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CONTINUOUS REVIEW BILLING DECISION SYSTEMS

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I. Introduction

The subject of this paper is the recurring set of retail billing decisions: the activities whereby the firm initiates the collection of receivables from its charge account customers. A model for this decision making activity is proposed in order to enhance the profitability of the firm's billing information system.

The approach adopted here for the billing decision is well motivated by the very strong analogy with the inventory replenishment decision in production management. The well known economic order quantity (EOQ) provides a decision rule that is essentially an optimal compromise between the working capital costs associated with holding inventory and the administrative costs of entering and receiving orders. In the accounts receivable environment, the "holding cost" is the interest expense incurred in extending credit to the customer, while the "reorder cost" arises from several system operations, including statement preparation, mailing and postage, file updating, and handling customer inquiries. In order to minimize the total system operating costs, it follows that, ceteris paribus, more active accounts should be billed more often
than relatively inactive accounts. Thus, our proposal is that the billing information system be designed to make billing decisions on an account-by-account basis, through continuous monitoring of their respective charge sales activities, rather than automatically billing all accounts at the same frequency -- usually monthly.

II. Description of the Model

The charge sales rate for each customer is assumed to be a known constant. With very little difficulty, it is possible to modify this assumption to reflect upward (downward) sales trends and seasonality factors. Also, recognizing the randomness inherent in forecasting individual account activity, a stochastic model may readily be invoked. In Benishay[1966] for example, the occurrence of credit sales is modeled as a stochastic point process, while the amounts charged are themselves independent random variables. Specifically, we should recommend using the compound Poisson process (see p. 23 of Ross[1970]) to account for this random behavior of an individual customer.

A very crucial assumption is that credit risks are nil. Essentially, this assumption means that every account balance surely shall be paid in full, as a consequence of the billing decision. Thus, credit decisions concerning such matters as credit limits, cash discounts, coercive collection measures, and bad debts are not

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1. As far as we know, no institution has yet to design a billing information system that explicitly incorporates such a model for routine support of billing decisions. Nevertheless, many systems employ control procedures and treat the "exceptional" accounts in a manner according to the prescribed rules of the model.
recognized in the model. The credit function might be integrated
directly into the model by assuming that the proportion of the
outstanding balance paid in the current billing cycle decreases both
with the magnitude of the unpaid balance and with its age. As a
starting point for this integrative task, we should attempt a
synthesis of the statistical sequential decision model, proposed
by Bierman and Hausman[1970] for the credit granting problem, and
the Markov chain model for the accounts receivable aging process,
originally proffered by Cyert, Davidson, and Thompson in their now
classic paper [1962]. We shall leave this difficult area for
future investigation.

Finally, one might argue that there exist a negative feedback
phenomenon, whereby customer purchase behavior is affected by the
duration of the billing cycle. However, we suggest that, so long
as the billing process and the terms of payment are consistent with
the credit agreement between firm and customer, no appreciable
change in her/his charge sales rate will appear. Obviously,
regardless of the billing cycle, if the customer is protected by,
say, a 30-day payment period, then we cannot expect any cash flow
from receivables less than one month old.

III. Comment on the Literature

There is a vast literature dealing with accounts receivable, but
there is little concern for action-timing problems, such as the
billing decision. The general thrust of most investigations is to
improve control: the system supports the credit manager through
continual monitoring of the receivables process and, with the use
of some diagnostic model (see Cyert et. al.[1962] or Lewellen and Edmister[1973]), he/she may then identify potential collection problems and, ultimately, forecast cash flow. An interesting deviation from this approach is given by Lieber and Orgler[1975], who present a decision model for maximizing the present value of net earnings from accounts receivable. They do consider timing decisions concerned with credit policy, viz. the discount period and the credit period, although they, too, later simplify their model by supposing that these periods are "given." Otherwise, with respect to the billing decision per se, we have been unable to locate any prior work.

IV. The Model

There are two costs which must be evaluated: \( b = \text{billing cost} (\$/\text{statement}) \), and \( c = \text{credit cost} (\% / \text{time}) \). The cost, \( b \), is fixed with each billing of each account and, as noted in the introduction, is attributable to several system operations. In particular, \( b \) is independent of the size of the account and also of the number of accounts billed concomitantly. The cost, \( c \), is determined by the effective interest on the funds required to carry customers' credit balances. The unit time period is, of course, arbitrary, but it will be convenient to set the time unit equal to the "standard" billing cycle. Thus, time will likely be measured in months, or 30-day periods.

