John Stonier identifies two problems that obstruct the wider use of real option analysis. The first problem is the lack of understanding of risk neutral valuation; the second is the inability to question a given constant discount rate for a project. We overcome these problems by demonstrating how to use the weighted average cost of capital (WACC) to perform real option valuation. Our WACC valuation is marginally more mathematically complex than risk-neutral valuation, but it is easy to implement in a spreadsheet. Our argument relies on the immunity of option valuation to choice of admissible discount rates; that is, different admissible discount rates must lead to identical option valuations (Arnold and Crack 2003ab). Contrasting the WACC valuation with the risk-neutral valuation leads us to conclude that the core issue regarding correct implementation of real option analysis is not choice of discount rate, but correct estimation of the volatility.

In Section 1, we show how to derive the Arnold and Crack (2003a) generalized one-period option pricing model (GOPOP) immediately from the Cox, Ross and

Rubinstein (CRR) one-period binomial tree model (1979). In Section 2, we use the WACC in the GOPOP model to give an example of non-risk-neutral real option valuation. Section 3 discusses the critical importance of volatility estimation in real option analysis. Section 4 concludes.

### **Section 1: The GOPOP Model**

The appendices of Arnold and Crack (2003a) give several derivations of the GOPOP model. It is a generalized version of the Cox, Ross, and Rubinstein (CRR) (1979) one-period binomial tree model and it allows any admissible discount rate to be used in option valuation. A short derivation is presented here.

Let  $V_0$  and  $V_1$  be the time-0 and time-1 values of an option. Assume that there are two states of the world at time-1, either  $V_1 = V_u$  (i.e., the “up state”), or  $V_1 = V_d$  (i.e., the “down state”). There is an underlying asset with values  $S_0$  and  $S_1$  at time-0 and time-1, respectively. The up and down states for option value  $V$  correspond to the two possible states for underlying asset value  $S$  at time-1: either  $S_1 = S_u = S_0 \times u$ , or  $S_1 = S_d = S_0 \times d$ , where  $u$  and  $d$  are multiplicative growth factors for the underlying asset value. The growth factors  $u$  and  $d$  are usually given as  $u = e^{\sigma\sqrt{\Delta t}}$  and  $d = e^{-\sigma\sqrt{\Delta t}}$ , where  $\sigma$  is the annual volatility of returns to the underlying asset and  $\Delta t$  is the length of the time period in years. Let  $r$  be the annualized continuously compounded riskless rate, and let  $R = e^{r(\Delta t)}$  be the riskless compounding factor, then the CRR model says that Equation (1) holds.

$$V_0 = \frac{1}{R} E_{RN}(V_1), \tag{1}$$

where  $E_{RN}$  is the risk-neutral expectation operator. We may rewrite Equation (1) by

introducing explicitly CRR's risk-neutral probability of the up state  $q = \left(\frac{R-d}{u-d}\right)$ , as in

Equation (2).

$$V_0 = \frac{1}{R} [qV_u + (1-q)V_d] = \frac{1}{R} \left[ \left(\frac{R-d}{u-d}\right)V_u + \left(\frac{u-R}{u-d}\right)V_d \right] \quad (2)$$

Let  $K = e^{k(\Delta t)}$ , where  $k$  is the expected continuously-compounded return (i.e., discount rate) for the underlying asset. We may introduce  $K$  by algebraic manipulation of Equation (2) as in Equation (3).

$$\begin{aligned} V_0 &= \frac{1}{R(u-d)} [(R-d)V_u + (u-R)V_d] \\ &= \frac{1}{R(u-d)} [(K-d)V_u + (u-K)V_d - (V_u - V_d)(K-R)] \\ &= \frac{1}{R} \left\{ \left[ \left(\frac{K-d}{u-d}\right)V_u + \left(\frac{u-K}{u-d}\right)V_d \right] - \left(\frac{V_u - V_d}{u-d}\right)(K-R) \right\} \\ &= \frac{1}{R} \left\{ [pV_u + (1-p)V_d] - \left(\frac{V_u - V_d}{u-d}\right)(K-R) \right\}, \end{aligned} \quad (3)$$

where  $p = \left(\frac{K-d}{u-d}\right)$  is the real world probability of the up state in the underlying asset.<sup>1</sup>

