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Rely to "Comment on 'Nonexistence of Certain Perfect Binary Arrays' and 'Nonexistence of Perfect Binary Arrays'"

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COMMENT

'NONEXISTENCE OF CERTAIN PERFECT BINARY ARRAYS' AND 'NONEXISTENCE OF PERFECT BINARY ARRAYS'

Y. X. Yang

The authors of Reference A found the nonexistence of the perfect binary array (PBA) of size $2 \times 2 \times 3 \times 3 \times 9$ by using a computer search and a combinatorial argument. By a considerably longer computer search, they also found the nonexistence of the PBA of size $4 \times 3 \times 3 \times 9$. In the following comments we point out that the nonexistence of the $4 \times 3 \times 3 \times 9$ PBA can be easily proved, thus the correspondingly longer computer search, in Reference A, becomes unnecessary. In addition, the nonexistence of the 9×36 PBA and 18×18 PBA will also be proved. All of these statements are based on the known nonexistence of the $2 \times 2 \times 3 \times 3 \times 9$ PBA and the following lemma.

(i) *Lemma:* If $A = [A(i_1, i_2, \dots, i_r)]$, $0 \leq i_1 \leq s_0 s_1 - 1$, $0 \leq i_k \leq s_k - 1$, $2 \leq k \leq r$, is an r-dimensional PBA of size $(s_0 s_1) \times s_2 \times s_3 \times \dots \times s_r$, then the following $(r+1)$-dimensional matrix $B = [B(j_0, j_1, \dots, j_r)]$ is an $(r+1)$-dimensional PBA of size $s_0 \times s_1 \times s_2 \times \dots \times s_r$, where for all $0 \leq j_k \leq s_k - 1$, $0 \leq k \leq r$, the entry $B(j_0, \dots, j_r)$ is defined by $B(j_0, \dots, j_r) = A(j_1 s_0 + j_0, j_{2,3}, \dots, j_r)$.

This simple lemma immediately leads to the following comments.

(ii) *Comment 1:* If A is a four-dimensional PBA of size $4 \times 3 \times 3 \times 9$, then by the lemma we can construct a five-dimensional PBA of size $2 \times 2 \times 3 \times 3 \times 9$, which is impossible due to the known nonexistence of the $2 \times 2 \times 3 \times 3 \times 9$ PBA. Thus we have easily proved the nonexistence of the $4 \times 3 \times 3 \times 9$ PBA. This proof is simply based on the nonexistence of the $2 \times 2 \times 3 \times 3 \times 9$ PBA, so the computer search, in Reference A, for the $4 \times 3 \times 3 \times 9$ PBA can be omitted.

(ii) *Comment 2:* From Reference B, we know that the existence of the 9×36 PBA and 18×18 PBA is undecided. Now, by repeatedly using the lemma and the known nonexistence of the $2 \times 2 \times 3 \times 3 \times 9$ PBA, it is easy to prove that there exist no PBAs of size 9×36 or 18×18. Therefore we can rule out another two undecided cases and reduce the list of undecided two-dimensional $s \times t$ PBAs to 10×40, 20×20, 8×72, 16×36, 15×60, 18×72, 36×36 and some other cases of $N \geq 20$.

(iii) *Comment 3:* From Reference A, the undecided higher (≥ 3) dimensional PBAs, with $N < 20$, are the PBAs of sizes $2 \times 2 \times 4 \times 5 \times 5$, $2 \times 8 \times 5 \times 5$, $4 \times 4 \times 5 \times 5$, $2 \times 2 \times 16 \times 9$, $4 \times 16 \times 9$, $8 \times 8 \times 9$, $4 \times 3 \times 3 \times 5 \times 5$, $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 9$, $2 \times 2 \times 4 \times 3 \times 3 \times 9$, $2 \times 8 \times 3 \times 3 \times 9$ and $4 \times 4 \times 3 \times 3 \times 9$. Combining the lemma and the undecided two-dimensional PBAs listed in comment 2, it is easy to see that the following relationships are satisfied by the two-dimensional PBAs and higher dimensional PBAs:

(*a*) The existence of the 10×40 PBA implies the existence of the $2 \times 2 \times 4 \times 5 \times 5$ PBA and $2 \times 8 \times 5 \times 5$ PBA or equivalently the nonexistence of the $2 \times 2 \times 4 \times 5 \times 5$ PBA or

$2 \times 8 \times 5 \times 5$ PBA implies the nonexistence of the 10×40 PBA.

