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The Cost-of-carry model and volatility : an analysis of gold futures contracts pricing

Edward M. Riley III
University of Richmond

The Cost-of-Carry Model and Volatility:
An Analysis of Gold Futures Contracts Pricing

By

Edward M. Riley III

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Advisors: Dr. Jerry L. Stevens, Dr. Andy Szakmary

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Abstract

The theoretical price of gold futures relies on the term structure of interest rates, the days to settlement, and the spot price of gold. However, when comparing the theoretical gold futures price and the observed price in the market, there is often a difference. The difference implies that the theoretical pricing model is incomplete. Though limited in explaining much of what causes a difference between the actual and theoretical futures prices, factors such as rise in the credit spread of interest rates and changes in the price of commodities are statistically significant and positively correlated determinants of the difference between theoretical and observed gold futures prices. Gold futures, primarily an investment and commercial instrument to hedge against inflation and unexpected changes in commodities, exhibit prices that differ from the theoretical price of its futures contract.

Introduction

Former Federal Reserve Chairman Ben Bernanke told the Senate Banking Committee in July of 2013 that "[N]obody really understands gold prices and I don't pretend to really understand them either." Indeed, gold prices are difficult to understand, and gold futures are at least as difficult to comprehend. However, futures have a theoretical framework to explain their price against the asset that underlies them, and this framework should apply particularly well for gold due to its trivial storage costs relative to many other commodities. However there are limitations to the theory. This paper will focus on explaining the daily deviations in gold futures prices from the price explained by the conventional theoretical framework. This paper explores what drives gold futures and hopes to use its discoveries to explain gold prices.

Background

Gold as an asset has existed for thousands of years, used primarily as a means of exchange since ancient civilizations, as well as a material used in artwork and medicine. Today, gold is used in dentistry and medical devices, jewelry, technology, and investing. According to the U.S. Bureau of Mines and the U.S. Geological Survey, jewelry and investing accounted for 36.3% and 38% of 6,882 metric tons of gold collected from 1975 to 2003, respectively. Gold is used in certain industries because of its unique physical properties. Dentistry uses gold for fillings due to its durability and has been used throughout history. Technology and industry also use gold because of its conductibility, malleability and resistance to corrosion.

However, these applications seem limited compared to some other elements found in the world, so why is gold so valuable? The answer lies in its scarcity. The World Gold Council estimates that the total amount of gold extracted from earth from ancient times to present day is 158,000 metric tons, which is equal to a 20.15-meter cube—approximately equal to a glass-encased tennis court filled to its maximum volume in gold. Therefore, along with some practical applications, gold's value in weight comes from its limited supply.

Gold's presence in the financial markets lies within investors hedging economic risk. Because of gold's scarcity, it appears to be a safe bet in times of crisis, however provides little return compared to financial assets. For instance, during the financial crisis of 2008, the S&P 500 fell 40% from 2008 to 2009, while gold increased 1.34%. The behavior of gold is not an anomaly. Gold consistently has ended at a higher price exiting a recession than when entering a recession, while other common assets such as equities and fixed income often lag. Coincidentally, futures contracts are also a hedging tool. Gold futures provide protection for investors without buying the asset outright.

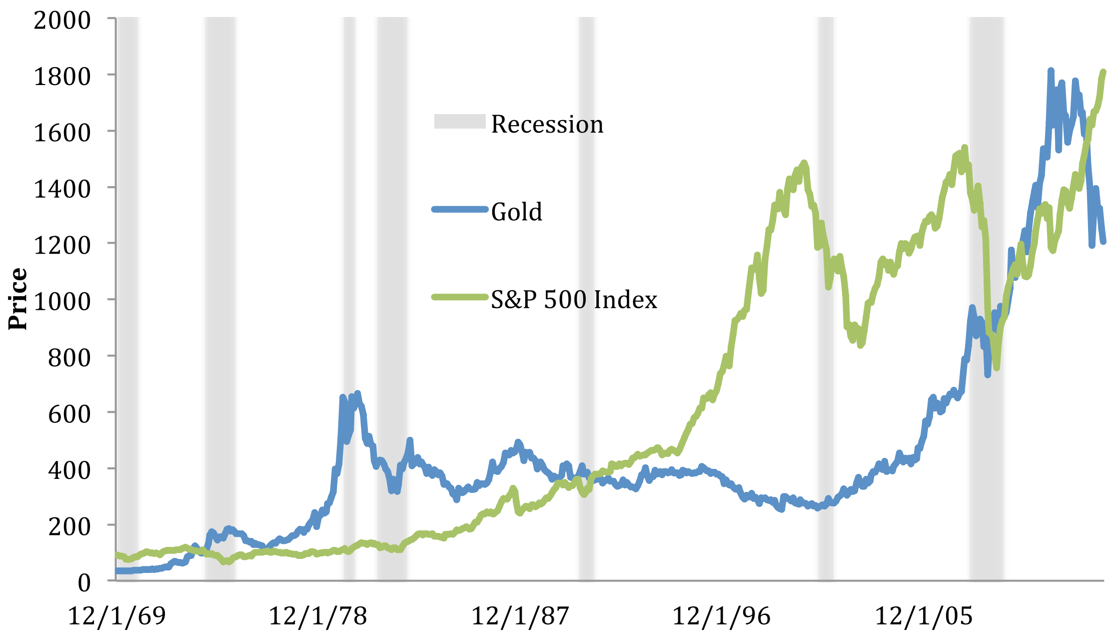

Figure 1: *Historical Performance of Gold vs. SPX*

The second important foundation for this paper is the concept of futures contracts. Futures are asset contracts between a buyer and a seller used to lock in a price on the exchange, paid in full on a later date. Futures are often used for assets such as wheat, oil, coffee beans, foreign currencies, and of course gold. The contracts differ by which asset is attached to the contract, which in futures terms is called the underlying asset, or the "underlying" for short. Futures are traded in an organized exchange, which requires traders of the exchange to place "margin"—a percentage of the total quoted price—in order to hold the contract before settling at the expiration of the contract.