Equation (4), the GOPOP model of Arnold and Crack (2003a), follows immediately from Equation (3).

$$V_0 = \frac{1}{R} \left[ E(V_1) - \left(\frac{V_u - V_d}{u-d}\right)(K-R) \right], \quad (4)$$

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<sup>1</sup> It is simple algebra to show that if  $E(S_1) = pS_u + (1-p)S_d$ ,  $S_u = S_0u$ ,  $S_d = S_0d$ , and  $E(S_1) = KS_0$ , then the real world probability of the up state is  $p = \left(\frac{K-d}{u-d}\right)$ .

where  $E$  is the real world expectation operator.<sup>2</sup> The GOPOP model in Equation (4) holds for any admissible  $K$ . Admissible  $K$  need only satisfy  $d < K < u$ , or equivalently,  $-\sigma/\sqrt{\Delta T} < k < \sigma/\sqrt{\Delta T}$ . Each admissible  $k$  produces an identical option valuation because the discount rate  $k$  determines the probability  $p$ , and by construction  $k$  and  $p$  offset each other to leave the option value unchanged. Note that in the special case when  $k = r$ , or equivalently when  $K = R$ , the GOPOP model in Equation (4) reduces to the risk-neutral CRR model in Equation (2). We present a numerical example using  $k = WACC$  in Section 2A.

Note that the discount rate  $k$  and the discount factor  $K$  are used to calculate the probabilities  $p$  appearing in Equation (3), and they drive the certainly equivalent adjustment in the GOPOP model in Equation (4), but they are not the discount rate and discount factor respectively for the option, but rather, for the underlying. Setting  $k$  to a particular value does, however, determine the expected return on the option,  $k_v$ , with respect to the probabilities determined by  $k$ , but we leave discussion of that for Section 2B.

## **Section 2: Using the WACC to Value a Real Option**

### **A. Numerical Example:**

Recklessly using the WACC to perform Net Present Value (NPV) project valuations can certainly lead to trouble. Suppose for example that a firm having a WACC of 12% can invest in two projects: (A) purchase \$10,000,000.00 worth of Treasury securities over the next ten years or (B) purchase \$10,000,000.00 worth of

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<sup>2</sup> A mathematically similar adjustment that produces a certainty equivalence relationship appears in Hodder, Mello, and Sick (2001).

lottery tickets over the next ten years. Project (A) is very safe and probably does not deserve a discount rate as high as the 12% WACC; Project (B), however, probably deserves a discount rate much higher than the 12% WACC.

Essentially, the WACC is only appropriate for a project NPV when the risk of the project is equivalent to the risk of the firm. As Stonier points out in the opening quote, through risk neutral valuation, real option analysis “gets the discount rate right.” The problem is that it can then generate a debate about risk neutral pricing *per se*. Our alternative is to go with real option valuation, but to use a dictated discount rate, e.g. the WACC or the WACC adjusted for risk, to produce a correct real option valuation and thereby avoid issues with risk neutral pricing.

Our WACC calculation is marginally more complicated than risk-neutral pricing, but that is a low price for appeasing “risk neutral valuation disbelievers” whilst retaining the real option method. The following numerical example using the WACC illustrates that the additional mathematical complexity is minor and that the option valuation produced is identical to that from a risk neutral valuation.

Suppose an oil field can be leased for one year at a cost of \$50,000.00. The oil in the ground is currently worth \$1,000,000.00 but would cost \$1,100,000.00 to extract—a sure loss. Is it worth purchasing the lease now to have the option to extract the oil in the event of a future oil price rise?

Assume that extraction costs will increase next year to \$1,150,000.00 (this will be the strike price for the real option analysis). Given an annual volatility for returns to oil prices of 20%, the GOPOP model (Equation (4)) using a WACC of 15% for  $k$  and a risk free rate  $r$  of 5% produces an option price of \$39,223.57 as in Equation (5).

$$\$39,223.57 = 0.95123 \left\{ \$60,839.93 - \frac{(\$71,402.75 - \$0.00)}{(1.22140 - 0.81873)} (1.16834 - 1.05127) \right\} \quad (5)$$

The option value obtained from taking the lease is less than the \$50,000.00 cost of the lease. Thus, the firm should not purchase the lease.

