Spin dependence of $K$ mixing, strong configuration mixing, and electromagnetic properties of $^{178}$Hf

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The combined data of two Coulomb excitation experiments has verified the purely electromagnetic population of the $K^\pi = 4^+, 6^+$, $8^-$, and $16^+$ rotational bands in $^{178}$Hf via $2 \leq \nu \leq 14$ $K$-forbidden transitions, quantifying the breakdown of the $K$-selection rule with increasing spin in the low-$K$ bands. The $\gamma$, $4^+$, and $6^+$ bands were extended, and four new states in a rotational band were tentatively assigned to a previously known $K^\pi = 0^+$ band. The quasi-particle structure of the $6^+$ ($t_2 = 77$ ns) and $8^-$ ($t_1 = 4$ s) isomer bands were evaluated, showing that the gyromagnetic ratios of the $6^+$ isomer band are consistent with a pure $\pi_2^+$ [404], $\pi_2^+$ [402] structure. The $8^-$ isomer band at 1147 keV and the second $8^-$ band at 1479 keV, thought to be predominantly $\nu_2^+$ [514], $\nu_2^+$ [624] and $\pi_2^+$ [514], $\pi_2^+$ [404], respectively, are mixed to a degree approaching the strong-mixing limit. Based on measured $\langle K^\pi = 16^+ \parallel E2 \parallel K^\pi = 0^+ \rangle$ matrix elements, it was shown that heavy-ion bombardment could depopulate the $16^+$ isomer at the $\sim 1\%$ level, although no states were found that would mediate photodeexcitation of the isomer via low-energy x-ray absorption.

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I. INTRODUCTION

Nuclei in the $A \approx 180$ region provide opportunities to study the interplay between collective and single-particle behavior in nuclear structure. In recent years, the study of electromagnetic (EM) excitation and deexcitation of high-$K$ isomeric states [1–6] has demonstrated significant violations of the $K$-selection rule in axially symmetric, quadrupole-deformed nuclei. An earlier Letter [4] showed that $K$-selection violations in $^{178}$Hf are mediated by a rapid increase in the $K$-forbidden transition matrix elements $\langle K_f \mid M_\lambda \mid K_i \rangle$ with increasing spin. The present work shows that the $K$-mixing occurs primarily in low-$K$ bands, and probes the mixing of high-$K$ components with increasing spin. The quadrupole moments and gyromagnetic ratios $g_K$ and $g_R$ measured from the in-band $E2/1M1$ branching ratios can be used to develop a picture of the collective and single-particle properties, revealing admixtures in the wave functions of rotational bands as well as the strength of the mixing. Changes in the moments of inertia of the nucleus provide additional evidence for band mixing, which is compared in the present work with the evolution of EM matrix elements as a function of spin.

Although the results of this work include several aspects of nuclear structure, the concepts and quantities related to the $K$-quantum number and its conservation require a more detailed introduction. If a deformed nucleus has axial symmetry, then $K$, the projection of the total spin $I$ on the symmetry axis, is a good quantum number, and the $K$-selection rule [7] forbids the EM transitions between two states $\mid I, M, K_i \rangle$ and $\mid I_f, M_f, K_f \rangle$ for which the forbiddenness $v \equiv |\Delta K| - |\lambda|$ is greater than zero, where $\lambda$ is the multipole order and $\Delta K \equiv K_f - K_i$. The degree of hindrance of a $K$-forbidden transition can be expressed in terms of the “reduced hindrance” $f_v \equiv |B(M_\lambda)_{W,u}/B(M_\lambda)|^{1/2}$, where $B(M_\lambda)_{W,u}$ is the Weisskopf single-particle estimate of the EM reduced transition probability. For $K$-forbidden transitions, $f_v$ is expected to be $\gg 1$.