The proliferation of futures is due to the benefits the financial instruments provide. Commodity production and marketing expose producers to risk. Wheat farmers, for instance, are exposed to falling wheat prices before the chance to reap their crop yield. However, the farmer can hedge—limit risk—by selling a futures contract for some of the crop. If the price of wheat does fall, the gains from selling futures mitigate the losses the farmer gets from holding wheat in the present—or “cash”—market. Conversely, a cereal producer knows that in the future he will need more wheat to produce cereal; however he does not need the input at the moment and would rather use his liquid assets for other purposes. Currently, the producer is at risk of rising wheat prices. The cereal producer could buy a futures contract to hedge his risk against rising wheat prices. In the event that wheat prices do rise, the producer can sell his contract for a profit, reducing the total cost of production. Alternatively, the wheat farmer and the cereal producer could simply organize a contract between each other so that the cereal producer takes delivery of the farmer’s wheat on an agreed upon price. In several ways, futures help facilitate commerce.

Futures have other benefits that explain their use for transferring risk. The futures exchanges standardize quality, quantity, and delivery time for each contract. The standardization of futures and existence of exchanges allows for greater liquidity in the market, so the contract holder can get out the contract should liquidity needs arise. Additionally, futures provide a transfer of default risk from the buyer and the seller to the financial intermediaries that organize the exchange because the exchanges are financially sound and legal counterparties of every contract.

Another benefit to futures is their minimal default risk, because exchanges require traders to adjust margin at the end of each trading day. Finally, futures promote specialization. Producers of goods focus on maximizing profit through production, while financial intermediaries and speculators focus on profit maximization through optimal pricing. Producers transfer risk to speculators.

The Cost of Carry Model

Due to their relative liquidity and standardized form, futures have a generalized pricing model that closely follows the prices represented in the market. The model is based on the cost associated with carrying the underlying asset until contract expiration. The costs associated with carrying the asset fall into four categories: storage costs, transportation costs, insurance costs, and financing costs. Commodities such as wheat or coffee must be stored before a buyer of the futures contract takes delivery of the asset, they must be insured to minimize the risk of spoiling due to water damage, theft, or other risks, and if there is a distance between the buyer and seller there is a cost associated with transporting the good. For gold, storage, transportation, and insurance costs are likely to be very small (Fama & French 1988). The largest cost, however, is financing the purchase of the asset and this is the most important cost when explaining the cost of carry model for gold.

The cost of carry model works by analyzing the opportunities for arbitrage—risk-free profit—in the futures market. The first form of arbitrage is cash-and-carry arbitrage. An arbitrageur wishes to buy gold to exploit the possibly overvalued futures contract at time $t = 0$ and sell at $t = T$. The arbitrageur can borrow cash to buy gold at $1,000 and use the asset as collateral. By doing so, he borrows at the risk-free rate of 5%.[1] When it is time to settle the contract the arbitrageur delivers the gold he is holding against the futures contract and is thereby absolved of any further obligation in the futures contract. The total cost the arbitrageur is obligated to pay is $1,050 at time $t = T$—the amount he borrowed and the interest he owes on the loan assuming no other charges. If the arbitrageur receives a futures contract settlement price

[1] Defining the risk-free rate will be discussed later in the paper. In the current example, risk-free rate is equal to the minimum amount of return any lender would expect for an investment with zero risk. Additionally, the interest rate used is a hypothetical example—using a larger number than current rates to illustrate costs of financing.

greater than $1,050, he succeeded in collecting a riskless profit. If there are numerous traders participating in the same strategy, then supply of futures contracts increases and the price must then fall to such a point that no arbitrageur could exploit the mispricing. Expressed as an inequality, the strategy's arbitrage-free equilibrium price yields:

$$(1) \qquad F_{0,T} \leq S_0(1 + C)$$

Where, $F_{0,t}$ is the futures price, S_0 is the spot price, and C is the cost of carrying the asset forward, represented as a fraction of the spot price.

Yet equation (1) shows that the futures contract could be *undervalued* relative to future value of the spot price of gold. If a contract is undervalued for a particular time interval from t = 0 to t = T, arbitrageurs use reverse cash-and-carry arbitrage. In order to exploit the mispricing, the arbitrageur borrows the physical gold from another seller and then sells the gold short at the spot price of $1,000.[2] The arbitrageur then buys a futures contract in gold, loans the money earned from selling gold short at the risk-free rate, receiving 5% interest. At the contract settlement date (time t = T), the arbitrageur collects his loan, uses the proceeds to take delivery of the physical gold at the futures contract price, and repays the short sale. If the futures contract settlement price was less than $1,050, then the arbitrageur collects a risk-free profit equal to the difference between the spot price where he sold the gold less the futures price. Again, many traders have an incentive to use the same strategy; because once the trade is completed there is no cost. The demand for futures then rises by arbitrageurs and so does the price, until there is no opportunity for risk-free profit using the reverse cash-and-carry. Expressed as an inequality:

$$(2) \qquad F_{0,T} \geq S_0(1 + C)$$

[2] Short-selling is when a trader sells an asset without owning it, with the expectation that the asset can be replaced at a cheaper price in the future and making a profit on the difference between the initial selling price and the replacement cost.

