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Frank Rudisill

Lewis A. Litteral
University of Richmond, llittera@richmond.edu

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CAPABILITY RATIOS: COMPARISON AND INTERPRETATION OF SHORT-TERM AND OVERALL INDICES

Frank Rudisill
Management Department
School of Business Administration and Economics
University of South Carolina Upstate
800 University Way
Spartanburg, SC 29303
(864) 503-5511
frudisill@gw.uscs.edu

and

*Lewis A. Litteral **
Management Department
Robins School of Business
University of Richmond
Richmond, VA 23173
(804) 289-8576
llittera@richmond.edu

* Please send all correspondence to the second author.

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ABSTRACT

The ability of a process to satisfy customer requirements is frequently measured by capability indices. The use and interpretation of these capability indices are often times misguided and or misunderstood by those involved in this aspect of statistical process control. Those who monitor and control processes and/or make decisions based on the reported values of these indices need to have a clear understanding of indices that are reported by or to them. This paper addresses the particular indices of C_p and P_p which indicate the capability of the process based only on its variability and C_{pk} and P_{pk} which indicate process capability considering both variability and location. A test is proposed to determine if there is significant non-common cause variability and a method to estimate the unstable component of the overall variability is presented. A table is provided to aid the analysis and examples based on the fiber characteristic of dyeability are given.

Introduction

Many industries (automotive, paper, computer chips, paint, electronics, etc.) typically use process capability indices to assess, monitor and communicate the capability of their processes and products to meet tolerances/specifications. Capability assessment and improvement are integral components of the ever-expanding Six Sigma philosophy (Breyfogle, 1999). Many companies use these as the fundamental requirements for supplier selection and certification. These indices also provide valuable internal information for setting priorities and identifying processes that need improving.

There are many numerical ways to quantify capability of variables-type data. Some of the more frequently used indices are; C_p, C_{pk}, P_p and P_{pk}. Each of these is defined carefully in the following section. Computer software (Minitab, Statgraphics, Statistica, QI Analysis, and more) calculates these as part of their capability analysis. C_p and P_p indicate the capability of the process based only on its variability. C_{pk} and P_{pk} indicate process capability considering both variability and location (average).

Bothe (1997) provides a comprehensive reference on process capability. He devotes a great deal of discussion to the relationship between the Six Sigma philosophy and process capability explaining that the often cited figure of 3.4 nonconformities per million opportunities is a result of viewing the process as being dynamic rather than static and recognizing that small shifts in the process average (less than +/ – 1.5 standard deviations) are often not detected. He provides tables that relate values of capability indices to stated nonconformities per million opportunities. For example, a C_p of 2.00 corresponds to a six sigma process when the process is dynamic. A review of recent work on process capability indices appears in Kotz and Johnson (2002). There is also an excellent series on the abuse and use of these indices in Gunter (1989a, b, c and d).

This paper proposes that practitioners who calculate, report and make decisions using these capability indices first perform a statistical test (the F test) to determine if there is a statistically significant component of non-common cause variability. If there is significant special cause variability, we show how to estimate the unstable component and calculate the relative percent contributions to the overall variability (this is useful in determining where to focus improvement efforts). A table is presented showing the impact that sample size has on the estimates. Besides, the use and interpretation of this procedure are illustrated with examples.

Some Capability Indices and their Estimates

C_p and C_{pk} reflect the capability of a process to meet specifications in the short run assuming the process is stable.

$$C_p = (USL - LSL) /(3\ \sigma_{st}) \tag{1}$$

$$C_{pk} = \min (C_{pk}\text{ upper}, C_{pk}\text{ lower}) \text{ where} \tag{2}$$

where

$$C_{pk}\text{ upper} = (USL - \mu)/ (3\ \sigma_{st}) \text{ and} \tag{3}$$

$$C_{pk}\text{ lower} = (\mu - LSL)/(3\ \sigma_{st}). \tag{4}$$

USL = upper specification limit

LSL = lower specification limit

μ = process mean

σ_{st} = short term standard deviation of the process

μ is estimated with the following formula where k is the number of in-control subgroups;

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{K} \bar{X}_i}{k}. \tag{5}$$

σ_{st} is usually estimated by:

1) pooled estimate of within subgroup variation from a control chart,
2) average range of within subgroup ranges or
3) average moving range of individuals.