For each individual account, we obtain a deterministic forecast of the credit demand parameter, \( s = \text{charge sales rate} (\$/\text{time}) \). \( s \) may
be extrapolated in various ways, and we assume that this rate is maintained into the near future. The decision variable associated with each account is $t = \text{length of the billing cycle (time)}$. Note that, when $t = 1$, then the account in question is billed on the standard schedule. Also, note that $t$ determines the maximum credit limit, $q = st$, which automatically initiates the billing/collection activity.

The decision criterion is to minimize the time-average cost attributable to each account. In one billing cycle, of duration $t$, the average balance is $q/2 = st/2$, which follows from the assumption that the previous bill has been, or shall have been, paid in full. So, the credit carrying cost incurred in one cycle is $cst^2/2$, and the average cost, per unit time, is then given by

\[ AC = \frac{b}{t} + \frac{cst}{2}. \]

Now, $AC$ is minimized at the critical value $t = t^*$, where

\[ t^* = \sqrt{\frac{2b}{cs}}. \]

The corresponding upper limit on the credit balance is $q^* = \sqrt{2bs/c}$, the well known EOQ formula obtained from inventory theory. The minimal average cost is obtained by substituting $t = t^*$ in (1):

\[ AC^* = \sqrt{2sb/c}. \]

From (1) and (3) respectively, we observe that both the frequency of the billing decision, and the average cost attributable to the account, increase proportionally with the square root of activity, $s$. 
Now, consider the magnitude of savings which might be realized through the use of the model. If an account of rate $s$ is billed at the standard frequency, the saving is obtained by subtracting (3) from (1), with $t = 1$.

\[(4) \quad \text{Actual Saving} = b + \frac{cs}{2} - \sqrt{2sbc} \]

To determine gross saving, it is necessary to average (4) over all accounts. A conservative approximation is presented in the Appendix; average saving $= .1144b$. Assuming that $t = 1$ denotes a monthly period, and letting $n = \text{number of charge accounts}$, then total annual saving is

\[(5) \quad \text{TAS} = 1.373nb.\]

For example, with billing costs $b = .73$, we get TAS $= n$; i.e., at least one dollar will be saved for each account. Of course, the saving could be considerably higher.

V. System Capacity Constraint

By following the model's prescription, (2), for the billing cycle for different accounts, depending upon their activity rates, $s$, the load placed on the billing information system may be altered considerably. If $n$ is the number of accounts, then presumably the system can accommodate $n$ billings per period, with possibly some slack capacity available for growth. Thus, we shall assume that the system capacity is $gn$ billings per period, where $g \geq 1$. By inverting (2), we obtain the burden (number of cycles per period)
attributed to a single account of rate $s$, so the total load is
given by $n \sqrt{c/2b} \, EV_s$, where the expectation operator, $E$, signifies
that the average value of $\sqrt{s}$ has been computed with respect to
the entire population of accounts. Now, if $E \sqrt{s} \leq g \sqrt{2b/c}$, the
system has adequate capacity. Indeed, in the event $E \sqrt{s}$ is small,
excess capacity may be freed for other operations. However, our
concern in this section is the overloaded system, in which

(6) \quad E \sqrt{s} > g \sqrt{2b/c},

which could arise when the accounts tend toward large balances and
when the interest rate, $c$, is relatively high compared to the fixed
billing cost, $b$.

When (6) holds, the solutions given by (2) are no longer feasible,
except possibly on a "crash" basis. Management may elect to stick
with the standard policy, or they may decide that the operational
saving, estimated in (5), warrants the investment in a more
efficient system. A third alternative uses constrained optimization
techniques. Billing decisions are still made on an account-by-
account basis in order to minimize the aggregate cost while,
simultaneously, respecting the constraint imposed by system capacity.

It will be helpful to index the accounts, so we let $s_i =$ charge
sales rate for the $i^{th}$ account and $t_i =$ billing cycle for the $i^{th}$
account, $i = 1, \ldots , n$. Using (1), we obtain the aggregate cost
per period,

(7) \quad \text{Agg. Cost} = \sum_{i=1}^{n} (b/t_i + cs_i t_i/2).
The problem is to minimize (7), subject to the capacity constraint,

$\sum_{i=1}^{n} \frac{1}{t_i} = gn$.  

In the Appendix, the method of Lagrange multipliers gives the optimal solutions,

$t_i^* = \frac{E\sqrt{s_i}}{\sqrt{\bar{s}}} \frac{1}{g}$.  

By inverting (9), we obtain $\frac{\sqrt{s_i}}{E\sqrt{s}} g$ as the load on the system -- the number of billings per period -- caused by the $i$th account. So, the billing frequency of any account is still proportional to the square root of its charge sales rate; the presence of $g/E\sqrt{s}$ in (9) merely insures that the constraint, (8), is satisfied.