Finally, the combination of the two rules yields:

$$(3) \qquad F_{0,T} = S_0(1 + C)$$

To summarize the outcome illustrated in equation (3), an arbitrageur should not make any profit or loss if he sells the gold futures contract short and is long[3] physical gold until the contract expires—or if the arbitrageur is long the gold futures contract and is short physical gold. This should make sense intuitively, because the only difference between the physical gold and the futures price of gold is the time value of money.

Assuming other costs besides the cost of financing are trivial for gold, the price of gold futures depend on three pieces of information. First, the spot price of gold. The physical market for gold is like any other commodity, traded every day and changing in response to new information reflecting supply and demand. The second piece of information is the risk-free rate. The risk-free rate is the lowest possible return a lender would expect on an investment with zero risk[4]. In reality, the closest estimate to a borrower with zero risk is the U.S. Government[5]. Therefore, the term structure of Treasury interest rates is used to determine the risk-free rate. Finally, the number of days to settlement completes the cost-of-carry model. An investor can estimate the approximate interest rate to use as the cost of financing, and then applies the ratio between the number of days to expiration T to the number of days in a year to estimate the total cost of carrying an underlying asset. The final equation is expressed as:

$$(4) \qquad F_{0,T} = S_0(1 + r)^{\frac{T}{365}}$$

[3] Long means that an investor has purchased the asset and intends to make a profit on the asset's price appreciating.
[4] The investment must be the risk-free rate because the arbitrageur is looking for risk-free profit. Using any other investment vehicle would mean the arbitrageur is taking risk. For instance, if the arbitrageur invests his short physical gold proceeds into a AAA-rated corporate bond, the corporation still is at risk to default on its loans and the arbitrageur does not receive his investment with interest back.
[5] The logic behind U.S. Treasuries as the risk-free asset is because the U.S. government backs their creditworthiness.

The rest of the paper focuses on comparing the cost-of-carry price to the observed futures price, identifying differences between the two prices, and offering possible explanations to the differences in price. Both market-specific and macroeconomic determinants will be tested for statistical significance and magnitude for explaining the variation in the futures price from the predicted price. Statistical results will determine the validity of a new theory, emphasizing an associated risk premium to the old pricing model of futures contracts.

Literature review

Paul Samuelson developed a hypothesis that futures prices increase in volatility as the contract nears its expiration date (1965). Samuelson argued that price competition drives futures prices to a level equal to the expected spot price at settlement date. His argument implies the best estimate for the future price is the current price; therefore the expected change in price is zero. In sum, futures follow a "random walk" in response to new information that will equal the expected spot price at contract expiration. Intuitively, futures contracts further from expiration have little information to estimate the expected spot price, and so volatility is low. However, as the contract nears its settlement date, information becomes increasingly relevant to the expected spot price. More information causes greater price changes, and so there is more volatility closer to expiration date.

Samuelson's paper is relevant to the hypothesis tested in this paper because it explains behavior of futures contracts beyond the cost-of-carry model. The primary assumption of Samuelson's model, market expectations, may be the driving force behind differences in the actual price of futures and the estimated cost-of-carry price of futures. Traders become less and less certain of the price of gold as they reach further into the future, and so the "discount rate" of information may be higher during the earlier days of the contract. Their assumptions may guide futures into higher or lower values than implied by the theoretical model, and the futures price becomes increasingly accurate to the cost-of-carry model when more information is available about the expected spot price.

A number of papers, specifically concerning forward currency markets, relate to Samuelson's theory. After Samuelson's paper on futures, Eugene Fama (1984) developed an empirical study on the unbiased forward exchange rate hypothesis. This hypothesis states that the forward exchange rate is an unbiased predictor of the future spot exchange rate, assuming that market participants are risk-neutral and have rational expectations. In other words, the difference between the expected spot rate and the current spot rate is equal to the difference between the forward rate and the current spot rate—ultimately suggesting that the forward rate must be equal to the expected spot rate[6]. Fama concluded that the regression coefficients in his empirical test were statistically significantly different from zero, suggesting that there is variation in the expected change in the spot price and forward price, as well as a difference in the forward price and future spot rate—so the cost of carry market leaves some mispricing left to explain.

Thomas Chiang (1988) also tests the unbiased forward rate hypothesis discussed in Fama's paper. Chiang's results show that the unbiased forward rate hypothesis does exist in data used from January 1974 to August 1983. Yet further tests on subsamples within the time period show that the estimators within his regression depend upon what time period he chose. He found that the constant coefficient and the one-period lagged forward-rate coefficient change through the availability of new information within the market and is time-variant. His results conclude that regressions for certain time periods had constant coefficients significantly different from

[6] To clarify, the *expected exchange rate* is the exchange rate traders anticipate in the future. The forward exchange rate is a mathematical function of the current exchange rate, time, and interest rates between two currencies.

zero and the coefficient for the difference between the estimated spot rate and the current spot rate significantly differ from one. Both conclusions were the same as Fama's results, casting further skepticism on the unbiased forward rate hypothesis.