P_p and P_{pk} reflect the capability of a process to meet specifications in the long run regardless of whether the process is stable.

$$P_p = (USL - LSL) /(3\ \sigma_{overall}) \tag{6}$$

$$P_{pk} = \min (P_{pk}\text{ upper}, P_{pk}\text{ lower}) \text{ where} \tag{7}$$

$$P_{pk}\text{ upper} = (USL - \mu)/(3\ \sigma_{overall}) \text{ and} \tag{8}$$

$$P_{pk}\text{ lower} = (\mu - LSL)/(3\ \sigma_{overall}). \tag{9}$$

$\sigma_{overall}$ is estimated by calculating the sample standard deviation (s) of all the data over a representative time period.

μ is estimated in the same way as for C_{pk} above. Values that are:

<1.33 imply the process fails to meet the minimum requirement for potential capability

=1.33 indicate the process just fulfills the minimum requirement

>1.33 mean the process surpasses the minimum requirement. The 1.33 is somewhat of an arbitrary value. Many industries require set C_{pk} requirements higher than 1.33, such as automotives (1.67) and computers/electronics (2.00).

Proper Use of the Indices

Short term variability, as measured by σ_{st}, is a component of the overall variability, as measured by $\sigma_{overall}$. If the process is stable over the time that data are collected the methods used to calculate the respective estimates give unbiased estimates of the same parameter the true sigma. In other words, if a process is stable the only source of variability is the common cause or short term component. This also includes the contribution of the measurement system. If there is only common cause variability the only meaningful indices are C_p and C_{pk}. If there is significant non-common cause variability (unstable), estimates of P_p and P_{pk} should be calculated and the unstable contribution should be estimated and its relative percent contribution to overall variability should be determined. The data requirements for precise values of these indices are quite large (at least 100).

Methodology

1) F Test (or converted to ratio of standard deviations instead of variances)

$$C_p = (USL - LSL) / (3\ \sigma_{st}) \tag{10}$$

$$P_p = (USL - LSL) / (3\ \sigma_{overall}) \tag{11}$$

$$\text{so } C_p/P_p = \sigma_{overall} / \sigma_{st} \tag{12}$$

Computer software simply calculates estimates of C_p and P_p, and does not make any comparisons. Typically C_p will exceed P_p (as it should) but if the process is stable it may not. This is confusing to a lot of users.

Same for C_{pk} and P_{pk}

$$C_{pk}/P_{pk} = \sigma_{overall} / \sigma_{st}. \tag{13}$$

This is intuitively appealing.

Thus, to determine if there is a statistical difference between C_p and P_p and C_{pk} and P_{pk} all we have to do is to compare $\sigma_{overall}$ with σ_{st}.

Comparisons of variances (assuming the underlying populations are fairly normal) involve the F ratio where the test statistic is the ratio of sample variances. The critical values found in an F table depend upon the degrees of freedom of each sample variance.

Let k = the number of subgroups

and n = the number to samples within each subgroup.

The degrees of freedom for $\sigma_{overall}$ is nk-1, the degrees of freedom for σ_{st} is k(n-1) assuming σ^2_{st} is the pooled within subgroup variance. If the range method is used the degrees of freedom for σ_{st} will be a different (less than) tabulated value. Also, if data are individuals, the 2 point moving range is used to estimate σ_{st}. This situation is easily considered.

If the F_{stat} does not exceed $F_{critical}$ we conclude that the only source of variability is short term (common cause). That means the process is stable or in control. This should be evident in a control chart. Recommend reporting P_p and P_{pk} as the measures of capability. Many companies tend to misuse these statistics because they do not have adequate sample sizes. A general rule of thumb is that the number of subgroups should be at least 100. With fewer than 100 subgroups the estimates of σ_{st} and $\sigma_{overall}$ will be imprecise and could lead to either overstating or understating capability.

2) Where is the variability?

If the F_{stat} exceeds the $F_{critical}$ there is statistically significant variability over and above the short term variability.

a) This can be estimated by using the following formula and the respective estimates of the variances.

$$\sigma^2_{unstable} = \sigma^2_{overall} - \sigma^2_{st} \tag{14}$$

b) The percentage breakdowns are determined by:

$$\text{stable \%} = \sigma^2_{st} / \sigma^2_{overall} * 100\% \tag{15}$$

$$\text{unstable \%} = \sigma^2_{unstable} / \sigma^2_{overall} * 100\% = 100\% - \text{stable \%}. \tag{16}$$

These percentages are useful to management and quality engineers in setting priorities for improvement. They indicate how successful the control plan is. If the unstable % is large (say 30 %) or more the control plan is not working.

