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Using GMM to Flatten the Option Volatility Smile

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Abstract

By using an over-identified Generalized Method of Moments (GMM) estimation procedure with careful consideration for data biases existing in the previous literature, parameters are estimated for a stochastic volatility jump diffusion option pricing (SVJ) model. The estimated parameters indicate a statistically significant highly negative infrequent jump process in the underlying security return distribution consistent with market crashes. When comparing to a stochastic volatility (SV) option pricing model, the SVJ is more robust but not always the superior model. The robustness of the models is further gauged by evaluating performance up to a year beyond the estimation data.

Keywords: Option Pricing, Stochastic Volatility, Jump-Diffusion, Generalized Method of Moments (GMM)

JEL: G12, G13

Section 1: Introduction

Understanding the statistical properties of financial securities has long been a critical component of financial economics. When multiple liquid options exist for a security, the option market generates more information about the underlying security than the single data point provided by the spot market for the security. Consequently, the estimation and statistical testing of option pricing models provides a unique framework for understanding the distributional properties of the option and the option's underlying security. To date, option research indicates that the distribution of equity returns exhibit fat tails (skewness and kurtosis) and is generally not symmetric.

Das and Sundaram (1999) demonstrate that return distributions with stochastic volatility and jump diffusion components incorporate skewness and kurtosis in the underlying security distribution in different ways. The effects of a jump diffusion process are short-term (about three months under plausible parameters) and the effects of stochastic volatility are long-term (several months, even years under plausible parameters). A number of recent papers combine both aspects into option pricing models: Bakshi and Chen (1997), Bates (1996), Bates (2000), Duffie, Pan, and Singleton (2000), and Scott (1997).

Empirical studies by Pan (2001) and Bakshi, Cao, and Chen (BCC; 1997) estimate complex option pricing models and determine that a stochastic volatility jump diffusion (SVJ) model is superior to a stochastic volatility (SV) model. However, given the work of Das and Sundaram (1999), the estimated parameters do not seem plausible for the underlying security (in both cases, the S&P 500 Index) even though the options are priced accurately. For example, a large speed-of-adjustment parameter (κ in equation

(2) and $\kappa + \lambda_y$ in equation (4); next section) allows stochastic volatility to behave like a jump process. With the addition of a jump component, one expects the speed-of-adjustment parameter to decrease in magnitude. The opposite effect happens in BCC. In Pan, the speed-of-adjustment parameter decreases but remains rather large. We believe the reasons for this inconsistency are due to data issues and estimation technique.

This study extends the work of Pan and BCC by estimating parameters for a stochastic volatility jump diffusion option pricing model (SVJ) and a stochastic volatility option pricing model (SV; nested within the SVJ) using S&P 500 options with an extensive cross-section of moneyness with two different maturities. By estimating all of the option prices simultaneously, the procedure incorporates all of the information the option market can provide. Further, we incorporate into our estimation the notion that a correct option pricing model has the same implied volatility for all options relative to term structure and moneyness (i.e. flattening the volatility smile) because different options should not produce conflicting information about the underlying security.

Previous studies do not incorporate such criteria in a statistically testable framework. The parameter estimates in BCC are not statistically testable and may be biased towards fitting in- and at-the money options, which comprise most of their data. The estimation by Pan only incorporates a single maturity class and at most two moneyness classes (in-the-money and at-the-money applied only to an SVJ model). In this paper, the more restrictive criteria produces better (more plausible) parameter estimations and more accurate model comparisons.

In section 2, an SVJ model variant of the Scott (1997) model is presented with its closed form solution and cumulant function (with appropriate appendices). Section 3 explains the data collection. Section 4 discusses the over-identified GMM estimation procedure. Section 5 discusses results and areas of potential future research. Section 6 concludes the paper.

Section 2: The Model

The SV and SVJ models in this study are adaptations of a general equilibrium model of Scott (1997) under the assumption that investors have a log utility function. The SV model is nested within the SVJ model.

The stochastic volatility jump process for the stock index is as follows:

$$\frac{dS}{S} = \left[\alpha + \lambda E\left(1 - e^{-X}\right)\right]dt + \left[\sigma_s \sqrt{y}\right]dw + \left[e^X - 1\right]dp \tag{1}$$

$$dy = \kappa[\theta - y]dt + \left[\sigma_y \sqrt{y}\right]dz \tag{2}$$

where:

$E[dw, dy] = \rho dt$

dp is a Poisson jump process with frequency λ

X is the magnitude of the jump and is normally distributed with mean μ_J and variance σ_J^2

To produce a risk neutralized valuation, the volatility and jump processes are adjusted. $\lambda_y * y(t)$ is brought into the equation as a risk premium determined by the covariability of $y(t)$ with the marginal utility of wealth. Assuming log-utility, λ_y is constant and equal to $\rho\sigma_y\sigma_s$. The jump parameters λ^* (jump frequency) and X^* (jump magnitude) represent the risk adjusted jump parameters. X^* is distributed normally with mean μ_J^* $(= \mu_J - \sigma_J^2)$ and variance σ_J^2. λ^* equals $\lambda[exp(-\mu_J + 0.5*\sigma_J^2)]$ (where $exp(*)$ is the exponential function) based on the expectation of the ratio of the marginal utility of wealth given that a jump has occurred over the marginal utility of wealth given that a jump has not occurred. One should note that in comparing the risk-neutral and non-risk-neutral jump parameters, the risk-neutral jump mean is more negative than the non-risk-neutral jump mean (i.e. $\mu_J^* = \mu_J - \sigma_J^2 < \mu_J$) and when assuming a negative jump mean, the risk neutral jump frequency is larger than the non-risk neutral jump frequency (i.e. $\lambda^* = \lambda[exp(-\mu_J + 0.5*\sigma_J^2)] > \lambda$). This provides some expectations for the regression results later in the paper. Further, σ_s is normalized to one for simplicity. Equations (1) and (2) change in the corresponding manner:

$$\frac{dS}{S} = \left[\alpha + \lambda^* E\left(1 - e^{-X^*}\right)\right]dt + \left[\sqrt{y}\right]dw + \left[e^{X^*} - 1\right]dq \tag{3}$$

$$dy = \left[\kappa\theta - \left(\kappa + \lambda_y\right)y\right]dt + \left[\sigma_y \sqrt{y}\right]dz \tag{4}$$

where:

$E[dw, dy] = \rho dt$

dq is a Poisson jump process with frequency λ^*

X^* is the magnitude of the jump and is normally distributed with mean μ_J^* and variance σ_J^2

The Scott model can also accommodate stochastic interest rates, however, a fixed interest rate is assumed in this study. Let r be the continuous fixed risk-free rate for a zero coupon bond with maturity T and a par value of one. Consequently, at time t (where $t < T$), the price of the bond is $exp(-r[T - t])$.