(*b*) The existence of the 20×20 PBA implies the existence of the $4 \times 4 \times 5 \times 5$ PBA and then the $2 \times 2 \times 4 \times 5 \times 5$ PBA or equivalently the nonexistence of the $2 \times 2 \times 4 \times 5 \times 5$ PBA or $4 \times 4 \times 5 \times 5$ PBA implies the nonexistence of the 20×20 PBA.

(*c*) The existence of the 8×72 PBA implies the existence of the $8 \times 8 \times 9$ PBA or equivalently the nonexistence the $8 \times 8 \times 9$ PBA implies the nonexistence of the 8×72 PBA.

(*d*) The existence of the 16×36 PBA implies the existence of the $4 \times 16 \times 9$ PBA and then the $2 \times 2 \times 16 \times 9$ PBA or equivalently the nonexistence of the $4 \times 16 \times 9$ PBA or $2 \times 2 \times 16 \times 9$ PBA implies the nonexistence of the 16×36 PBA.

(*e*) The existence of the 15×60 PBA implies the existence of the $4 \times 3 \times 3 \times 5 \times 5$ PBA or equivalently the nonexistence of the $4 \times 3 \times 3 \times 5 \times 5$ PBA implies the nonexistence of the 15×60 PBA.

(*f*) The existence of the 18×72 PBA implies the existence of the $2 \times 8 \times 3 \times 3 \times 9$ PBA, then the $2 \times 2 \times 4 \times 3 \times 3 \times 9$ PBA and then the $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 9$ PBA or equivalently the nonexistence of the $2 \times 8 \times 3 \times 3 \times 9$ PBA, $2 \times 2 \times 4 \times 3 \times 3 \times 9$ PBA or $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 9$ PBA implies the nonexistence of the 18×72 PBA.

(*g*) The existence of the 36×36 PBA implies the existence of the $4 \times 4 \times 3 \times 3 \times 9$ PBA, then the $2 \times 2 \times 4 \times 3 \times 3 \times 9$ PBA and then the $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 9$ PBA or equivalently the nonexistence of the $4 \times 4 \times 3 \times 3 \times 9$ PBA, $2 \times 2 \times 4 \times 3 \times 3 \times 9$ PBA or $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 9$ PBA implies the nonexistence of the 36×36 PBA.

(iv) *Comment 4:* From Reference A, it is clear that a direct computer search for the five-dimensional $2 \times 2 \times 3 \times 3 \times 9$ PBA is much simpler than that for the four-dimensional $4 \times 3 \times 3 \times 9$ PBA. The lemma suggests the following computer search approach for the nonexistence of some PBAs: when the size of the PBA is given, we first try to search for those higher dimensional PBAs with dimension of small order, then by the lemma, if the nonexistence of the higher dimensional PBAs is found then the nonexistence of lower dimensional PBAs follows.

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REPLY

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Yang's comment [C] is based on a lemma which claims to construct an $s_0 \times s_1 \times s_2 \times \ldots \times s_r$ perfect binary array (PBA) from an $s_0 s_1 \times s_2 \times \ldots \times s_r$ PBA.

Unfortunately this lemma is not correct. A counter-example is the nonexistence of a $2 \times 2 \times 3 \times 3 \times 9$ and $4 \times 3 \times 3 \times 9$ PBA [D] despite the existence of a 9×36 and 18×18 PBA [E]. The resulting comments 1, 2 and 4 of Reference C are also incorrect.

We can however use the following result of Lüke *et al.* [4]:

(i) *Result 1:* Let $\gcd(s_0, s_1) = 1$; then there exists an $s_0 s_1 \times s_2 \times \ldots \times s_r$ PBA if and only if there exists an $s_0 \times s_1 \times s_2 \times \ldots \times s_r$ PBA.

Some of the implications of comment 3 [C] follow from result 1, for example the existence of a 10×40 PBA is equivalent to the existence of a $2 \times 8 \times 5 \times 5$ PBA.

In Reference C, Arasu *et al.* construct a $2^a 3^b \times 2^{a+2} 3^b$ and $2^{a+1} 3^b \times 2^{a+1} 3^b$ PBA for all $a, b \geq 0$. In particular there exist PBAs of size 9×36, 18×18, 18×72, 36×36, 54×54 and 72×72. From Reference 6, there remain the following 21 undecided cases for an $s \times t$ PBA with $s, t \leq 100$:

10×40, 20×20, 8×72, 16×36, 15×60, 16×100, 20×80, 40×40, 22×88, 32×72, 25×100, 52×52, 56×56, 40×90, 60×60, 44×99, 68×68, 78×78, 80×80, 88×88, 100×100.

There remain 11 undecided cases for an $s_1 \times \ldots \times s_r$ PBA with $\prod_i s_i < 1600$, as stated in Reference D.

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