The degrees of freedom for short term are based on using the ranges (or moving ranges) to estimate sigma short term. One could also use the pooled estimates of within subgroup variances.

In Table I, column F is the square root of column E. All one has to do is to divide C_p by P_p and compare the result with column F. If the capability ratio is larger than the critical values in column F there is significant unstable variability.

Table I. **The critical values (CCR) for the ratio of Cp/Pp and Cpk/Ppk**

A # of Subgroups	B n Per Subgroup	C df Short Term	D df Overall	E F Critical 0.05	F CCR (Critical Capability Ratio)
30	1	26	29	1.91	1.38
40	1	34	39	1.75	1.32
50	1	43	49	1.64	1.28
60	1	52	59	1.57	1.25
70	1	61	69	1.51	1.23
80	1	70	79	1.47	1.21
90	1	78	89	1.44	1.20
100	1	87	99	1.41	1.19
150	1	131	149	1.32	1.15
200	1	175	199	1.27	1.13
30	2	27	59	1.79	1.34
40	2	35	79	1.65	1.29
50	2	44	99	1.56	1.25
60	2	53	119	1.50	1.22
70	2	62	139	1.45	1.20
80	2	71	159	1.41	1.19
90	2	79	179	1.39	1.18
100	2	88	199	1.36	1.17
150	2	132	299	1.28	1.13
200	2	176	399	1.24	1.11
30	3	55	89	1.51	1.23
40	3	73	119	1.43	1.19
50	3	91	149	1.37	1.17
60	3	109	179	1.34	1.16
70	3	128	209	1.31	1.14
80	3	146	239	1.28	1.13
90	3	164	269	1.26	1.12
100	3	182	299	1.25	1.12
150	3	273	449	1.20	1.09
200	3	364	599	1.17	1.08
30	4	82	119	1.41	1.19
40	4	110	159	1.34	1.16
50	4	137	199	1.30	1.14
60	4	165	239	1.27	1.13
70	4	192	279	1.25	1.12

80	4	219	319	1.23	1.11
90	4	247	359	1.21	1.10
100	4	274	399	1.20	1.10
150	4	411	599	1.16	1.08
200	4	548	799	1.14	1.07
30	5	109	149	1.35	1.16
40	5	145	199	1.29	1.14
50	5	181	249	1.26	1.12
60	5	218	299	1.23	1.11
70	5	254	349	1.21	1.10
80	5	290	399	1.20	1.09
90	5	326	449	1.19	1.09
100	5	362	499	1.18	1.08
150	5	543	749	1.14	1.07
200	5	724	999	1.12	1.06
30	6	134	179	1.31	1.14
40	6	179	239	1.26	1.12
50	6	224	299	1.23	1.11
60	6	268	359	1.21	1.10
70	6	313	419	1.19	1.09
80	6	358	479	1.18	1.09
90	6	402	539	1.17	1.08
100	6	447	599	1.16	1.08
150	6	671	899	1.13	1.06
200	6	894	1199	1.11	1.05

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Source: Computations are based Wheeler (1990, p. 302)

Examples and Implications

The method described above is helpful to practitioners that use capability ratios to evaluate the capability of their suppliers or their internal processes. The following examples illustrate the application and interpretation of these ratios of capabilities ratios. The following data and graphical examples are presented for illustrative purposes only. The data were generated via computer simulation to reflect four different scenarios encountered by the authors in over 25 years of consulting experience. Fiber dyeability is used as the variable of interest, but this could be any key process or product variable. Dyeability is a critical characteristic of fiber. Whether the end product is automotive upholstery, residential or commercial carpet or apparel garments, it is important that fiber producers control the dyeability of their materials. Fiber dyeability is frequently measured and reported as a relative percentage of standard value. A 100% result indicates that the production sample dyes the same as the reference standard. A 110% result indicates that the production sample dyes 10% darker than the reference standard. Results less than 100% indicate that the production sample dyes lighter than the standard.

In each of the following examples, the assumed customer requirements for fiber dyeability are 100% +/- 20% for each sample. The typical test protocol requires that four random samples from each production run (lot) be samples and individually tested for dyeability.