Both the EM excitation and decay of high-$K$ states can be greatly hindered by conservation of the $K$ quantum number. Electromagnetic excitation probabilities decrease by many orders of magnitude with increasing multipole order, whereas for Coulomb excitation (EM excitation during nuclear collisions) the probability of multiple-step excitations decreases approximately exponentially with the number of steps, making the EM excitation of high-$K$ states from the ground state unlikely. For this reason, the Coulomb excitation of the $8^-$ isomer in $^{178}$Hf has remained a mystery during the two
has had some success in describing cranking (TAC) model \[ 19 \]. This alternative to of successive rotational alignments, using the tilted-axis
Calculations have been made \[18\] based on a statistical process
the collected in Gammasphere \[ 24 \]. Matrix elements coupling
\( \gamma \) one mechanism may need to be considered \[ 2,10–12\]. The
have been made based on a number of mechanisms, and
others \[11,14,15\]. Projected shell-model (PSM) calculations
overpredicted and underpredicted transition probabilities for
path through the nuclear states and the EM matrix elements
in kinematic coincidence using CHICO \[ 23\], Rochester’s
transition in the four-quasiparticle \( K\pi \) \( 178\)Hf, showing the overall influence of the
et al. et al. decades since it was first reported by Hamilton \[ 8\] and since its verification by Xie et al. \[9\] in 1993. The unexpected population of high-\(K\) isomers to measurable levels by Coulomb excitation has brought into question the validity or “goodness” of the \(K\) quantum number.

Predictions of the hindrance values of \(K\)-isomer decays have been made based on a number of mechanisms, and a variety of measurements have suggested that more than one mechanism may need to be considered \[2,10–12\]. The \(\gamma\)-barrier tunneling model has been successful in reproducing measured hindrances for some nuclei \[1,13\], but has both overpredicted and underpredicted transition probabilities for others \[11,14,15\]. Projected shell-model (PSM) calculations have been proposed to treat softness to \(\gamma\) deformation \[16,17\]. Calculations have been made \[18\] based on a statistical process of successive rotational alignments, using the tilted-axis cranking (TAC) model \[19\]. This alternative to \(\gamma\) tunneling has had some success in describing \(K\)-isomer decay in \(178\)Hf. Neither the PSM nor the TAC approach has been fully developed at present.

A Coriolis \(K\)-mixing calculation for \(K\)-forbidden transitions of apparently low hindrance has been demonstrated \[20\] to recover the “Rusinov rule” \[21\]. \(f_\nu \sim 100\), by adjusting the forbiddenness \(\nu\) for the \(K\) admixtures in the isomer state. However, at least one quantitative measurement has shown a high-\(K\) isomer to be very pure in \(K\) \[22\]. Allusions to the admixture of high-\(K\) components in the wave functions of states in the yrast band have been under consideration for many years \[2\], and recent measurements reiterate the possibility (e.g., Refs. \[12,20\]). \(K\)-mixing calculations for high-\(K\) isomer states, based on density of states considerations, have reproduced some, but not all, of the observed systematics \[10\].

The Coulomb excitation work of Hamilton \[8\] and Xie \[9\] measured only the yield of the \(K^\pi = 8^-\) isomer state of \(178\)Hf, showing the overall influence of the \(K\)-selection rule in terms of the total isomer excitation cross sections. To find the path through the nuclear states and the EM matrix elements connecting the ground-state band (GSB) to the \(8^-\) isomer band, it is necessary to measure the accompanying excitation of the individual rotational states built on the isomer. The pair of experiments in the present work will be shown to determine the excitation paths of the \(8^-\) isomer as well as the \(6^-\) and \(16^-\) isomers in \(178\)Hf. The rate at which the onset of \(K\)-selection violations occurs with increasing spin is determined, and this is shown to be consistent with a rotational alignment picture.

This work comprises two separate Coulomb excitation experiments. In the first, a \(650\)-MeV \(136\)Xe beam was incident on a \(178\)Hf target. Projectile and target ions were counted in kinematic coincidence using CHICO \[23\], Rochester’s parallel plate avalanche counter (PPAC), and \(\gamma\)-ray data were collected in Gammasphere \[24\]. Matrix elements coupling the \(K^\pi = 0^+, 2^+, 4^+, 6^+_{\text{isom}}(t_1 = 77\) ns), \(8^+_{\text{isom}}(t_2 = 4\) s), and \(16^+_{\text{isom}}(t_2 = 31\) yr) bands were fit to prompt and delayed \(\gamma\)-ray yields. Unexpectedly high yields from the \(19^{K=16}_K \rightarrow 19^{K=16}_K\) transition in the four-quasiparticle \(K^\pi = 16^+\) band were measured to be \(\sim 10^{-3}\) of the \(8^+_{\text{GSB}} \rightarrow 6^+_{\text{GSB}}\) transition. In the second experiment, directed at verifying the EM population of the \(16^+\) isomer, a \(179\)Hf beam was activated into the \(16^+_{\text{isom}}\) state at five beam energies and trapped in natural tantalum foils. An activation function for the isomer covering \(73–86\%\) of the Coulomb barrier \(E_{\text{Coul}}\) was used to find a set of matrix elements coupling the GSB to the \(K = 16\) isomer band.