Let $C(S(t), y(t), t, T, r, K)$ represent an option price based on the above processes with a maturity T and a strike price of K. The given option pricing model must satisfy the following partial differential equation.

$$C_t + \frac{1}{2} y S^2 C_{ss} + \frac{1}{2}\sigma_y^2 y C_{yy} + \rho \sigma_y y S C_{Sy}$$

$$+ [r - \lambda^* E(e^{x^*} - 1)] SC_s + (\kappa\theta - \kappa y - \lambda_y y) C_y - rC \tag{5}$$

$$+ \lambda^* E[C(S(t)e^{x^*}, y(t), t, T, r, K) - C(S(t), y(t), t, T, r, K)] = 0$$

where subscripts denote partial derivatives

For a European-style call option, the pricing formula must also satisfy the boundary condition *C(S(T), y(T), T, T, r, K) = max [0, S-K]* as t approaches expiration. As with the SV model of Heston (1993), a closed-form solution for the call option exists by applying Fourier inversion to the characteristic function generated from the underlying security's probability distribution (see Scott (1997) for details regarding the evaluation of the Fourier inversion). Scott derives the appropriate characteristic functions, Φ_1 and Φ_2 (the adapted versions are available in Appendix A of this paper) yielding the solution:

$$\begin{aligned} C[S(t), y(t), t, T, r, K] &= S(t)\left[\frac{1}{2} + \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_1(u)\frac{e^{-iu\ln K}}{iu}\,du\right] \\ &- K\,e^{-r(T-t)}\left[\frac{1}{2} + \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_2(u)\frac{e^{-iu\ln K}}{iu}\,du\right] \end{aligned} \tag{6}$$

By removing the jump process, a stochastic volatility (SV) option pricing model nested within the SVJ model emerges. A put option version of the formula is created using Put-Call Parity.

Statistical moments relative to the underlying security return process and the volatility process are computed from the associated cumulant function of the joint distribution. The applicable cumulant function for this study is available in Appendix B and is used to generate moment conditions for the GMM estimation discussed later in the paper.

Section 3: The Data

Three sources of data are necessary for this analysis: a proxy for the risk-free interest rate, the Standard and Poors' 500 Index Option (SPX) with synchronous index level, and the dividends for the Standard and Poors' 500 Index (S&P 500). The data sources are as follows:

- Risk-free interest rate: *The Wall Street Journal* (1987 through 1992)
- SPX option with synchronous index level: magnetic data tape available through the Chicago Board Options Exchange
- S&P 500 dividends: *S&P 500 Information Bulletin* and *CRSP* for 1987

A weekly series of data is created using observations for only Wednesdays of a given week. Wednesday is chosen (as with many other studies) to avoid any potential "day of the week" effects and because holidays are few. Should data (SPX or risk free rate) not be available for a given Wednesday, Tuesday data is used (one occurrence) or Thursday data is used (four occurrences). However, portions of data are genuinely missing: between 9/16/92 and 10/7/92 and are not available from other sources. Table 1 displays the availability of given days of the week within the sample and also display the average call and put option volume relative to the day of the week. Relative to average volume, Fridays tend to be rather active with Mondays relatively inactive. However, the transaction data in Table 1 only includes trading up to 3:00 p.m. (CST) because most empirical investigations exclude trades at the close of the session, 3:15 p.m. (CST). By omitting the closing trades, the true average volume on a given day is under-estimated.

==== Insert Table 1 here ======

The risk free rate is determined as a weighted average between Treasury bills that straddle the option expiration date. This risk free rate is used as a model input and to present value any dividends that occur before option expiration. The dividends are reported daily in the *S&P 500 Information Bulletin* after 1987. To estimate the 1987 dividend stream, the difference between the daily *CRSP* value-weighted index level with and without dividends is taken. The dividends are accumulated for the quarter and compared to S&P 500 Index quarterly dividend reports. A "multiplier" is created by taking the S&P 500 actual quarterly accumulated dividend over the *CRSP* value-weighted quarterly accumulated dividend. This multiplier is then applied to the *CRSP* daily dividend stream to adjust it to the S&P 500 dividend stream. As a check for robustness, the procedure is also evaluated against data when daily dividends are available and found to be robust (results not reported).

On a given day of data, SPX option quotes and the contemporaneous S&P 500 Index level (adjusted for dividends in a manner similar to BCC (1997)) for a given maturity are taken and sorted into "moneyness" categories based on a "moneyness ratio" defined as the dividend-adjusted index level over strike price. There are seven moneyness categories based on the following moneyness ratio values: greater than or equal to 1.09, 1.06 to 1.09, 1.03 to 1.06, 1.00 to 1.03, 0.97 to 1.00, 0.94 to 0.91, and less than or equal to 0.91. The moneyness category "1.00 to 1.03" is considered to be "at-the-money". Similar to Pan (2001), call and put options are not separated for the analysis in order to provide as complete a dataset as possible. As option quotes appear throughout the day, option quotes of a particular maturity are separated into the seven different moneyness categories and replace existing stale quotes. As a consequence, each moneyness category for a particular maturity will only have a single representative current option quote (assuming one exists) at a given point in time.

For this study, only the two nearest to maturity options that are greater than nine days to expiration are gathered. More precisely, the maturity classes become: 10 to 38 days to expiration and 38 to 129 days to expiration (the latter options are primarily less than 60 days to maturity). Further, for each of the two maturities, only one set of quotes contained in the moneyness categories are taken to represent the given day's data. Thus, a given day's data consists of fourteen quotes representing two option maturities and seven moneyness categories. A particular effort is taken to select the fourteen quotes with the least amount of time between the oldest quote and the newest quote. All quotes are taken between 11:00 a.m. and 3:00 p.m. (CST) with most quotes occurring between 2:00 p.m. and 3:00 p.m. The average time between the oldest and newest quote is 30 minutes.

Before extracting data, standard arbitrage conditions are met and opening/closing quotes are excluded. Over the sample period, not all moneyness/maturity classes are represented, as seen in Table 2. The data from 1987 through 1991 is used to estimate parameters for a given option pricing model. A separate estimation is also performed without the year 1987 to prevent a bias relative to the 1987 Crash. The remaining year, 1992, provides an out-of-sample analysis for the different models under different parameter estimates.

===== Insert Table 2 here ========

By allowing as equal as possible a representation of moneyness/maturity classes, moment conditions implemented in the GMM parameter estimation force the volatility across all classes to be the same without a bias towards fitting one particular group of options. The degree to which volatility is not the same relative to all option classifications is statistically tested using the "goodness-of-fit" achieved by a given model. This element of equal option representation for estimation purposes distinguishes this study from previous studies.

BCC's estimation data for call options (as reported in Table 1 of BCC) is dominated by in-the-money options (49% of the data) followed by at-the-money options (28% of the data). Because a minimum squared error criteria is used to parameterize a given model, the domination of a particular moneyness class (or maturity class) within the data skews the parameter estimates towards fitting those particular options.

Pan uses primarily 15 to 30 day at-the-money options (one option drawn randomly between a 10:00 and 10:30 a.m. (CST) window every five trading days to generate a time series) for estimation and in one case uses in-the-money and at-the-money options. Pan combines call options and put options (converted to call options) and uses an average of the index level for the given day instead of a simultaneously reported index level. In the one model estimation that uses both in- and at-the-money options, there is an equal representation between the two moneyness classes. However, neither out-of-the-money options nor a second maturity class are considered.