Supplier A

Figure 1 shows the X-bar and R dyeability control charts based on the most recent 100 lots of fiber produced by Supplier A. Note that both X-bar and R indicate that the dyeability is in control. The estimated σ_{st} is 5.07 and the estimated $\sigma_{overall}$ is 5.12. $C_p = 1.32$ and $P_p = 1.30$.

The capability ratio is

$$C_p/P_p = C_{pk}/P_{pk} = \sigma_{overall} / \sigma_{st} = 5.12/5.07 = 1.01. \qquad (17)$$

The critical ratio from Table 1 (with 100 subgroups of size 4) is 1.10. Thus, there is no statistically significant special cause contribution to variability. Supplier A needs to maintain it's level of control and devote resources to reduce the common cause variability.

Supplier B

Now consider Supplier B's X-bar and R charts as shown in Figure 2. Supplier B has several points that indicate out-of-control conditions. The estimated σ_{st} is 5.07 (the same as Supplier A) and the estimated $\sigma_{overall}$ is 5.66. $C_p = 1.32$ and $P_p = 1.18$.

The capability ratio is

$$C_p/P_p = C_{pk}/P_{pk} = \sigma_{overall} / \sigma_{st} = 5.66/5.07 = 1.12. \qquad (18)$$

This ratio is greater than the critical value of 1.10 in Table 1. Thus, there is a statistically significant special cause contribution to variability.

The estimates of the percent contributions for stable and unstable components are:

$$\text{stable \%} = \sigma^2_{st} / \sigma^2_{overall} * 100\% = 5.07^2 / 5.66^2 * 100\% = 80\%. \tag{19}$$

$$\text{unstable \%} = \sigma^2_{unstable} / \sigma^2_{overall} * 100\% = 100\% - 80\% = 20\%. \tag{20}$$

Thus, Supplier B's special causes contribute 20% to his total variability. Supplier B needs to reduce his common cause variability as well as improve his control system.

Supplier C

Next, consider Supplier C's who has not devoted much attention to process control. Supplier C's X-bar and R charts are as shown in Figure 3. There are numerous indications of out-of-control conditions. The estimated σ_{st} is 5.07 (the same as for Suppliers A and B) and the estimated $\sigma_{overall}$ is 7.09. The results are $C_p = 1.32$ and $P_p = 0.94$, respectively.

The capability ratio is

$$C_p/P_p = C_{pk}/P_{pk} = \sigma_{overall} / \sigma_{st} = 7.09/5.07 = 1.40. \tag{21}$$

This ratio is greater than the critical value of 1.10 in Table 1. Thus, there is a statistically significant special cause contribution to variability.

Supplier C's estimates of the percent contributions for stable and unstable components are:

$$\text{stable \%} = \sigma^2_{st} / \sigma^2_{overall} * 100\% = 5.07^2 / 7.09^2 * 100\% = 51\%. \tag{22}$$

$$\text{unstable \%} = \sigma^2_{unstable} / \sigma^2_{overall} * 100\% = 100\% - 51\% = 49\%. \qquad (23)$$

Thus, Supplier C's special cause contribution is half of his total variability. Supplier C definitely needs to improve his control plan and then work to reduce his common cause variability.

Supplier D

To carry the example further, consider Supplier D's X-bar and R charts as shown in Figure 4. Supplier D is hard-pressed to find any points that indicate a stable condition. The estimated σ_{st} is 5.07 (the same as for Suppliers A, B and C) and the estimated $\sigma_{overall}$ is 9.00. Supplier D's C_p = 1.32 (respectable) and P_p = 0.74 (disappointing).

The capability ratio is

$$C_p/P_p = C_{pk}/P_{pk} = \sigma_{overall} / \sigma_{st} = 9.00/5.07 = 1.78. \qquad (24)$$

This ratio is greater than the critical value of 1.10 in Table 1. Thus, there is a statistically significant special cause contribution to variability.

The estimates of the percent contributions for D's stable and unstable components are:

$$\text{stable \%} = \sigma^2_{st} / \sigma^2_{overall} * 100\% = 5.07^2 / 9.00^2 * 100\% = 32\%. \qquad (25)$$

$$\text{unstable \%} = \sigma^2_{unstable} / \sigma^2_{overall} * 100\% = 100\% - 32\% = 68\%. \qquad (26)$$

Thus, the majority of Supplier D's variability is due to special causes. Supplier D needs to significantly overhaul his control system since the current system is clearly not effective.