Sections II–III of this article detail the two experiments and their analyses, including a significantly extended level scheme with a possible extension to the lowest known \(K = 0\) band obtained from experiment I. The fit of matrix elements that populate the \(K^\pi = 16^+\) isomer band was accomplished through a combined analysis of the first and second experiments, described in Sec. IV. Section VA covers the measurements of the EM properties of the nucleus, including measured quadrupole moments, the gyromagnetic ratios of the \(K^\pi = 6^+\) isomer band, and the band mixing between the two \(8^-\) bands at \(1147\) keV and \(1479\) keV. Conclusions regarding the goodness of the \(K\)-quantum number constitute the majority of Sec. V.

II. EXPERIMENT

A. \(178\)Hf\((136\)Xe,\(136\)Xe\)\(179\)Hf coulomb excitation

To resolve \(\gamma\) rays in complex spectra, the \(\gamma\)-ray energies must be measured to an accuracy of the order of \(1\%.\) To achieve this in a thin-target experiment, where the \(\gamma\) rays are emitted in flight, the detector system must support event-by-event correction of the Doppler-shifted \(\gamma\)-ray energies. This was accomplished in the first of the two experiments described herein by utilizing 100 of 110 elements of the \(4\pi\) Gammasphere array and CHICO. Gammasphere was fitted with 0.002-in.-thick Ta and 0.010-in.-thick Cu absorbers over the germanium faces to attenuate low energy (\(\leq 100\) keV) photons and prevent the flooding of the detector with atomic \(x\) rays, resulting in a \(p-p-\gamma\) (two particles and at least one \(\gamma\) ray) trigger rate of 4 kHz. The total photopeak detection efficiency for a \(\gamma\) ray in Gammasphere is \(9\%\) at \(1.3\) MeV, so that \(\gamma\)-ray triples (triple-coincidence \(\gamma\)-ray events) can be collected with good statistics. CHICO consists of 20 iso-butane-filled PPACs, which are capable of detecting light and heavy ions, including \(\alpha\) particles, and identifying the scattered particles by kinematic coincidence with a typical mass resolution of \(5\%\), an angular coverage of \(12^\circ < \theta < 85^\circ\) and \(95^\circ < \theta < 168^\circ\) in the polar angle, and an acceptance of \(2.8\)\(\pi\) sr or \(69\%\) of the sphere. The ATLAS superconducting linac at Argonne National Laboratory provided a \(650\)-MeV \(136\)Xe beam for a total of \(\approx 76\) hours, incident on a \(0.51\) mg/cm\(^2\), \(89\%\) enriched \(178\)Hf target supported by a \(0.035\) mg/cm\(^2\) carbon foil. A \(1\) \(\mu\)s sweeper interrupted the beam, allowing the detection of delayed \(\gamma\) rays from isomer decays with \(10\) ns \(\leq t_2 \leq 1\) \(\mu\)s.

The target comprised six species in the following molar fractions: \(<0.05\%)\,(174\)Hf, \(0.52(5\%)\,(176\)Hf, \(4.3(6\%)\,(177\)Hf, \(89.14(10\%)\,(178\)Hf, \(2.90(5\%)\,(179\)Hf, \(3.07(5\%)\,(180\)Hf.

Approximately \(5 \times 10^8\) usable \(p-p-\gamma\) events were recorded (two particles coincident detected by CHICO plus at least one \(\gamma\) ray detected by Gammasphere). Scaled-down singles (\(p-p\) events with no \(\gamma\) ray required) were not utilized, which necessitated the normalization of all \(\gamma\)-ray yields to the strong \(8^+_{\text{GSB}} \rightarrow 6^+_{\text{GSB}}\) transition. Because the normalization was not absolute in terms of the cross section, these are referred to below as “relative yields.” The \(\gamma\)-ray detection efficiency was
measured as a function of $E_γ$ using data from the GSB $γ$-ray cascades of four isotopes: $^{238}$U, $^{152}$Eu, $^{170}$Er, and $^{178}$Hf. The $^{238}$U and $^{170}$Er efficiency data were obtained from a 1358-MeV $^{238}$U on $^{170}$Er experiment [25,26] that ran immediately after the present Hf measurement, and the $^{152}$Eu data were obtained from a $\approx 5$-$\mu$Ci source calibration run.