Section 4: The Estimation of Model Parameters

A. Statistical Testing:

The two models estimated/tested are an SVJ model and an associated SV model nested within the SVJ model (Scott (1997)). Operationally, the SVJ model in its risk neutral form requires the estimation of nine parameters. However, there is also the ability to estimate two non-risk neutralized parameters relative to the jump frequency and the jump size mean (λ and μ_J from Section 1). By adding these two parameters to the estimation, a total of eleven parameters are estimated for the SVJ model. Eliminating the jump diffusion from the model (yielding an SV model), only six parameter estimates are necessary.

An over-identified Generalized Method of Moments (Hansen (1982)) (GMM) estimation corrected for auto-correlation and heteroskedasticity (Newey-West procedure (1987A)) is used to determine and test the parameter estimates for both models over a weekly data series of SPX options from 1987 to 1991. A second estimation is performed for the same data series excluding 1987 (the Crash year) to be consistent with prior research.

The GMM framework allows a Chi-square "goodness of fit" test for the null hypothesis of the given model conforming to the data. The number of degrees of freedom for this statistic is the difference between the number of moment conditions less the number of parameters estimated assuming over-identification (discussed later in this section). This test is rather powerful if one can reject the null hypothesis, but not very powerful when one accepts the null hypothesis. In particular, the researcher can "game" the statistic (see Khan and Zhang (1999)) by using additional (possibly even non-applicable) moment conditions to increase the statistic's degrees of freedom causing acceptance of the null. By including the non-risk neutral jump parameters in this study, two degrees of freedom are actually lost biasing against acceptance of the null.

Additional tests are performed to determine the statistical significance of individual parameters. These are standard t-tests relative to the null hypothesis of the parameter being zero. Further, by considering the SV model as a restricted version of the SVJ model, Newey and West (1987B) develop a Chi-square statistic for the null hypothesis that the restricted model conforms to the data (referred to as the "D-statistic").

The GMM estimation is very dependent upon the moment conditions that are theoretically zero under the correct parameterization of the model. Consequently, the selection of moment conditions must be judicious. However, before discussing the selection of moment conditions, a discussion about estimation biases is necessary.

Poteshman (2000), Bates (1996,2000), and Hilliard and Reis (1999) all discuss some aspect of multiple parameterizations conforming to a given set of data or of time series data being too short for the underlying asset for proper estimation. Some of the discussion is relative to using a least squared error approach like BCC (1997) and is not applicable to this estimation. However, some of the discussion is relative to more sophisticated estimation techniques such as maximum likelihood and is applicable to a GMM estimation.

In particular, moment conditions that only involve mean, variance, skewness, and kurtosis relative to the underlying security are subject to multiple parameterizations conforming to the data. Further, additional moment conditions based upon the mean and variance of the volatility process and the correlation between the volatility process and the return process also suffer from possible multiple parameterizations. Pan (2001) performs a GMM estimation that relies greatly upon moment conditions of this type. However, using a Monte-Carlo comparison for an SVJ model (see appendix E.3 of Pan), Pan's parameter estimations appear to be robust for the short maturity options used in the study. Given the short maturity length of the options in Pan's study (making the options close to being priced at intrinsic value; this is also the reason Pan justifies discarding an adjustment for dividends), the robustness of Pan's Monte-Carlo results may not apply to the longer maturity options in this investigation.

To address the issue of multiple parameterizations, additional moment conditions are created in this study based on the idea that an option price generated by a particular parameterization of a given model is equal to the actual option price within a bid ask spread. By forcing a given parameterization to fit actual option data across a span of moneyness and maturity classes simultaneously (and assuming a flat volatility smile), the mitigation of multiple solutions appears to be possible.

However, there is still the possibility that a given model may be incorrect and produces multiple parameterizations as a result. The use of out-of-sample data to determine the stability of a given set of parameters provides some evidence that a given set of parameters is correct. In addition, applying the results of Das and Sundaram (1999) as to the appropriateness of a given parameter estimate provides further criteria.

B. Development of Moment Conditions:

The GMM estimation procedure in this study is related to the "Implied-State GMM" or IS-GMM estimation performed in Pan (2001). The 10 to 38 day option series for a moneyness ratio between 1.00 and 1.03 is used to imply a volatility time series for a given set of model parameters. More specifically, it is the quote midpoint that is used for implying the option's volatility. This series of implied volatilities is considered the correct volatility for all options (i.e. the volatility smile will be forced to be flat across moneyness and maturity). The synchronous S&P 500 Index level relative to this option is considered to be the time series for the underlying index (whereas, Pan uses a daily average of the index).

As noted in Pan, the model parameterization and index time series (as well as other contract specific variables) affect the volatility time series and vice versa. This affects the GMM estimation since both the index and volatility time series are incorporated into moment conditions. In appendix C of Pan, the large sample properties of the IS-GMM estimator are established to be strongly consistent and asymptotically normal with fairly non-restrictive assumptions. In this study, an over-identified GMM estimation is performed based on Pan's IS-GMM estimation technique. The departure from Pan's study is evidenced from the selection of moment conditions and the method of accounting for illiquidity within the option quotes.

Using the cumulant function (available in Appendix B), joint conditional moments of index returns and the volatility of index returns are generated. The term ΔlnS_t represents the change in the log of the index level between time t and $t - \Delta t$ (i.e. $lnS_t - lnS_{t-\Delta t}$). The cumulant function is in terms of ΔlnS_t and the volatility process $y(t)$.

Using $\Psi(u_1, u_2)$ to symbolize the cumulant function, the following distributional moments are generated (where u_1 refers to ΔlnS_t and u_2 refers to $y(t)$).

$$\frac{\partial \psi(u_1,u_2)}{\partial u_1}\Big|_{u_1=u_2=0} = E(\Delta \ln S_t) = \overline{K}_1(\Delta \ln S_t)$$

$$\frac{\partial^2 \psi(u_1,u_2)}{\partial u_1^2}\Big|_{u_1=u_2=0} = E[\Delta \ln S_t - E(\Delta \ln S_t)]^2 = \overline{K}_2(\Delta \ln S_t)$$

$$\frac{\partial^3 \psi(u_1,u_2)}{\partial u_1^3}\Big|_{u_1=u_2=0} = E[\Delta \ln S_t - E(\Delta \ln S_t)]^3 = \overline{K}_3(\Delta \ln S_t) \qquad (7-13)$$

$$\frac{\partial^4 \psi(u_1,u_2)}{\partial u_1^4}\Big|_{u_1=u_2=0} + 3\left[\frac{\partial^2 \psi(u_1,u_2)}{\partial u_1^2}\Big|_{u_1=u_2=0}\right]^2 = E[\Delta \ln S_t - E(\Delta \ln S_t)]^4$$

$$= \overline{K}_4(\Delta \ln S_t) + 3[\overline{K}_2(\Delta \ln S_t)]^2$$

$$\frac{\partial \psi(u_1,u_2)}{\partial u_2}\Big|_{u_1=u_3=0} = E[y(t)] = \overline{K}_1(y(t))$$

$$\frac{\partial^2 \psi(u_1,u_2)}{\partial u_2^2}\Big|_{u_1=u_3=0} = E[y(t) - E[y(t)]]^2 = \overline{K}_2(y(t))$$

$$\frac{\partial^2 \psi(u_1,u_2)}{\partial u_1 \partial u_2}\Big|_{u_1=u_3=0} = \overline{K}(\Delta \ln S_t, y(t)) = Cov(\Delta \ln S_t, y(t))$$