Discussion and Conclusion

The proposed method of calculating the ratios of capability indices, C_p and P_p, testing for statistical significance and then estimating the relative percent contributions of common cause and special cause variation is useful in a variety of scenarios by practitioners.

1. Process engineers may use this technique to evaluate their internal processes. If the F test indicates statistically significant special cause variation then the control plan is not effective and it should be improved. If the F test indicates no statistical significance, then the C_p and C_{pk} values should be used to assess the need to reduce variation or center the process (as they are intended).
2. This technique provides supply chain managers and engineers with a tool to evaluate current and potential suppliers. If the F test indicates statistically significant special cause variation then the supplier's control plan is not effective. Procuring product from this supplier may lead to problems even if the supplier reports capable C_p and C_{pk} values. If the F test indicates no statistical significance, then the supplier has an effective control plan and supplier capability can be evaluated based on the C_p and C_{pk} values.

The purpose of this paper was to present a method for comparing two routinely calculated capability indices (C_p and P_p or C_{pk} and P_{pk}) and using this comparison to determine the relative percent contributions of common and special causes to the total variability. The table of critical ratios, the calculation of the percent contributions and the examples of four supplier scenarios were presented to assist quality practitioners who frequently calculate the ratios but are not sure of the best way to interpret and use them to improve their processes. The techniques presented above are not stand alone methods. They should be used in conjunction with other sound quality and process control techniques.