For the risk neutral valuation, we need only substitute the risk free rate  $r$  for  $k$  in the GOPOP equation. Be sure to remember that this changes the probabilities in the expectation operator, producing Equation (6).

$$\$39,223.57 = 0.95123 \left\{ \$41,234.61 - \frac{(\$71,402.76 - \$0.00)}{(1.22140 - 0.81873)} (1.05127 - 1.05127) \right\} \quad (6)$$

The WACC and risk-neutral real option valuations are identical, as asserted.

### B. A Clarification

As mentioned implicitly at the end of Section 1, in our example the WACC is not being used as the option's discount rate. Rather, the WACC enters the GOPOP model as the discount rate  $k$  for the underlying asset, and not as the discount rate  $k_v$  for the option.

The discount rate for the option in our example can be found by algebraically rearranging Equation (7) to yield Equation (8):

$$E(V_1) = V_0 e^{k_v(\Delta t)} \quad (7)$$

$$k_v = \ln \left[ \left( \frac{E[V_1]}{V_0} \right)^{\frac{1}{\Delta t}} \right] = \ln \left[ \left( \frac{\$60,839.93}{\$39,223.57} \right)^{\frac{1}{1}} \right] = 43.8968\% \quad (8)$$

We can also find  $k_v$  without explicit calculation of  $V_0$ , as in Equation (9).

$$k_v = r + \ln \left[ \left( \frac{E(V_1)}{E_{RN}(V_1)} \right)^{\frac{1}{\Delta t}} \right] = 0.05 + \ln \left[ \left( \frac{\$60,839.93}{\$41,234.61} \right)^{\frac{1}{1}} \right] = 43.8968\% \quad (9)$$

The discount rate for the option is much higher than the discount rate for the underlying asset because the option is a leveraged investment in, and is thus riskier than, the underlying asset.

If we move to a multi-period binomial tree by breaking the life of the option up into smaller time periods, the option's discount rate  $k_V$  changes from period to period; increasing (decreasing) as the option becomes more out-of-the-money (in-the-money). This is explored in detail in Arnold and Crack (2003a) in their generalized multi-period option pricing model (GEMPOP).

Unlike the option's real world discount rate  $k_V$ , the underlying asset's real world discount rate  $k$  does not change from period to period in a multi-period tree. This “stability” makes  $k$  a prime candidate for a dictated constant discount rate. However, making  $k$  equal to the WACC is not the same as making  $k_V$  equal to the WACC. If the goal is to value the real option with the WACC as the discount rate for the real option (i.e.  $k_V$  equals the WACC),  $k$  needs to be adjusted to produce a  $k_V$  equal to the WACC.

So, what discount rate  $k$  would give us a WACC of 15% for  $k_V$  in our one-period tree? We may rearrange Equation (10) to get Equation (11) for  $k$  in terms of  $k_V$ .

$$V_0 * e^{k_V \Delta t} = p(V_u) + (1-p)V_d = V_d + p(V_u - V_d) = V_d + \left[ \frac{e^{k(\Delta t)} - d}{u - d} \right] (V_u - V_d) \quad (10)$$

$$k = \ln \left\{ \left[ \frac{(V_0 * e^{k_V \Delta t} - V_d)(u - d)}{(V_u - V_d)} + d \right]^{\frac{1}{\Delta t}} \right\} \quad (11)$$

In our example,  $k_v = 15\%$  implies via Equation (11) that  $k = 7.2997\%$ . Using  $k$  equal to  $7.2997\%$  produces an identical option value to that already calculated, as shown in Equation (12).

$$\$39,223.57 = 0.95123 \left\{ \$45,571.29 - \frac{(\$71,402.76 - \$0.00)}{(1.22140 - 0.81873)} (1.07573 - 1.05127) \right\} \quad (12)$$

Do note, however, that in a multi-period tree, the underlying discount rate  $k$  would have to differ at each period if it is to maintain a constant WACC of  $15\%$  per annum for  $k_v$ . Forcing  $k$  to change in this fashion from period to period so as to maintain a constant  $k_v$  equal to the WACC, though perfectly feasible, seems unduly artificial to us.