To apply the theoretically correct statistical moments to the data, the following variables are defined (the "hat" on the volatility variable *y* indicates that the volatility is implied and will vary as it is re-adjusted throughout the estimation of the model parameters):

$$m_1(t) = \varepsilon_t = \Delta \ln S_t - \overline{K}_1(\Delta \ln S_t \mid \hat{y}[t-\Delta t]) \qquad (14-23)$$

$$m_2(t) = \varepsilon_t^2 - \overline{K}_2(\Delta \ln S_t \mid \hat{y}[t-\Delta t])$$

$$m_3(t) = \varepsilon_t^3 - \overline{K}_3(\Delta \ln S_t \mid \hat{y}[t-\Delta t])$$

$$m_4(t) = \varepsilon_t^4 - \overline{K}_4(\Delta \ln S_t \mid \hat{y}[t-\Delta t]) - 3\{\overline{K}_2(\Delta \ln S_t \mid \hat{y}[t-\Delta t])\}^2$$

$$m_5(t) = \eta_t = \hat{y}[t] - \overline{K}_1(y[t] \mid \hat{y}[t-\Delta t])$$

$$m_6(t) = \eta_t^2 - \overline{K}_2(y[t] \mid \hat{y}[t-\Delta t])$$

$$m_7(t) = \varepsilon_t \eta_t$$

$$m_8(t) = \eta_t \hat{y}[t-\Delta t]$$

$$m_9(t) = \varepsilon_t \hat{y}[t-\Delta t]$$

$$m_{10}(t) = (\eta_t \eta_{t-\Delta t}) - \overline{K}_2(y[t] \mid \hat{y}[t-\Delta t])e^{\kappa \Delta t}$$

These ten moment conditions, under the correct model parameterization (assuming the model is correct) have expectations of zero. The first seven conditions are the same conditions employed in Pan (2001). The last three conditions emerge from the dynamics of the volatility process. As mentioned previously, similar to Pan, the at-the-money (shorter maturity in this case) option supplies the volatility process.

Except for one estimation (of an SVJ model), Pan does not incorporate options beyond the at-the-money option. When Pan incorporates an in-the-money option time series, the moment condition becomes the difference between the midpoint of the in-the-money option and the model price divided by the bid-ask spread. The reason for incorporating the bid-ask spread is to compensate for assumed error (based on different sensitivities to volatility and jump risks) in the pricing of these options that is further assumed to be proportional to the bid-ask spread. In this study, the bid-ask spread is assumed to compensate for illiquidity and any effects from the market being segmented from the markets for other options (be it due to informational efficiency, different reactions to a given piece of information, *etc.*). Consequently, the "correct" option price is believed to be within the bid-ask spread regardless of moneyness/maturity.

Incorporating this logic into moment conditions generates thirteen additional moment conditions based on thirteen option time series differentiated by maturity and moneyness. Although there are fourteen option time series available, the at-the-money short-term option is not considered since it is used to generate the volatility time series.

$$m_{11-23}(t, T, MR) = \hat{C}(t, T, MR, \Omega) - C(t, T, MR) \tag{24}$$

where "T" indicates expiration, "MR" indicates moneyness ratio, and Ω indicates a given parameterization of a given model

To incorporate the bid-ask spread, the error between the actual option price and model price is measured in excess of the bid-ask prices (i.e. a model price above the ask price has an error measure of [model price less ask price] and a model price below the bid price has an error measure of {- [bid price less model price]}). The incorporation of these moments makes the estimation process more difficult, but potentially mitigates the problem of multiple solutions for model parameters.

The estimation of the parameters follow the process outlined in Hamilton (1994) and Arnold and Crack (2000). Each moment has an expected value of zero assuming a correct parameterization under a correct model. Given a set of parameters for a given model, *Ω*, a vector *g(Ω)* (column matrix) is defined with elements corresponding to the *j-th* moment condition.

$$g_j(\Omega) = \frac{1}{N} \sum_{t=1}^{N} m_j(t) \tag{25}$$

where N is the number of observations in the time series

The criterion function, $J(\Omega)$, minimized is based on a weighted sum of the square errors of the moment conditions contained in $g(\Omega)$.

$$J(\Omega)=\frac{1}{2}g'(\Omega)W(\Omega)g(\Omega) \tag{26}$$

The weighting matrix, $W(\Omega)$, is first replaced by an identity matrix to find initial parameters for the estimation. After the initial parameters are retrieved, a new estimation begins with the weighting matrix being the inverse of the covariance matrix associated with the moment conditions defined as $L(\Omega)$. An individual element of the matrix is defined as follows:

$$L_{i,j}(\Omega)=E\left[\frac{1}{N}\sum_{t=1}^{N}m_i(t)m_j(t)\right] \tag{27}$$

To prevent the possibility of auto-correlation and cross-correlation, a Newey-West (1987A) correction is applied to the covariance matrix using twenty lagged periods.

The efficient GMM estimator, $\hat{\Omega}_{GMM}$, (Hansen (1982)) is asymptotically normal with the following properties:

$$\sqrt{N}\left[\hat{\Omega}_{GMM}-\Omega_0\right] \sim \text{Normal}(\,0,\, V_{GMM}) \tag{28}$$

$$V_{GMM}=\left[\Gamma'(\Omega_0)L(\Omega_0)^{-1}\Gamma(\Omega_0)\right]^{-1}$$

$$\Gamma_{i,j}(\Omega_0)=E\left[\frac{1}{N}\sum_{t=1}^{N}\frac{\partial m_i(t,\hat{y}_t)}{\partial\Omega_j}-\frac{\partial\hat{y}_t}{\partial\Omega_j}*\frac{\partial m_i(t,\hat{y}_t)}{\partial y_t}\right]$$

where "i" represents a row associated with a particular moment condition and "j" represents a column associated with a particular parameter. The zero subscript represents a vector containing the "correct" parameter specification assuming a correct model. $\hat{y}$ *indicates the volatility implied from the at-the-money short-term option.*

The second term in the element of the Jacobian matrix, $\Gamma(\Omega_0)$, is due to the dependence of the implied volatility upon a given parameterization (it is this aspect of the estimator that produces the IS-GMM estimation of Pan (2001)). Moment conditions m_6 through m_{10} do not have this second term applied sacrificing some efficiency with the estimator.

The standard errors necessary for parameter t-tests are found using the elements of V_{GMM} divided by N.

Section 5: Results

===== Insert Table 3 here =======

In Table 3, parameter estimation, performed with or without 1987, produces an SV model that conforms to the data (significant at the 95% level). All of the SV model parameters are statistically significant at the 99% level. Consistent with Das and Sundaram (1999) and relative to the BCC (1997) estimation for short-term options (under 60 days to expiration), the speed of adjustment parameter in the volatility process is very high (4.36 and 4.99 when viewing the relevant risk-neutral parameter $\kappa + \lambda_y$) allowing for high levels of skewness and kurtosis for relatively short maturities (holding periods). Whether the parameters are plausible relative to the underlying security is somewhat debatable. Pan (2001) finds the speed of adjustment parameter to be 7.1 and 5.3 under different versions of the SV model. Given the even shorter-term nature of the options in the Pan study, the higher parameter values are credible given what the model can effectively fit. However, a robust model that can fit multiple maturity options does not seem to exist in the guise of an SV model due to this dependency of the speed of adjustment parameter on maturity.