Figure 1
Xbar/R Chart for Supplier A

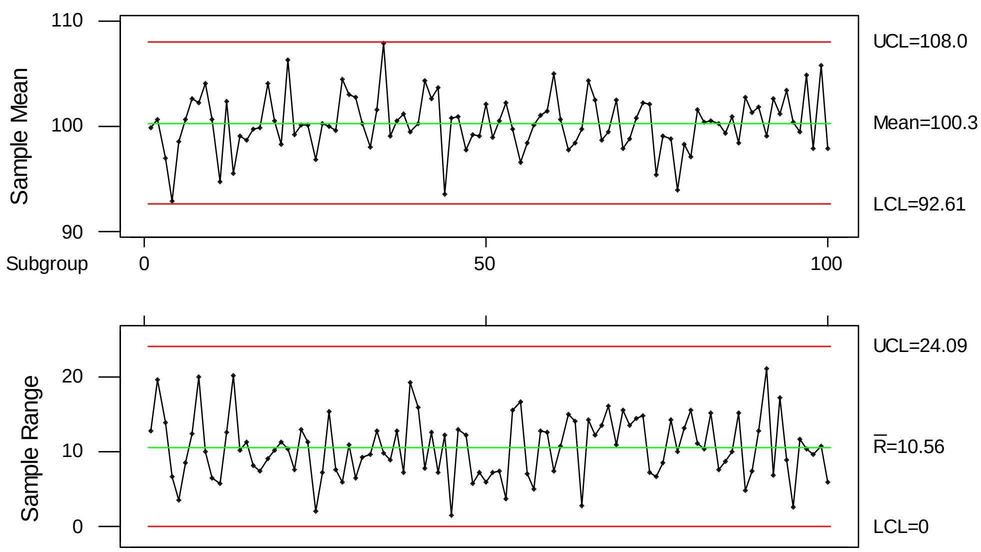


Figure 2
Xbar and R Chart for Supplier B

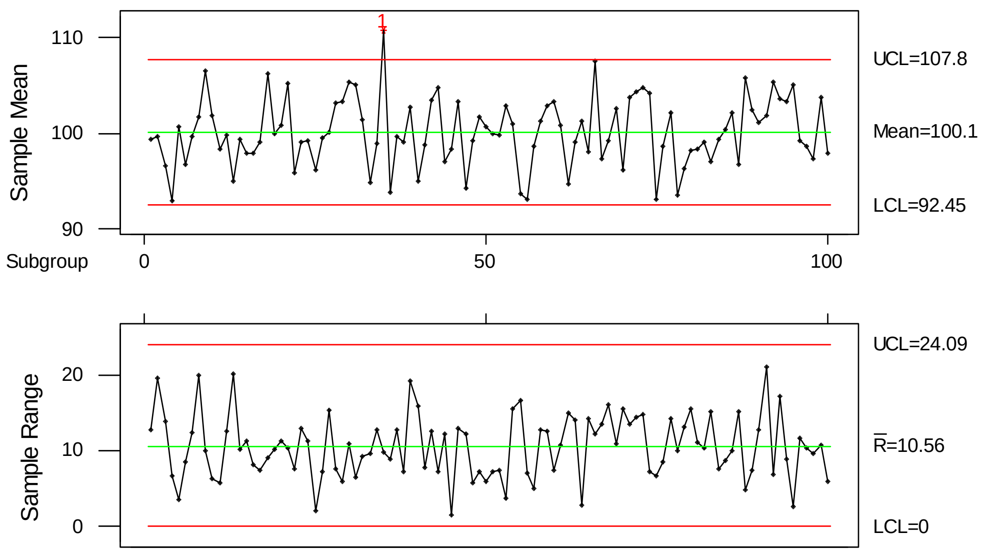


Figure 3
Xbar and R chart for Supplier
C

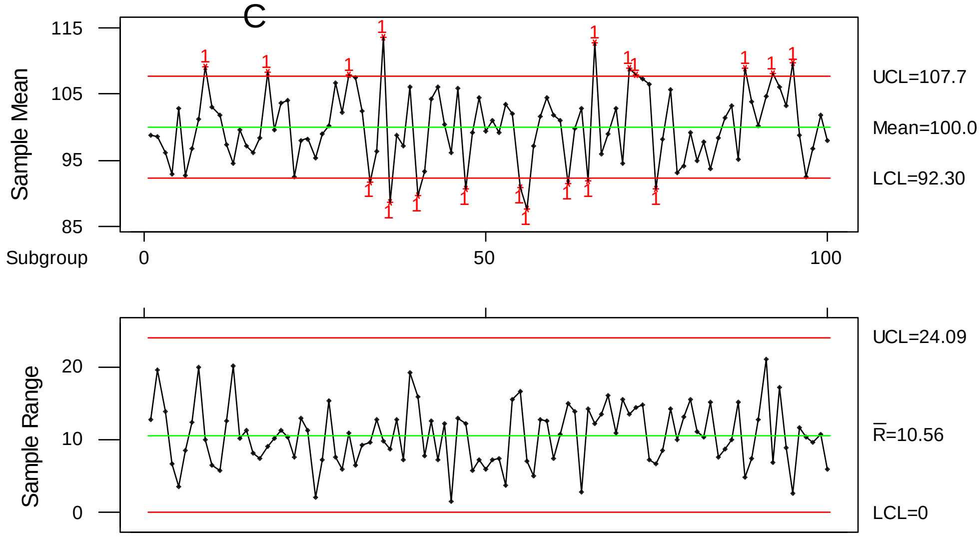
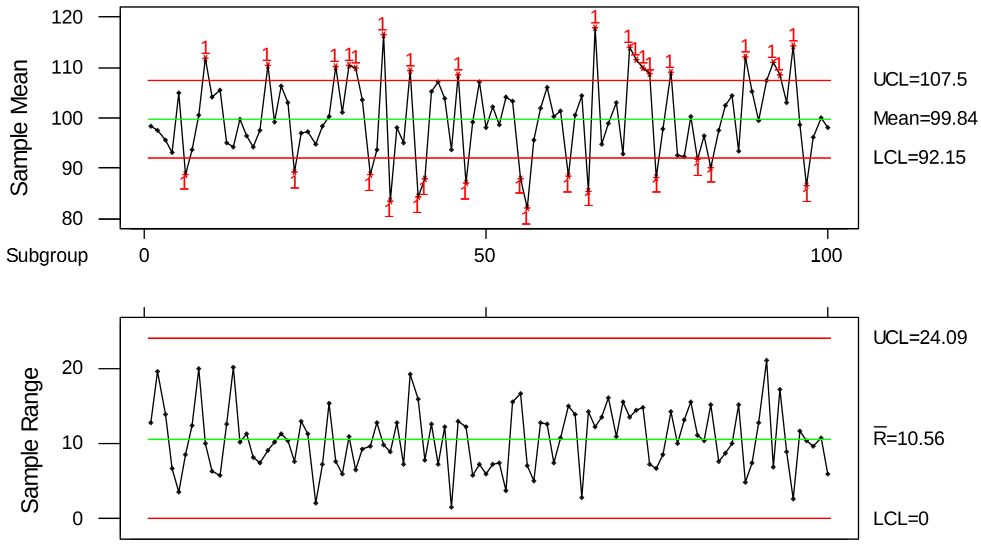


Figure 4
Xbar and R chart for Supplier D



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