The studies on forward rates (Fama, Chiang, Naka & Whitney 1995) and the theory presented by Samuelson relate to the hypothesis being tested in this paper. If information and time can influence expected prices in ways that differ from the spot price observed in the market, then the cost-of-carry model may not incorporate fundamental parts that determine futures contract prices. However, the inconsistent results of such studies may also present opportunities to test the theory of unbiased forward rates in a new asset such as gold.

Fama and French's (1985) research on "convenience yield" in metal commodities refers to gold futures and the cost-of-carry model (1985). Convenience yield relates to the storage and transportation cost that persists in some metal assets' cost-of-carry model. Iron ore, for instance, has more value in the present than in the future to market participants, and so the futures price may be lower than the implied cost-of-carry price. Fama and French contend that marginal convenience yield on inventory rises as inventory decreases, modeling the effect of warehousing cost and the benefits of storing the commodity on the difference between a commodity's futures contract price and spot price. One of the conclusions of the study is precious metal storage costs are low relative to the value of the asset, and demand for gold as an investment asset limits variation in convenience yields. Fama and French's conclusion makes sense, considering the demand for gold is primarily driven by investment. As long as investors can be assured their rights of ownership to gold, the value of physically holding the asset compared to housing it in another location is negligible.

This paper relates to the study by Fama and French because both studies examine the potential causes of the difference between the spot price and futures price, beyond the simple pricing model that relies solely on interest rates. Additionally, based on their results, it is assumed that warehousing costs are not a significant determinant of price changes in the gold futures market when developing this paper's econometric model. However, this thesis focuses on the expectations effect on pricing gold futures, as opposed to the storage costs and convenience yield in Fama and French's paper.

Another factor important in determining the risk premium on gold futures is uncertainty in interest rates. Kenneth N. Kuttner's research in federal funds rate futures and the term structure of benchmark Treasury rates provides background information on the systematic risk of futures (2000). The study is important to this paper because it illustrates further the effect of interest rate shocks within the futures market. Kuttner hypothesizes that a change in the Federal Funds rate drives changes in the benchmark interest rate. The paper's results show that unexpected FOMC Federal Funds rate changes drive changes in the term structure of interest rates, yet expected changes in the Federal Funds rate do not. Additionally, Treasury bonds are more sensitive to unexpected changes in interest rates, while shorter-term interest rates are less sensitive to changes in the Federal Funds rate. The study's conclusion indicates that uncertainty factors into interest rate determination, to a certain degree. However, since the short-term Treasury rates are the interest rates most important in determining the theoretical price of gold futures, interest rate uncertainty may vary in magnitude by the number of days to settlement. As a futures contract reaches its settlement date, the difference between the futures contract price and the spot price becomes smaller. Therefore, unexpected changes in interest rates will create larger fluctuations for contracts further down the gold futures term structure, similar to benchmark rates term structure.

The issue with Kuttner's paper is the way the Federal funds futures settlement price is calculated by taking the average of the month's effective overnight Federal funds rate, instead of the rate that coincides with the settlement date. Additionally, the contract is not settled on the Fed's target rate, but rather the effective Federal funds rate. These complications illustrate how the Federal funds futures contract may be the incorrect asset to analyze market uncertainty of Fed policy. This paper will improve upon Kuttner's analysis by incorporating the three-month T-Bill prorated for each trading day instead of the Federal funds rate.

Melvin and Sultan's paper on the variation in gold futures and exogenous forces such as South African political unrest and oil prices is also related to this thesis (1990). According the Melvin and Sultan, South African political unrest and oil prices drive the spot price of gold. If there is uncertainty or expectations regarding these factors in the future, there will be a premium associated with the futures price of gold. The hypothesis adds a risk premium factor to the theoretical futures price of gold, assuming that both factors positively affect the price of gold. South African political unrest and oil price changes are significant determinants of the conditional variance of the estimated spot gold price error. Both this paper and Melvin & Sultan identify causes that create a difference between the observed futures price and the estimated spot price, based on the assumption that futures pricing model for gold futures is incomplete. Yet the coefficients of the price of oil and South African political unrest were small in magnitude and left much of the variation in the futures and spot price left unexplained. This paper will improve upon Melvin and Sultan's model to increase the pricing ability of the theoretical futures contract model.

The New Cost of Carry Model

The futures contract cost-of-carry model shows that the future price is a function of the spot price, the risk free interest rate, and the duration of the futures contract—accounting for other costs besides financing. Based on previous papers (Fama & French 1988), gold does not have significant transportation or storage costs, so the only true cost is financing. This means that gold futures follow to the simplest form of the futures pricing model, as explained earlier in equation (4). However, the model may not translate into the real world, even in the gold futures market where financing costs hold the most importance to maintain the cost-of-carry relationship. The factor is the risk of uncertainty in the financial system. Mentioned earlier in the paper, futures provide the ability to hedge risk by locking in a price. If a speculator equally shorts the same amount of gold in the futures market as he holds in the physical market, then he is fully hedged and carries zero risk. However, there may be default risk within the futures market. As mentioned earlier, one of the benefit of futures is they are organized on an exchange. Financially sound exchanges minimize risk of default because the exchange serves as a legal counterparty to every transaction. Yet default risk may be a factor regardless outside of the futures market.