The time-of-flight difference $\Delta t$ and the angle $θ$ between the scattered beam and recoiling target ions was measured by CHICO and transformed into mass $m$ vs. $θ$ (Fig. 1), where the two kinematic solutions are clearly separated. Event-by-event Doppler correction resulted in a 0.5% $ΔE_γ$ resolution, sufficient to measure $γ$-ray yields to the $10^{-3}$ level relative to the $^{8}$GSB $→ ^{6}$GSB yield.

Figure 2 shows the level scheme derived from this experiment. The p-p-$γ$ singles spectra were effective in measuring the $≥1$% yields of strongly populated states in the GSB and the lower levels of the $γ$ band. Prompt-prompt $γ$-$γ$ matrices were effective in measuring the $γ$-band decays to the GSB. Prompt-delayed $γ$-$γ$ matrices were constructed from the data in $9^\circ$ intervals in the projectile (Xe) scattering angle $θ_{scat}$. The prompt-delayed matrices provided a high selectivity for measuring the yields of the $K = 6^+$ isomer band, where the $r_2 = 77$ ns half-life permitted correlations between the prompt intraband transitions and the isomer decays. Symmetric cubes of threefold $γ$-ray events were analyzed using the RADWARE package [27] to measure the yields of the remaining bands, populated below the $10^{-2}$ probability level.

B. $K^\pi = 16^+$ isomer activation experiment

The Hf$^{178}$(Xe,$^{136}$Xe)$^{178}$Hf Coulomb excitation experiment measured a remarkably high $19^+ \rightarrow 18^+$ yield (Fig. 2) in the $K^\pi = 16^+$ isomer band. This second experiment was devised to measure the Coulomb excitation of the $16^+$ isomer by its activity as a function of collision energy to confirm that the $16^+$ band population followed Coulomb excitation predictions and to extract a set of $(I_{K=16^+}||E2||I_{GSB})$ matrix elements. A stack of five 1 mg/cm$^2$ natural Ta targets was irradiated by a $\approx 10$-pA $^{178}$Hf$^{24+}$ beam from ATLAS, providing an excitation function over a centroid bombarding energy range of $73\%$ (target 5) to $86\%$ (target 1) $E_{Coul}$.

The incident beam energy of 858 MeV was chosen to give $E_{beam} ≲ 80\% E_{Coul}$ for the third, fourth, and fifth targets, so that nuclear effects would be small or insignificant in all five Ta targets [28–30].

The targets were arranged normal to the beam, separated by 0.59-cm-long, 1.0-cm-diameter hollow cylindrical 42-mg/cm$^2$ tantalum “catchers” to collect scattered Hf ions over $40^\circ ≤ θ_{scat} ≤ 90^\circ$, so that $≤1\%$ of the nuclei in the $16^+$ state were lost or embedded in downstream targets. The target stack was biased at +4 kV to prevent a scattered electron current in the Faraday cup. A 1 mg/cm$^2$ Ta scattering foil was used to scatter beam and target particles into a silicon monitor detector mounted at $θ_{scat} = 45^\circ$. The rate was reduced to $\approx 30$ Hz using a collimator mounted on the face of the silicon detector. Faraday cup and silicon data provided an absolute measurement of the total beam dose of 1.7(2) particle-mC (particle-milli-Coulombs) to normalize the absolute activation cross sections. A germanium detector monitored the prompt online $γ$ rays emitted by Hf nuclei stopped in the Faraday cup to measure target ablation as a function of time and to monitor the beam species. The 25-cm-long, 3-cm-diameter Faraday cup was mounted with its downstream end 43 cm from the center of the target chamber.

Deviations of the beam position on target and small-angle scattering did not introduce significant errors into the activation rate measurement. Calculations using SMSCAT [31] showed that $\sim 10\%$ of the Hf ions scattered on the first target would be displaced by $≤ 0.2$ mm at the end of the target stack, and the errors introduced by electronic scattering would be small compared to other effects, such as target ablation.