Other parameter differences between the SV model in this study and those in the BCC and Pan studies exist as well. The relevant long-run mean estimations for the volatility process are larger in the current study (about twice the size of the Pan estimates and ten times the size of the BCC estimates). The risk adjustment parameter needs to be taken in absolute value to provide a correct comparison to Pan’s estimates (-3.8 and –3.1 compared to 8.6 and 4.4 in Pan with no parameter comparable in BCC). The estimation of the correlation coefficient between the volatility process and the return generating process (approximately -0.43 under either set of estimation data in the current study) is more positive than under Pan (-0.53 and -0.57) and BCC (-0.76). However, the largest parameter estimation difference is relative to the volatility of the volatility process (approximately 2.1 in this study), whether 1987 data is included or not. Pan estimates the parameter to be 0.32 and 0.38, while BCC arrive at an estimate of 0.44.

Relative to the variety of parameter estimators, a clear difference between the studies is the time series over which the given parameters are estimated. Given the 1987 Crash and its effects into early 1988, a high "volatility of volatility" parameter in the current study may be understandable. Aside from this distinction, given the lack of robustness that the SV model appears to display relative to option maturity (see the discussion in the first paragraph of this section) along with the evidence that an SVJ model cannot be restricted to being an SV model statistically (D-statistic in Table 3 significant at the 99% level in rejecting the null hypothesis that given parameter restrictions are not binding), making comparisons in what appears to be an inappropriate model is rather meaningless.

When comparing the SVJ model estimations across the previous studies, the current study (using the risk-neutral estimates measured per annum) provides evidence of highly negative infrequent jumps (λ^* of 0.19, μ_J^* of –0.35, and σ_J of 0.11 with 1987 data; λ^* of 0.24, μ_J^* of –0.24, and σ_J of 0.10 without 1987 data), which is distinctly different relative to previous estimations. BCC estimate the jump frequency as 0.61 with the mean jump being –0.09 with a jump volatility of 0.14. Pan estimates the jump frequency as 12.3 (finding evidence that setting the risk-neutral and non-risk-neutral jump frequency equal is viable) with a mean jump of –0.192 and a jump volatility of 0.04.

Consistent with the notion of risk-neutral pricing (see Section 1 discussion), risk-neutral jump parameter estimates are more frequent and more negative in magnitude relative to non-risk-neutral jump parameter estimates. Although the differences between the risk-neutral and non-risk-neutral jump parameters are consistent with what is expected, the non-risk-neutral jump parameters are highly insignificant in this study. These highly insignificant results are also found in Pan. However, Pan has difficulty in finding a non-risk-neutral jump frequency that is less than the risk-neutral jump frequency. Contrary to Pan's conclusion of setting the non-risk-neutral and risk-neutral jump frequencies equal as being appropriate, this study argues that theoretically the risk-neutral and non-risk-neutral jump parameters are different in a particular fashion. However, the empirical findings are inconclusive although theoretically correct.

Again, estimation data differences may explain some of the parameter estimation differences. Certainly 1987 exhibits a large negative jump. However, given the idea of "crash fear" (Bates (2000)), the current study's relatively infrequent jump estimation seems more appropriate relative to the BCC and Pan jump frequency estimates to a crash scenario. The intensity of a crash, measured by the jump mean parameter, is subject to being risk-neutral (i.e. potentially being more negative) may further favor the current study's estimate. Assuming a market correction to be considered a –10% adjustment. The BCC jump mean estimate of -9%, which becomes less negative in the actual probability measure, makes the BCC estimation not as likely to fit the crash scenario relative to the current study's estimations even though BCC's jump frequency is much lower than that estimated by Pan.

The volatility process within the SVJ models estimated by Pan and BCC yield distinct differences as well. In the BCC estimation, much of the volatility process parameters are unchanged aside from the correlation coefficient (between the volatility process and security return process) becoming –0.52 from –0.76 and the volatility speed of adjustment changing from 1.62 to 3.93. The correlation coefficient compares well with that estimated by Pan (-0.53, which is virtually unchanged relative to the SV model estimations), but the higher speed of adjustment parameter does not compare with Pan's estimate of 6.4. Further, with the incorporation of a jump into an SV model, in theory, the speed of adjustment parameter should decrease as the jump process accounts for some of the skewness/kurtosis. This is not the case in BCC.

Similar to Pan, in the current study, the addition of the jump process takes explanatory power away from some of the skewness/kurtosis sensitive parameters in the volatility process (best exhibited by a lower speed of adjustment parameter within the volatility process). The current study's parameter estimates for the speed of adjustment are 2.48 with a correlation of –0.44, with 1987 data, and 0.94 and –0.66 respectively, without 1987 data. The reduction of the skewness/kurtosis sensitivity in the volatility process upon the inclusion of a jump process is logical. Consequently, the increase in the speed of adjustment parameter by more than two-fold in the BCC estimation does not appear to be valid estimation of the true parameter. Further, the high skewness/kurtosis sensitivity still displayed by the Pan volatility process estimation appears to possibly be the result of the very short maturity option data upon which the parameters are estimated. Pan's hypothesis of other jump processes existing in the data is plausible, but possibly not for longer maturity options and options in general given this study's results.

Finally, the glaringly excessive volatility of volatility parameter estimates in the current study's SV model is more reasonable in the SVJ model, 0.1382 (with 1987 data) and 0.1682 (without 1987 data). Thus, given an "appropriate" model with the estimation scheme introduced in this paper, a better model parameterization over previous studies appears possible. Further, the use of a very over-identified system apparently mitigates the potential for an alternate "more correct" parameterization to exist.

However, many of the volatility process parameters remain statistically insignificant. Perhaps, due to the inclusion of the 1987 Crash and its residual effects in 1988, the jump process estimations pull too much explanatory power away from the volatility process (relative to other data samples). Another possibility is that SV models are better adapted for longer maturity options given their ability to fit skewness/kurtosis over longer holding periods. Consequently an SV model does not substitute for the "perfect" option pricing model, one that is invariant to moneyness and maturity (assuming such a model exists), but it certainly makes for a very pragmatic choice of a model given longer maturity. Neither possibility is explored further in this paper but has the potential for interesting future research.

To further explore the ability of a given model to fit the data, Table 4 provides a comparison between the model price and actual market price under a given parameterization (assuming the at-the-money short-term option midpoint provides the correct volatility; the Black-Scholes model (BS) (1973) is included for comparison). The average absolute pricing errors are reported in excess of the quoted spread.