For instance, if an arbitrageur pursues the cash-and-carry strategy there are many ways in which default risk appears. First, he must post margin when shorting the futures contract. As the volatility of gold increases, the margin on gold futures increases and the frequency of margin calls increases as well. Treasuries can be placed in the margin account, so an arbitrageur can still receive a risk-free rate of interest. However, if the arbitrageur has to pay a higher interest rate on the short sale than the risk-free rate he is assuming a risk greater than zero. Additionally, the arbitrageur needs to finance his purchase of physical gold by borrowing. Once again, if the arbitrageur must borrow at a higher interest rate than the risk-free rate the cash and carry trade becomes more difficult. Finally, if there is a greater uncertainty in gold prices, the trade becomes even more difficult—especially if the uncertainty in gold prices coincides with greater market turmoil and borrowing spreads increase for arbitrageurs against the benchmark Treasury rates.

The same default issues arise when discussing the reverse cash and carry strategy. Since the arbitrageur shorts gold and invests the proceeds in T-bills, posting margin costs nothing to him. Yet selling gold short might challenge the arbitrageur, since an owner of physical gold would most likely need either collateral or compensation for the trade. Once again, the cost of borrowing physical gold may cost more than the risk-free rate and will rise as market volatility increases.

In sum, the risk premium is the arbitrageur's default risk relative to a given settlement date.

Expressed as a function:

$$(5) \qquad F_{0,T} = S_0[1 + (r + \rho)]^{\frac{T}{365}}$$

where rho (ρ) is the additional spread on the risk-free rate. Rho is expected to be positive, and changes through market perception of new information. If default risk, borrowing costs, and uncertainty in asset prices are indeed a factor, then rho will have a direct relationship with asset volatility.

Within this paper, the primary focus is the risk premium. This study will begin with a regression model similar to the unbiased forward rate hypothesis regression, and will conduct additional regressions to determine whether or not there is a risk premium within the price of gold futures. Uncovering what determines the risk premium will explain the deviation of futures prices from its theoretical price, which will ultimately aid in explaining the difference between gold futures and their theoretical prices. Determinants to be examined include credit spreads (TED spread), commodities index volatility (GSCI volatility), and equity volatility (VIX).

The General Regression Model

The empirical test is to see whether certain measures of volatility relate to the divergence of the actual futures price from the theoretical price. Therefore, the dependent variable (Y_i) is the absolute value of the actual futures price less the estimated futures price, and the independent variables (X_i) will be measures of volatility and other exogenous risk factors. The total 5,574 observations are further split into two groups, one is the occurrence of negative deviations of the observed price from the theoretical price, and the other is a group of positive deviations from the observed price. Two separate regressions are used on the groups. The purpose of separating the two groups is to observe the cash and carry and the reverse cash and carry arbitrage strategies individually and infer any differences between the two strategies. The regression is a stochastic model that includes a constant term (α_i) and an error term (ε_i). The resulting model is expressed as:

$$(6) \qquad Y_{it} = \alpha_{it} + \beta \mathrm{X}_{it} + \varepsilon_{it}$$

The independent variables regressed against the difference in futures prices include the TED spread, implied equity volatility, and historical commodity volatility. The Treasury to Eurodollar (TED) spread is the number of basis points[7] between the three-month Treasury bill and the ninety-day London Interbank Offered Rate (LIBOR)[8]. The TED spread describes the risk

[7] Basis Point is equal to 1/100 of a percentage point

[8] LIBOR is the rate at which banks can borrow from other banks in the London interbank market. Recent controversy over global financial institutions manipulating LIBOR in the late 2000's could reduce the legitimacy of using the TED spread as a measure of interest rate market risk

premium associated with the most capitalized borrowers in the world—financial institutions—and the United States government. The TED spread captures short-term interest rate market risk, which will test the assumption that overall market risk creates fluctuations in futures prices. Higher TED spreads imply higher borrowing rates against arbitrageurs seeking financing for their trades.

The Chicago Board Options Exchange Volatility Index (VIX) reflects a market estimate of future volatility, based on the weighted average of the implied volatilities for a wide range of option strikes. The VIX may drive gold futures prices upward, as an expected increase in equity volatility increases investor risk-aversion. Once again, the VIX is a form of market risk measurement, which could suggest higher pricing disparity between the theoretical and observed gold futures contract price.

Other commodities are correlated with gold prices, which suggests additional non-diversifiable risk when owning a commodity. For instance, oil prices are correlated with gold prices, possibly due to oil producers using the commodity as a hedging tool (Melvin & Sultan 1990). So the Goldman Sachs Commodity Index (GSCI) historical thirty-day volatility will be included as another independent variable.

Data

Obviously the most important data is gold futures prices. Gold futures prices are from Datastream, with support from Bloomberg data, collected from the Chicago Mercantile Exchange Group Commodity Exchange Division (CME). Each contract is for one hundred troy-ounces of gold, so a contract priced at $1,300 troy ounces is equal to $130,000 total. Quality and quantity are consistent throughout contracts, reducing statistical errors due to changes in exchange policy. The contracts used are the futures contract closest to its settlement date (nearby contract), and the contract second-closest to settlement (first deferred), These contracts are used because the closer a contract is to expiration, the more liquid and active it will be. Liquidity is important because this study is concerned with determining whether tumultuous markets and other factors affecting the underlying asset are significant independent variables that influence the futures price. Insufficient liquidity could disrupt otherwise clearer results. By analyzing a contract with little open interest[9], the price may indicate liquidity as a significant determinant of price differences above all else, weakening results.