The activities in the targets and catcher foils were counted 5 months after the activation at Yale University’s Wright Nuclear Structure Laboratory. Offline counting of the isomer decays via the cascade shown in Fig. 3 provided the cross section measurements. The targets were positioned between two four-crystal “clover” Ge detectors, which were arranged with their front faces 1 mm apart and shielded by several inches of lead on all four sides. The detectors were composed of four approximately cylindrical crystals that were read individually. A target foil and its catcher were positioned between the clovers, equidistant from all eight crystals and counted for times ranging from 16.5 to 237 h (Table I). In every case, the target foil and its catcher foil (unrolled and flattened from its original cylindrical shape) were counted simultaneously. (Target 2, excited at 83% $E_{Coul}$, was not measured.) The foil and catcher pairs were positioned overlapping at the center of the detector pair so that they resembled point sources as closely as possible, simplifying the analysis. Both the foil and the catcher were thin enough to cause negligible $γ$-ray attenuation. Relative $γ$-ray efficiency data were taken using a $^{152}$Eu source.

The quiet background and unexpectedly high count rate in the clover detectors showed remarkably prominent $^{178}$Hf...
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FIG. 2. Master level scheme showing all of the levels observed in the $^{176}$Hf($^{136}$Xe,$^{136}$Xe)$^{176}$Hf experiment. New levels and isomer states are shown as bold horizontal lines. Dashed horizontal lines represent tentatively placed states.

peaks due to the isomer activity (Fig. 4). The GSB peaks are well above background, as well as some $K = 8^-$ in-band transitions and the 88-keV decay of the $8^-$ isomer to the $8^+_K=0$ state. Ideally, the activity could be measured by counting either the $8^-$ band cascades or the GSB cascades, but the latter provided more accurate results because no knowledge of branching ratios was required. The peak-to-background ratio of the single-$\gamma$ spectrum was not high enough (4% for target 5) to accurately measure the activation of the lower energy targets. Better statistics were obtained from the n > 1-fold (twofold and higher) matrix by gating on a GSB transition and counting the coincident $\gamma$ rays in the GSB (Fig. 5).

To correct for the target ablation over time and to measure the total integrated dose for the entire second experiment, the Si detector was used to calibrate the Faraday cup early in the experiment, and the integrated current from the Faraday cup was used to measure the dose in the remaining runs. The absolute cross sections for excitation to the $K = 16^+$ isomer state were calculated from the total beam dose and the initial average areal density of the scattering target, $\rho_A = 1.0(1) \text{ mg/cm}^2$.

![Gamma Band](image1.png)

![Ground State Band](image2.png)

![Excitation Energy](image3.png)

FIG. 3. A partial level diagram for $^{176}$Hf built from the data of the Hf(Xe,Xe)Hf and Ta(Hf,Hf)Ta experiments. The known 12.7-keV transition was not observed. The strongest decay cascade (bold arrows) is known from branching ratios. The $\Delta J = 1$ branch from the $13^-$ level accounts for only ~10% of the total decay width.

![Energy Graph](image4.png)

TABLE I. Count times and raw count rates of GSB transitions (uncorrected for counting efficiency) for each target. Count rates are from $\gamma$-ray singles.

<table>
<thead>
<tr>
<th>Target</th>
<th>Counting time (h)</th>
<th>Single-$\gamma$ count rates ($h^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.5</td>
<td>195(12) 173(10) 123(9)</td>
</tr>
<tr>
<td>2a</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>17.5</td>
<td>67(9) 80(6) 73(7)</td>
</tr>
<tr>
<td>4</td>
<td>105.6</td>
<td>29(14) 24(2) 17(2)</td>
</tr>
<tr>
<td>5</td>
<td>237</td>
<td>6.9(16) 11(1)</td>
</tr>
</tbody>
</table>

aTarget 2 was not counted.
16 h of postactivation data collection are presented. The data from the activity of target 1 after 8 to the 10 CHICO is evident not only from the ability to measure yields $K\pi$ tentatively connected to a previously known in Fig. 2 and described below. In addition, intermediate states confirmed, but a 22 $^+\text{Hf}$ $^{178}$ were discovered.

A. Levels and bands

The power of the combination of Gammasphere and \text{CHICO} is evident not only from the ability to measure yields to the 10$^{-4}$ level in the $^{178}$Hf target nucleus but also from the data gleaned from isotopic impurities at the 0.5–4% level, using $p-p-\gamma-\gamma$ triples data. Data from $\gamma$-ray singles ($p-p-\gamma$) and gated doubles and triples were used to extend several previously known rotational bands in $^{178}$Hf [32–34] as shown in Fig. 2 and described below. In addition, intermediate states tentatively connected to a previously known $K^\pi = 0^+$ band [35–38] were discovered.