=== Insert Table 4 here ===========

In viewing the model fit in this manner, the economic significance (i.e. differences in error beyond a 1/8th tick) between using one model instead of another model is evaluated. With or without the 1987 data, the SV model and SVJ model perform better than the BS model (except for the longer-term at-the-money contract, where it is only slightly better than the SVJ model), particularly for options that are not at-the-money. In viewing the near-term options, the SV model fairs better than the SVJ model but never by more than one tick and generally only for "away from the money" options. The same is true for the longer term options except that the SV model performs especially poor when options are close to being at-the-money. Essentially, the SV model has difficulty in explaining options of different maturities simultaneously. The SVJ model has some difficulty as well. However, it produces either the least or next to least amount of average absolute pricing error for every moneyness/maturity classification.

As to including or not including 1987 data, one cannot say that the performance of one set of parameters is consistently superior to the other set of parameters based on the average absolute pricing error. When taking the correlation between the models using the implied volatilities for the short-term at-the-money option, the correlation between the BS model and SVJ model are slightly higher (0.942 with 1987 data and 0.872 without 1987 data) than the respective correlation between the BS model and SV model (0.930 with 1987 data and 0.857 without 1987). Even though both correlations are more pronounced when 1987 data is included, it does not constitute a validation of either set of parameters over the other set of parameters. However, the parameter estimates for the SVJ model that include 1987 data do seem to have "crash effects" in that the volatility process has much fewer statistically significant parameters relative to the other SVJ model parameterization. Based on the latter criteria, 1987 data may justifiably be excluded but there is no justification for its exclusion based on affecting the model's ability to fit the data.

When the average absolute pricing errors are computed using non-estimation period data from 1992 (Table 5), the same conclusion emerges again relative to the inclusion of 1987 data since the performance of either set of parameters is not superior to the other.

======= Insert Table 5 here =======

What is interesting about the out-of-sample performance is the manner in which the SV model begins to fail. Large amounts of pricing error emerge in both sets of maturities indicating the instability of the model. This leads one to conclude that the SV model is only robust on the data on which its parameters have been estimated. The SVJ model maintains its pricing ability for the shorter maturity options (as would be expected based on Das and Sundaram (1999)) but shows signs of failing relative to the longer maturity options, specifically in moneyness categories 1.00 to 1.03 and 0.97 to 1.00. The superiority of the SVJ to the SV model is demonstrated in previous work. However, the robustness of the SVJ model to out-of-sample data for an extended period of time is not examined in the previous literature.

Empirically, the SVJ model is the better model relative to the SV and the BS models. However, it is not uniformly the best model. This can potentially be related to sample size. Using a larger sample is a possibility. Or possibly a change in perspective, if one considers a given piece of information to be reacted upon by options of different maturity in a unique manner relative to maturity (i.e. different informational efficiencies based on maturity), then it is plausible that estimating parameters based on options of different maturity simultaneously is inappropriate.

This is different than the liquidity arguments posed in other papers for excluding certain maturities but related in that liquidity may be indicative of efficiency. The ability of the bid-ask spread in this study to mitigate liquidity issues (and possibly differences in informational efficiency) may be enhanced by separating parameter estimation relative to maturity. Such an exercise may be a better testing venue for both the SV and SVJ models and in fact may demonstrate the SV model to superior under certain conditions/markets. As to the theoretical justification for performing such an investigation, the issue is debatable and is left for future research.

Section 6: Conclusion

This study demonstrates the superiority of the SVJ model relative to the SV model (nested within the SVJ model). The SVJ model parameters are more plausible than in previous studies. This is attributable to the use of an over-identified GMM estimation that incorporates the bid-ask spread and an equally-represented option time series of different maturity and moneyness. The imposed GMM estimation mitigates a number of issues in the evaluation of option pricing models and demonstrates great potential for future testing of option pricing models.

Further, the estimated models are evaluated based on the ability to fit out-of-sample data over an extended period of time. Out-of-sample, the SVJ model tends to be robust relative to shorter maturity options (10 to 38 day maturity) but does not perform as well when pricing longer maturity options (38 to 129 day maturity).

Appendix A: Characteristic Functions

$$\Phi_1(u) = \left[\frac{2\gamma_1 e^{\frac{1}{2}[\kappa+\lambda_y-\gamma_1](T-t)}}{D_1(u)}\right]^{\frac{2\kappa\theta}{\sigma_y^2}} * \exp\{iu[lnS(t)+r(T-t)]\}$$
$$* \exp\left\{\kappa\theta(T-t)S_{21} + \frac{(1-e^{-\gamma_1(T-t)})[\sigma_y^2 S_{21}^2 - 2S_{12} + 2(\kappa+\lambda_y)S_{21}]y(t)}{D_1(u)}\right\}$$
$$* \exp\left\{-iu\lambda^*(T-t)[e^{\mu_J^*+\frac{1}{2}\sigma_J^2}-1] + \lambda^*(T-t)g_1[e^{h_1}-1]\right\}$$

$$\Phi_2(u) = \left[\frac{2\gamma_1 e^{\frac{1}{2}[\kappa+\lambda_y-\gamma_1](T-t)}}{D_2(u)}\right]^{\frac{2\kappa\theta}{\sigma_y^2}} * \exp\{iu[lnS(t)+r(T-t)]\}$$
$$* \exp\left\{\kappa\theta(T-t)S_{22} + \frac{(1-e^{-\gamma_1(T-t)})[\sigma_y^2 S_{22}^2 - 2S_{12} + 2(\kappa+\lambda_y)S_{22}]y(t)}{D_2(u)}\right\}$$
$$* \exp\left\{-iu\lambda^*(T-t)[e^{\mu_J^*+\frac{1}{2}\sigma_J^2}-1] + \lambda^*(T-t)g_2[e^{h_2}-1]\right\}$$

where

$$D_1 = 2\gamma_1 e^{-\gamma_1(T-t)} + (\kappa+\lambda_y+\gamma_1+\sigma_y^2 S_{21})(1-e^{-\gamma_1(T-t)})$$

$$D_2 = 2\gamma_2 e^{-\gamma_2(T-t)} + (\kappa+\lambda_y+\gamma_1+\sigma_y^2 S_{22})(1-e^{-\gamma_2(T-t)})$$
$$* \exp\left\{-iu\lambda^*(T-t)[e^{\mu_J^*+\frac{1}{2}\sigma_J^2}-1] + \lambda^*(T-t)g_2[e^{h_2}-1]\right\}$$

$$\gamma_1 = \sqrt{(\kappa+\lambda_y)^2 + 2\sigma_y^2 S_{11}} \qquad \gamma_2 = \sqrt{(\kappa+\lambda_y)^2 + 2\sigma_y^2 S_{12}}$$

$$S_{11} = \frac{1}{2}\rho^2 + \frac{1}{2}(1-\rho^2)u^2 - \frac{\rho\hat{\sigma}}{\sigma_y}(\kappa+\lambda_y) - iu[\frac{1}{2} + \frac{\rho\hat{\sigma}}{\sigma_y}(\kappa+\lambda_y) - \rho^2]$$

$$S_{12} = \frac{1}{2}(1 - \rho^2)u^2 - iu\left[-\frac{1}{2} + \frac{\rho\hat{\sigma}}{\sigma_y}(\kappa + \lambda_y) \right]$$

$$S_{21} = -\frac{\rho\hat{\sigma}}{\sigma_y}(1 + iu) \qquad S_{22} = -\frac{\rho\hat{\sigma}}{\sigma_y} iu$$

$$g_1 = e^{\mu_J^* + \frac{1}{2}\sigma_J^2} \qquad h_1 = iu(\mu_J^* + \sigma_J^2) - \frac{1}{2}\sigma_J^2 u^2$$

$$g_2 = 1 \qquad h_2 = iu\,\mu_J^* - \frac{1}{2}\sigma_J^2 u^2$$

Note: exp{*} refers to the exponential function.