Again, the magnitude of potential cost saving depends upon the relative distribution of activity, $s$, among the different accounts. Substituting (9) into (7) gives the minimum aggregate cost,

$MAC = nbg + (E\sqrt{s})^2 nc/2g$.  

Setting $t_1 = 1$ in (7) gives the aggregate cost for the standard policy: $nb + ncs/2$. For purposes of comparison, we shall let $g = 1$ in (10), so that exactly the same number of billing transactions, $n$, occur in the optimal policy as occur in the standard policy. Letting $g = 1$, which implies that the system is already operating at full capacity, is also fairly conservative and will tend to underestimate the potential saving. Aggregate saving per period is obtained by subtracting and averaging the above expressions, yielding

$Cost Saving = \frac{nc}{2} [Es - (E\sqrt{s})^2]$.  

An evaluation of (11) is provided in the Appendix, resulting in an approximation for total annual saving of

\[
TAS = 1.333nb. 
\]

Note that (12) compares favorably with (5), the saving achieved when no constraint on system capacity is imposed, although we again remind the reader that (5) is very likely to understate the potential benefit to be derived from using the model.

Appendix

Approximation of Average Saving per Account per Period:

In the "group billing decision problem," all accounts are billed at the same frequency, 1/t. Following the usual derivation, we find that the optimal group billing cycle is \( t_G = \sqrt{\frac{2b}{cEs}} \). Now, \( Es \), the average sales rate, depends upon the relative distribution of \( s \) among the entire population of accounts. In order to obviate this dependence, we make a very conservative assumption, where "conservative" means that the ensuing calculations tend to understate the saving potential.

The standard policy may be interpreted as group billing with \( t = 1 \), and we shall assume that it is optimal. Setting \( t_G = 1 \) yields

\[
Es = \frac{2b}{c}.
\]

In practice, we might expect to find \( Es < \frac{2b}{c} (t_G > 1) \), and the economic rationalization of the standard policy would follow from inclusion of credit risks.
In order to compute $E\overline{V_s}$, it is now necessary to assign some
particular distribution to the parameter, $s$. Perhaps the simplest
assignment is to let

\[(A2) \quad s \sim \text{Unif}[0,4\beta/c].\]

$(A2)$ preserves $(A1)$, allows for completely inactive accounts,
although not for extremely active ones, and, because of the
relatively larger variance -- compared to the mean -- associated
with the uniform distribution, will also compensate for the
conservative assumption that $t_G = 1$. Anyhow, using $(A2)$, $E\overline{V_s} = 2\sqrt{2}/3\sqrt{Es}$. Then, from $(A1)$ and $(4)$ we get

Average Saving $= 2b - 4\sqrt{2}b/3 = .1144b.$

Solution of the Constrained Problem (7) and (8):

With $\lambda$ = the customary Lagrange multiplier, which should be
interpreted as the marginal value of system capacity, let $L$ be
the Lagrangian,

\[L = \sum_{i=1}^{n} \left( \frac{b}{t_i} + \frac{c s_i t_i}{2} \right) - \lambda \left( \sum_{i=1}^{n} \frac{1}{t_i} - gn \right).\]

Differentiating $L$ with respect to the $t_i$ and $\lambda$, and equating the
results to zero yields the $n+1$ equations

\[
\frac{\partial L}{\partial t_i} = -\frac{b}{t_i^2} + \frac{cs_i}{2} + \frac{\lambda}{t_i^2} = 0, \quad i = 1, \ldots, n,
\]

\[
\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} \frac{1}{t_i} - gn = 0.
\]
Solving the above system, we obtain \( t_i = \sqrt{\frac{2(b-\lambda)}{c s_i}} \), so
\[
\sum_{j=1}^{n} \sqrt{c s_i / 2(b-\lambda) } = g_n. \]
Solving for \( 2(b-\lambda) \) we have \( \sqrt{2(b-\lambda)} = \sqrt{c \text{EVS}/g} \); and, substituting into the expression for \( t_i \) gives (9).

An Evaluation of Cost Saving in the Constrained Problem:

To evaluate (11), let \( E_s = 2b/c \). In view of (6), this is quite conservative, for, by Jensen's inequality (see p. 148 of Ross[1970]), with \( g = 1 \), \( E_s > (\text{EVS})^2 > 2b/c \). Using the uniform distribution again, we have \( \text{EVS} = 2\sqrt{2} E_s/3 = .9428 E_s \). Hence, from (11), the saving per period is \(.05555ncE_s = .1111nb \). Multiplying by 12 gives the annual basis, (12).

REFERENCES