In addition to the futures price, another value is a calculated futures price based on the cost of carry model. This value derives from the gold nearby futures contract and the three-month T-Bill rate. In addition to gold futures data, the Treasury bill data and associated index data come from Datastream and Bloomberg. The nearby gold futures price is used instead of the gold spot price because of timing differences in determining market price. Treasury bill rates are also based on the consensus price of all contributors available on Datastream. Days to settlement are assumed to be constant because the model compares nearby futures to first deferred futures, so each daily calculation should have a constant sixty-day spread between the two. With all of the data available, the theoretical futures pricing model is used to calculate a theoretical value for the futures price. The contracts roll over bi-monthly, as gold futures contract open interest transfers from the current calendar month to the nearest calendar month as first notice of delivery approaches. The reason why there is a roll over period from one contract to another leading up to the earliest contract's expiration is because most speculators have no interest in owning the physical asset. Extremely small differences will be expected to occur in calculations because the

[9] There is no such thing as "volume" in futures markets. Instead, liquidity is measured by "open interest," which is the number of contracts available for trading for a specified settlement date.

interest rate is assumed to be constant over three months, whereas in the actual term structure of interest rates may only slightly differ.

The reason why the gold futures price is used to estimate the first-deferred contract with the cost of carry model because there is a timing differences between gold spot and the nearby contract. Reporting for both gold spot and Treasury bill rates occurs at 5:00PM ET. However, the price of the gold futures contract is not at the same time, and is rather a combination of the open outcry closing price and the electronic trading price. Open outcry is a form of trading in which buyers and sellers trade in-person with one another—often involving hand-signals and yelling out prices, hence the name. Open outcry for gold futures closes at 1:30PM ET, while electronic trading closes at 5:15PM ET. Datastream uses a consistent form for posting gold futures prices. The variation between the predicted and actual futures price due to timing would make poor estimations, which is why nearby futures contract price is used to estimate the first deferred contract.

Table 1: Summary Statistics

Series	Observations	Mean	Std Error	Minimum	Maximum
Day	5574	2787.500	1609.220	1.000	5574.000
Nearby	5574	633.136	445.636	252.800	1889.000
Deferred	5574	638.025	446.986	253.700	1889.700
Theoretical Def'd	5574	634.078	445.293	253.788	1889.016
Deviation	5502	0.288	1.006	-11.836	39.029
AbsVal Deviation	5502	0.484	0.927	0.000	39.029
LIBOR	5574	3.359	2.178	0.235	6.869
T-Bill	5574	2.850	2.047	0.000	6.240
TED	5574	0.508	0.398	0.088	4.579
VIX	5574	20.119	8.288	9.310	80.860
GSCI	5574	19.741	8.674	6.260	66.180

Anticipated Estimation Issues

The regression used is a standard OLS model, however the model uses an adjusting calculation—Newey-West—to correct standard error issues in a heteroskedastic and autocorrelated residuals in the model. The general concept behind Newey-West is to incorporate a lagged adjustment to heteroskedastic and autocorrelated error terms. In the regressions used for this paper, there are nine lagged time-series variables that adjust the error terms. Other models such as Generalized Autoregressive Corrective Heteroskedasticity (GARCH), Common Filter, and other econometric techniques could provide alternative results. Based on multiple previous studies (Hess & Kamara 2005, Frankel 1988) and prior knowledge, financial asset studies have generally accepted the concept of non-constant, correlated variance over time. Newey-West adjusts these issues found in the parameter estimates to accurately interpret OLS results. Futures have many statistical features such as leptokurtic distribution as well as autocorrelation, which require further statistical tests and adjustments to correct for these problems (Kolb & Overdahl 2007).

Leptokurtosis or the probability of having more observations outside more than one standard deviation compared to normal distribution is a common issue when dealing with futures over time. This problem is commonly known as distributions having "fat tails." Though advanced econometric models correct for this issue, it is still important to know that making inferences of variables with such distributions is more risky.

In addition to leptokurtosis, futures are first-order autocorrelated (Kolb & Overdahl 2007, Fama, 1984). First-order autocorrelation means that a variable has a tendency to be either positively or negatively correlated to the observation immediately before it. Autocorrelation creates biased estimates of the standard errors within an OLS model, and so models such as Newey-West, ARIMA, and GARCH minimize the bias within estimated parameters.

Finally, futures are heteroskedastic. Heteroskedasticity exists when a variable assumes a non-constant distribution. As mentioned in the Samuelson article, volatility tends to increase within futures prices as contracts reach their settlement date (2000). Therefore, futures prices display heteroskedasticity, conditional over time. Heteroskedasticity does not cause biased estimators, however it does cause the estimates of standard errors to be biased. With biased standard errors, testing for statistical significance in regressions is flawed and results will not be as reliable as a regression with a constant variance. This issue is resolved using the Newey-West error correction technique.