Appendix B: Cumulant Function

The cumulant function is available through Scott (upon request) and Arnold (1998). The notation has been changed and solutions for the joint distributional moments computed from the cumulant function are available in Arnold (1998). The relevant variables within the cumulant function are "$\Delta \ln S_t$" and "y". The cumulant function is the natural log of the moment generating function where "u_1" refers to "$\Delta \ln S_t$" and u_2 refers to "y_t".

$$\Psi(u_1,u_2) = \frac{2\kappa\theta}{\sigma_y^2}\left\{\frac{1}{2}(\kappa-\gamma)\Delta t + \ln\left[\frac{2\gamma}{D(S_2)}\right] - \frac{1}{2}\rho\sigma_y \Delta t u_1\right\}$$
$$+ y(t-\Delta t)\left\{-\frac{\rho u_1}{\sigma_y} + \frac{\left(1-e^{-\gamma\Delta t}\right)\left[(\kappa-\gamma)S_2 - 2S_1\right] - 2\gamma S_2 e^{-\gamma\Delta t}}{D(S_2)}\right\}$$
$$- \lambda \Delta t u_1\left(1 - e^{-\mu_J + \frac{1}{2}\sigma_J^2}\right) + \lambda\Delta t\left(e^{\mu_J u_1 + \frac{1}{2}\sigma_J^2 u_1^2} - 1\right) + \alpha_1 \Delta t u_1$$

where

$$S_1 = -\frac{1}{2}\left(1-\rho^2\right)u_1^2 - u_1\left(-\frac{1}{2} + \kappa\frac{\rho}{\sigma_y}\right)$$
$$S_2 = -\frac{\rho u_1}{\sigma_y} - u_2$$
$$\gamma = \sqrt{\kappa^2 + 2\sigma_y^2 S_1}$$
$$D(S_2) = 2\gamma e^{-\gamma\Delta t} + \left[\kappa + \gamma + \sigma_y^2 S_2\right]\left(1 - e^{-\gamma\Delta t}\right)$$

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Table 1

Day of the Week Frequency with Average Option Volume

Panel A: 1 to 30 Day Options				
Day of the Week:	**Average Call Option Volume:**	**Average Put Option Volume:**	**Total Average Option Volume:**	**Frequency of Day of the Week:**
Monday	3090	2977	6067	286
Tuesday	3289	3191	6480	306
Wednesday	3250	2963	6213	306
Thursday	3155	3137	6292	301
Friday	3219	3096	6315	292

Panel A: 31 to 60 Day Options			
Day of the Week:	**Average Call Option Volume:**	**Average Put Option Volume:**	**Total Average Option Volume:**
Monday	1627	1500	3127
Tuesday	1669	1651	3320
Wednesday	1592	1789	3381
Thursday	1700	1701	3401
Friday	2033	2257	4290

Volume is measured as the number of contracts traded prior to 3:00 p.m. (CST). Trading actually continues through 3:15 p.m. (CST). The decision to stop collecting volume after 3:00 p.m. is to be consistent with the Bakshi, Cao, and Chen (1997) data criteria of using all outstanding quotes as of 3:00 p.m.

Table 2

Number of Observations Available for each Moneyness Category per Year per Option Maturity

Panel A: 10 to 38 Day Maturity Option Quotes by Year and Moneyness Ratio [a]

Year:	**Moneyness Ratio:**						
	≥ 1.09	**1.06 to 1.09**	**1.03 to 1.06**	**1.00 to 1.03***	**0.97 to 1.00**	**0.94 to 0.97**	**≤ 0.94**
1987	41	51	52	52	52	51	25
1988	49	51	52	52	52	51	48
1989	40	45	52	52	52	48	35
1990	43	52	52	52	52	52	43
1991	49	52	52	52	52	47	31
1992 [b]	49	46	48	49	49	48	25
Total:	271	297	308	309	309	297	207

Panel B: 38 to 129 Day Maturity Option Quotes by Year and Moneyness Ratio

Year:	**Moneyness Ratio:**						
	≥ 1.09	**1.06 to 1.09**	**1.03 to 1.06**	**1.00 to 1.03**	**0.97 to 1.00**	**0.94 to 0.97**	**≤ 0.94**
1987	35	43	52	52	52	52	28
1988	41	51	52	52	52	51	46
1989	37	45	52	52	52	52	30
1990	44	50	52	52	52	52	46
1991	49	48	52	52	52	51	39
1992 [b]	43	44	46	46	46	44	29
Total:	249	281	306	306	306	302	218

[a] Moneyness Ratio = (Dividend-Adjusted Index Level) / (Strike Price)
[b] 1992 only has 49 weeks of observations available
* Indicates the at-the-money option used to provide the volatility series.
Options contained in the moneyness ratio categories can be either call or put options. Put options are transformed to call options via put-call parity.

Table 3

GMM Parameter Estimations Based on Sample Period With Standard Errors

		SVJ Model		SV Model	
Process:	**Parameter:**	**Data: 1987 to 1991**	**Data: 1988 to 1991**	**Data: 1987 to 1991**	**Data: 1988 to 1991**
Underlying Drift:	α_1	0.1446***	0.0907*	0.2011***	0.1441***
		(0.0428)	(0.0514)	(0.0333)	(0.0331)
Stochastic Volatility:	θ	0.0229	0.0640***	0.0490***	0.0486***
		(0.0146)	(0.0207)	(0.0026)	(0.0026)
	λ_y	1.4015	0.8369***	-3.7846***	-3.0926***
		(1.3079)	(0.2479)	(0.5364)	(0.5041)
	κ	1.0776	0.0985	8.1431***	8.0850***
		(0.9626)	(0.2408)	(0.1109)	(0.0754)
	σ_y	0.1382	0.1682	2.1004***	2.0646***
		(0.2399)	(0.1903)	(0.0363)	(0.0392)
	ρ	-0.4379	-0.6637	-0.4273***	-0.4309***
		(0.4790)	(0.4144)	(0.0099)	(0.0089)
	$\kappa+\lambda_y$	2.4791***	0.9355***	4.3585***	4.9924***
		(0.6097)	(0.2204)	(0.4171)	(0.2874)
	$\kappa\theta$	0.0247	0.0063	0.3990***	0.3928***
		(0.0271)	(0.0156)	(0.0222)	(0.0215)
Jump Process:	λ	0.0045	0.0220		
		(7191.11)	(874.07)		
	μ_J	-0.0043	-0.0077		
		(869.24)	(57.567)		
Jump Process (Risk-Neutral):	λ^*	0.1924***	0.2469***		
		(0.0705)	(0.0853)		
	μ_J^*	-0.3469***	-0.2379***		
		(0.0844)	(0.0564)		
	σ_J	0.1084	0.1038*		
		(0.0665)	(0.0583)		
Chi-square:		14.7260[a]	17.0996[a]	19.8819[a]	16.8687[a]
D Statistic:		441.0094[b]	583.3726[b]		