We now ask the following question. If real option valuation is immune to assumptions about the discount rate, then questions about the legitimacy of risk neutral valuation are a non-issue, so what is the critical parameter for real option valuation?

### **Section 3: The Critical Issue is Volatility**

Suppose management has a clear idea about the success of an attractive project: there is a  $10\%$  chance of extremely good performance, and a  $90\%$  chance of good performance. Then  $p = 0.10$  is known, and performance is extremely good, and good respectively in our two states of the world. Suppose that management also has estimated values for the possible future performance of the project (i.e.  $V_u, V_d$ ), and consequently,  $E[V_I]$  is known. What management wants to know is whether it is worth investing in the project (i.e. is the project value greater than the cost).

In this case, because of management's expertise, many of the parameter values for the option model (GOPOP or CRR) are already known. The only missing parameter

values are  $k$ ,  $r$ ,  $u$ , and  $d$ . We can unwind  $p$  to find  $k = \ln\left\{ [p(u-d) + d]^{\frac{1}{\Delta t}} \right\}$ ,  $r$  is a matter of record, and  $d$  is one divided by  $u$ . Thus, we need to find  $u$ . The parameter  $u$  can be found only if the volatility is known or if the current value of the project is known. The current value of the project is what is being sought, so, an estimation of volatility becomes the ultimate missing piece of information.

Thus, even if you know the payoffs to the option one period ahead, and the real world probabilities with which they will occur, you cannot value the option without an estimate of volatility. There are two cases: if you are using risk-neutral valuation (CRR), then you still need the risk-neutral probabilities before discounting at  $r$ , and you cannot find these probabilities without an estimate of volatility; if you are using non-risk-neutral valuation (GOPOP), then even if you have the non-risk-neutral probabilities, you still need the discount rate, and this is a function of volatility. That is why volatility is the critical issue.

Finally, note that if management uses NPV analysis, increased risk (in a CAPM sense) increases the discount rate. Thus, overestimating risk is conservative; it biases you toward rejecting a project due to an overly large discount rate. In real option analysis, however, increased risk (in the volatility sense) has the opposite effect on valuation. If management increases a volatility estimate (due to real or perceived uncertainty), the real option increases in value. Whereas an overestimate of risk may make you too conservative in NPV analysis, an overestimate of risk can make you very daring in real option analysis. A sensitivity analysis with regard to volatility is therefore crucial in making project decisions in real option analysis.

#### **Section 4: Conclusion**

We present a WACC-based real option valuation as an alternative to risk-neutral based real option valuation. Given the opening quote to our paper, it may be that this method is preferable to educating associates and clients about risk neutral pricing.

Deeper inspection reveals, however, that because the real option valuation is immune to the choice of admissible discount rates (as per Arnold and Crack 2003a), management should focus on estimating the volatility parameter correctly because even when management has accurate forecasts of future events, the volatility still needs to be estimated. Management should also realize that an over-estimation of volatility (a natural tendency) is not conservative, but risk-seeking in real option valuation.

#### **Acknowledgements:**

We thank the editor and an anonymous reviewer.

## References:

Arnold, Tom, and Timothy Falcon Crack. 2003a. "Option Pricing in the Real World: A Generalized Binomial Model with Applications to Real Options." Unpublished working paper available on Social Sciences Research Network (April).

Arnold, Tom, and Timothy Falcon Crack. 2003b. "The Irrelevance of Risk-Adjusted Discount Rates in Pricing Options: An Intuitive Approach." Unpublished working paper.

Copeland, Thomas E., and Vladimir Antikarov. 2001. *Real Options: A Practitioner's Guide*. New York, NY: Texere.

Cox, John C., and Mark Rubinstein. 1985. *Options Markets*. Englewood Cliffs, NJ: Prentice-Hall, Inc.

Cox, John C., Stephen Ross, and Mark Rubinstein. 1979. "Option Pricing: A Simplified Approach." *Journal of Financial Economics*, vol. 7 (September): 229–263.

Hodder, James E., Antonio S. Mello, and Gordon S. Sick. 2001. "Valuing Real Options: Can Risk-Adjusted Discounting Be Made to Work?" *Journal of Applied Corporate Finance*, vol. 14 (Summer):90–101.