Based on all of the issues presented, regression models for gold futures must be taken into account the errors such as autocorrelation and heteroskedasticity. The Newey-West standard error correction provides standard errors that do not reflect such issues. Another model worth using for the paper would be GARCH. The GARCH model can correct estimators for heteroskedasticity and autocorrelation, and test results must take into account the possibility of leptokurtic distributions (Wang 2011).

Results

Table 2A: Regression Results

Dependent Variable: Absolute Value of Positive Deviations				
Variable	Coefficient	Standard Error		
Constant	0.18171	0.13031	Observations	3698
TED Spread	0.37462***	0.05907	Degrees of Freedom	3694
VIX	0.01241**	0.00561	Durbin-Watson	1.3752
GSCI	-0.00247	0.00374	R-Squared	0.05145

*** = 99% confidence interval
** = 95% confidence interval

Table 2B: Regression Results

Dependent Variable: Absolute Value of Negative Deviations				
Variable	Coefficient	Standard Error		
Constant	0.03838	0.13057	Observations	1804
TED Spread	0.55131**	0.22790	Degrees of Freedom	1800
VIX	-0.00842***	-0.00260	Durbin-Watson	1.0744
GSCI	0.00923	0.00565	R-Squared	0.05157

*** = 99% confidence interval
** = 95% confidence interval

The multiple regression results, shown in Table 2, prove supportive of the hypothesis; market risk does indeed reduce the efficiency of cost of carry arbitrage. The first independent variable, the TED spread, appears to be significant, both statistically and in magnitude. This result supports the hypothesis that the cost of financing increases disparities in the theoretical and actual futures prices. Furthermore, in cases of where there are negative and positive price deviations, there is a significant and positive relationship between the TED spread and futures mispricing. Using Newey-West standard errors, the TED spread variable is statistically significant at the 95% confidence level for positive price deviations, and the variable is statistically significant at the 99% level when there are negative deviations. Interpreting the coefficient, one standard deviation change in the TED spread (0.398) is expected to an average deviation of $0.18—calculated taking the average of the positive and negative coefficients. To put in terms of a single futures contract, a single "tick"—or price move of a futures contract is ten cents, and one contract is equal to 100 troy ounces of gold upon delivery. Therefore, one tick is equal to a ten-dollar price move. One must also consider that futures are bought on margin with leverage ratios ranging from 1:4 to 1:25. And so an approximate $0.18 difference is $18.43 in contract terms, and adding the natural leverage that exists within futures creates a potential profit or loss ranging from approximately $73.72 to $460.75.

Applying these calculations today, it is clear that the TED spread coefficient is significant in magnitude as well. Current margin requirements for a single gold futures contract closest to expiration is $7,975. As of March 8, 2014, the nearest contract trades at $1,339.50, and so the leverage ratio for gold futures is currently about 16.67. Considering current market conditions, a one percent increase in the TED spread would risk $307.23 of capital—a market loss of 3.94% for the smallest possible change in price differences.

The positive, significant coefficient suggests that the risk-free borrowing rate is unattainable in the real world for even the most well capitalized financial institutions in the world that borrow at LIBOR. Additionally, regardless of whether or not contracts are relatively overpriced or underpriced relative to the cost of carry model, an increase in the TED spread will always increase the cost of arbitrage. The assumption of the cost of carry model is the arbitrageur could borrow at the risk-free rate to buy physical gold, and use the purchased gold as collateral on the loan. The empirical results illustrate that an arbitrageur will have to borrow at a spread greater than the T-Bill rate, even if he has gold as his collateral.

Another statistically significant variable is the CBOE thirty day Implied Volatility Index. The multiple regression results show that the VIX is statistically significant at the 95% confidence level in cases of positive deviations between the observed and calculated price of futures contracts. Using the same calculations as the TED spread to determine economic significance, one standard deviation increase the in VIX (8.288) is expected to make a loss of a single contract equal to $17.14 under the condition that the arbitrageur performs a cash and carry trade. The results of the regression support the hypothesis that greater market volatility cause cost of carry arbitrage to weaken as an effective valuation and strategy.

However, in circumstances when the market price is less than the cost of carry price, an increase the VIX is expected to cause price deviations to decline—holding all other variables constant. The expected decrease in the mispricing would be equal to $11.63, given the standard deviation of the VIX and the regression coefficient. The negative value on the regressor is particularly interesting. One possible explanation is because gold is considered a safe-haven asset, when the equity markets experience greater volatility investors buy gold and gold-linked derivatives. The increase in demand for gold and its futures would then push prices upward, reducing the negative disparity between market and cost of carry prices.

The GSCI coefficient in the multiple-regression is not statistically significant. Comparing the multiple regression results to the simple linear model results in Table 3 (shown in Appendix)—with GSCI as the regressor—it is clear that adding other measurements of market risk change the significance of the variable. In a simple OLS model with Newey-West standard errors, GSCI volatility does appear to be statistically significant, however it is only statistically significant in cases where the market price is greater than the cost of carry price. A single unit increase in the GSCI is expected to widen the positive deviation between the theoretical first-deferred gold futures contract and the actual contract by approximately $0.014. Once again, when interpreting the magnitude by actual investment losses or gains, the coefficient becomes quite a large economic effect when assuming a standard error change in the volatility of the GSCI (8.674).