SVJ: Stochastic Volatility Jump Diffusion Model, SV: Stochastic Volatility Model, [a] fail to reject "goodness of fit" at the 95% level, [b] reject the null of restricted model (i.e. SVJ restricted to SV) conforming to the data at the 99% level,* 90% level, **95% level, and ***99% level

$$\frac{dS}{S} = \left[\alpha + \lambda^* E\left(1 - e^{-X^*}\right)\right]dt + \left[\sqrt{y}\right]dw + \left[e^{X^*} - 1\right]dq \text{ with } dy = \left[\kappa\theta - \left(\kappa + \lambda_y\right)y\right]dt + \left[\sigma_y \sqrt{y}\right]dz$$

where: $E[dw, dy] = \rho dt$, dq is a Poisson jump process with frequency λ^*, and X^* is the magnitude of the jump which is normally distributed with mean μ_J^* and variance σ_J^2

Table 4

Estimation Sample Analysis
Average Absolute Pricing Errors** for Each Moneyness Category

Panel A: 10 to 38 Day Option Parameter Estimation (1987 - 1991 Data)

Model:	Moneyness Ratio[a]:						
	≥ 1.09	1.06 to 1.09	1.03 to 1.06	1.00 to 1.03*	0.97 to 1.00	0.94 to 0.97	≤ 0.94
BS	0.4287	0.4411	0.2971	0.0000	0.4175	0.4270	0.2732
SV	0.2906	0.2848	0.2203	0.0000	0.3450	0.2360	0.2010
SVJ	0.3504	0.3525	0.2168	0.0000	0.3159	0.2924	0.2193

Panel B: 38 to 129 Day Option Parameter Estimation (1987 – 1991 Data)

Model:	Moneyness Ratio:						
	≥ 1.09	1.06 to 1.09	1.03 to 1.06	1.00 to 1.03	0.97 to 1.00	0.94 to 0.97	≤ 0.94
BS	1.1348	1.1757	0.8539	0.4632	0.7681	0.8685	0.8081
SV	1.0949	1.0640	0.9283	0.7182	0.7301	0.7039	0.5878
SVJ	1.1278	1.0229	0.7641	0.4691	0.6971	0.7472	0.6891

Panel C: 10 to 38 Day Option Parameter Estimation (1988 – 1991 Data)

Model:	Moneyness Ratio:						
	≥ 1.09	1.06 to 1.09	1.03 to 1.06	1.00 to 1.03*	0.97 to 1.00	0.94 to 0.97	≤ 0.94
BS	0.4314	0.4816	0.3330	0.0000	0.4554	0.4115	0.2016
SV	0.2839	0.2554	0.2025	0.0000	0.3290	0.1942	0.1412
SVJ	0.3301	0.3408	0.2260	0.0000	0.3257	0.2443	0.1357

Panel D: 38 to 129 Day Option Parameter Estimation (1988 – 1991 Data)

Model:	Moneyness Ratio:						
	≥ 1.09	1.06 to 1.09	1.03 to 1.06	1.00 to 1.03	0.97 to 1.00	0.94 to 0.97	≤ 0.94
BS	1.2256	1.2838	0.9635	0.4525	0.8139	0.8581	0.7893
SV	1.0961	1.0370	0.9688	0.7116	0.7260	0.6644	0.5396
SVJ	1.1429	1.0125	0.8111	0.4562	0.7639	0.7602	0.6739

*The implied volatility from the 10 to 38 Day At-the-Money Option (Ratio:1.00 to 1.03) is used as the volatility input for all Moneyness categories for both maturities

**Pricing Errors are only calculated when the Model Price is outside of the Bid and Ask Prices and is always equal to the Model Price minus the Actual Price

[a] Moneyness Ratio = (Dividend-Adjusted Index Level) / (Strike Price)

SVJ: Stochastic Volatility Jump Diffusion Model

SV: Stochastic Volatility Model

BS: Black-Scholes Model

Table 5

Out of Estimation Sample Analysis for the Year 1992:
Average Absolute Pricing Errors[**] for Each Moneyness Category

Panel A: 10 to 38 Day Option Parameter Estimation (1987 - 1991 Data)

Model:	Moneyness Ratio[a]:						
	≥ 1.09	1.06 to 1.09	1.03 to 1.06	1.00 to 1.03*	0.97 to 1.00	0.94 to 0.97	≤ 0.94
BS	0.3191	0.4068	0.4127	0.0000	0.5668	0.3858	0.1732
SV	0.3019	0.5829	0.5187	0.0000	0.3877	0.3690	0.1272
SVJ	0.3467	0.3852	0.2770	0.0087	0.3569	0.1884	0.1880

Panel B: 38 to 129 Day Option Parameter Estimation (1987 – 1991 Data)

Model:	Moneyness Ratio:						
	≥ 1.09	1.06 to 1.09	1.03 to 1.06	1.00 to 1.03	0.97 to 1.00	0.94 to 0.97	≤ 0.94
BS	1.4487	1.6454	1.2066	0.4558	0.9841	0.9355	0.5759
SV	0.9682	1.0308	1.2615	2.0021	1.9036	0.7060	0.7028
SVJ	1.1437	1.0412	0.8492	0.6563	1.0530	0.7607	0.5296

Panel C: 10 to 38 Day Option Parameter Estimation (1988 – 1991 Data)

Model:	Moneyness Ratio:						
	≥ 1.09	1.06 to 1.09	1.03 to 1.06	1.00 to 1.03*	0.97 to 1.00	0.94 to 0.97	≤ 0.94
BS	0.3191	0.4068	0.4127	0.0000	0.5668	0.3858	0.1732
SV	0.2905	0.5650	0.5066	0.0000	0.3848	0.3507	0.1234
SVJ	0.3182	0.3722	0.2778	0.0066	0.3486	0.1827	0.1532

Panel D: 38 to 129 Day Option Parameter Estimation (1988 – 1991 Data)

Model:	Moneyness Ratio:						
	≥ 1.09	1.06 to 1.09	1.03 to 1.06	1.00 to 1.03	0.97 to 1.00	0.94 to 0.97	≤ 0.94
BS	1.4487	1.6454	1.2066	0.4558	0.9841	0.9355	0.5759
SV	0.9642	1.0443	1.2932	2.0477	1.9494	0.7129	0.6836
SVJ	1.1077	1.0284	0.8511	0.6938	1.1510	0.7572	0.5425

*The implied volatility from the 10 to 38 Day At-the-Money Option (Ratio:1.00 to 1.03) is used as the volatility input for all Moneyness categories for both maturities
**Pricing Errors are only calculated when the Model Price is outside of the Bid and Ask Prices and is always equal to the Model Price minus the Actual Price
[a] Moneyness Ratio = (Dividend-Adjusted Index Level) / (Strike Price)
SVJ: Stochastic Volatility Jump Diffusion Model
SV: Stochastic Volatility Model
BS: Black-Scholes Model