However, GSCI historical volatility loses both statistical significance and economic significance when added to the multiple regression. The TED spread and VIX provide a stronger relationship between cost of carry price deviation. There may be multicollinearity between the variables, which could explain the loss of statistical significance in the GSCI coefficient. Multicollinear regressions still provide unbiased estimators—assuming the all other requirements of OLS are satisfied—and so the GSCI could possibly be a significant indicator, though not economically significant when positive price deviations occur.

Conclusion

Based on the multiple regression model's results, it is clear that volatility weakens the cost of carry model's accuracy of pricing futures contracts. Most notably, the TED spread creates the largest variation in prices compared to the other regressors. This particular result makes sense, considering the emphasis of financing costs associated with the cost of carry model. If the risk-free rate is not attainable for financial institutions, then the interest rates offered to arbitrageurs is at least as expensive as the financial institutions' borrowing rates. Loans for purchasing gold, interest on margin accounts, all of these factors could expose an arbitrageur to higher interest rates. These higher interest costs could drive an arbitrageur to buying riskier assets for higher interest payments, making the cost of carry trade a speculative bet than a risk-free arbitrage strategy.

Equity volatility also contributes to wider positive deviations between the market price and gold futures cost of carry price. This result suggests that other forms of market risk also make arbitrage more difficult to execute. The GSCI historical volatility is not statistically significant, however the coefficients of the GSCI when the market price is less than cost of carry price still illustrates the relationship between market risk and cost of carry pricing.

The primary argument of this paper was to determine whether or not the cost of carry model has inconsistencies that weaken its credibility as the model for futures pricing. Based on the results of the regressions, it is clear that the cost of carry model may not account for the numerous instances when interest rates and default risk increase the cost of arbitrage. Much like the failure of Long Term Capital Management during the Asian financial crisis and Russian financial crisis, arbitrage is effective in low market volatility and costs of borrowing can be anticipated. However, when volatility rises, uncertainty drives rapid price fluctuations, raises interest rates, and the amount of leverage an investor assumes can exacerbate the loss of capital—combining these factors creates a feedback effect and mispricing becomes greater and more prevalent. In the instance of gold futures, the amount of leverage is inherently high, and uncertainty is non-diversifiable in a global financial market. The undertones of market risk keep borrowing rates high for any borrower besides the U.S government, and so the cost of carry arbitrage price is unattainable for even the most well capitalized financial institutions in the world.

In order to have more accurately price gold futures, credit risk of an investor must be taken into consideration. Therefore, the effective time value of money interest rate in the cost of carry model should be an interest rate that fluctuates higher in volatile markets such as ninety-day LIBOR, instead of the U.S. T-Bill rate which generally falls as investors flood safe-haven assets[10].

Finally, some improvements to this paper would be to include dummy variables for years when price deviations were particularly high. This addition would be to examine whether statistically significant price differences only exist in times of financial crisis. Also, incorporating exogenous factors beyond credit risk and volatility may improve the explanatory power of the model. Other additions include using different estimation models, besides OLS with Newey-West error terms, and comparing the results to check parameter estimations. Also, incorporating daily margin requirements as another independent variable in the model to see if higher capital requirements cause barriers to market entry and widen price deviations.

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[10] Safe-haven assets are liquid assets that appreciate in price in times of crisis, due to investors' increase in risk-aversion.

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Appendix

Table 3: Separate Simple Linear Regression Model of Each Independent Variable

3.1A

Dependent Variable: Absolute Value of Positive Deviations				
Variable	Coefficient	Standard Error		
Constant	0.3301205375***	0.039580724	Degrees of Freedom	3696
TED Spread	0.4782032614***	0.076418288	R-squared	0.0443525

3.1B

Dependent Variable: Absolute Value of Negative Deviations				
Variable	Coefficient	Standard Error		
Constant	0.041444733	0.096996843	Degrees of Freedom	1802
TED Spread	0.5160365283**	0.219896742	R-Squared	0.0432279

3.2A

Dependent Variable: Absolute Value of Positive Deviations				
Variable	Coefficient	Standard Error		
Constant	0.154488814	0.113690909	Degrees of Freedom	3696
VIX	0.0204358694***	0.006023624	R-Squared	0.0317278

3.2B

Dependent Variable: Absolute Value of Negative Deviations				
Variable	Coefficient	Standard Error		
Constant	0.271721604	0.059712618	Degrees of Freedom	1802
VIX	0.001441212	0.003393197	R-Squared	<.001

3.3A

Dependent Variable: Absolute Value of Positive Deviations				
Variable	Coefficient	Standard Error		
Constant	0.279587326	0.114732321	Degrees of Freedom	3696
GSCI	0.013594121	0.005609035	R-Squared	0.014094

3.3B

Dependent Variable: Absolute Value of Negative Deviations				
Variable	Coefficient	Standard Error		
Constant	0.195675378	0.081650887	Degrees of Freedom	1802
GSCI	0.006541689	0.00546651	R-Squared	<.001

Table 4: Correlation Coefficients Matrix

Sample size	3698			
		TED	*VIX*	*GSCI*
TED	**Correlation Coeff.**	*1.*		
	R Standard Error			
VIX	**Correlation Coeff.**	*0.50777*	*1.*	
		0.0002		
GSCI	**Correlation Coeff.**	*0.43102*	*0.63958*	*1.*
	R Standard Error	*0.00022*	*0.00016*	