1. The GSB and the $\gamma$ band

The 20$^+_\text{GSB}$ level in $^{178}$Hf found by Mullins et al. [32] was confirmed, but a 22$^+$ level could not be found. The $\gamma$ band, with a band head at 1175 keV, previously known up to the $6^+$ level, was extended to the 14$^+$ level with tentative $15^+$ and $16^+$ states. The $\gamma$ band’s even and odd signatures have nearly equal moments of inertia and do not diverge up to spin 16$\hbar$, in contrast with the strong signature splitting and diverging moments seen in $^{180}$Hf [39,40] (Fig. 6). This indicates a major change in the interaction between the GSB and the $\gamma$ band on the addition of two $i_{13/2}$ neutrons.

2. A possible extension to the $K^\pi = 0^+\pi^+$ band

Four intermediate levels and a tentative fifth level in the unidentified band “A” (Fig. 2) were observed feeding into the GSB, suggesting a small $K$ value. The spacing of the observed states, as well as the decay pattern into the GSB, indicate that the observed intraband decays of band A are likely $\Delta I = 2$ transitions. The relative yields, normalized to the $8^+_\text{GSB} \rightarrow 6^+_\text{GSB}$ transition for a scattering angle range of $52^\circ \leq \theta_{\text{lab}} \leq 78^\circ$ were measured using the $\gamma$-ray triples (Fig. 7). The branching ratios needed to correct for effects due to coincidence gating were not measurable, but one intraband yield and three lower limits were found. From the decay pattern and the level spacing, possible spin-parity assignments for the lowest observed state are $10^+, 9^-$, and $8^+$. Odd parity is assumed for spin 9, because only one signature of the band is seen, more likely the natural parity states. Dominant $\Delta I = -2 \gamma$ decays from lower states to the GSB were not observed and may have been obscured by the large Doppler-broadened 1.3 MeV $^{130}\text{Xe} 2^+ \rightarrow 0^+$ transition.

An exhaustive search for the complete set of lower levels for the band was undertaken using the present $\gamma$-ray data and the known levels and bands in $^{178}$Hf (e.g., Refs. [32,34,37,41,42]) and testing all possible values of $K$ and spin-parity. The measured moments of inertia of Fig. 6 and comparison to the
suggest that band A could be the continuation of the $K^\pi = 0^+_2$ band (Fig. 6). The $I^\pi = 0^+_2$ state in $^{178}$Hf was characterized as a $\beta$ vibration by Nielsen et al. [35,36], but the most recent measurement of $0.3 < B(E2; 2^+_2 \rightarrow 0^+_0^{\text{GSB}}) < 1.5$ W.u., a strength typical of single-particle transitions, shows that the vibrational character of the $I^\pi = 0^+_2$ state is an open question [38]. Nonetheless, the similarity between the moments of inertia of band A and the $K^\pi = 0^+_2$ band of Refs. [39,40] supports the $K = 0$ identification of band A.

Because the moment of inertia for a particular level is very sensitive to the intraband decay energies, an accurate interpolation of the missing levels’ moments would give a precise indication of the expected intermediate $\gamma$-ray energies if band A is a continuation of the $K^\pi = 0^+_2$ band. Figure 8 shows a parabolic fit to the moments of the $2^+ \leftrightarrow 4^+$ levels of the $K^\pi = 0^+_2$ band and the $12^+ - 16^+$ levels of band A, as well as linear extrapolations from the $K^\pi = 0^+_2$ and A bands. None of the sets of interpolated or extrapolated $\gamma$-ray energies was found in the data, presumably because the interband decay branches dominate the decay. No other coincident sets of $\gamma$ rays were found which gave the correct sum energy.

The tentative identification of the $6^+_1$ level of the $K^\pi = 0^+_2$ band at 1731 keV in previous work [41] was based on a multiply placed transition, which some authors have suggested feeds the $2^+_0$ level [34]. If the $6^+_1$ level energy is correct, and band A is the $K^\pi = 0^+_2$ band, this implies that the moment of inertia of the band oscillates by $\approx 5\hbar^2$/MeV between the $6^+$ and $10^+$ levels, to satisfy the correct $\gamma$-ray sum energy. This seems unlikely, considering the measured moments shown in Fig. 8.

If the $K^\pi = 0^+_2$ bands in $^{178,180}$Hf share the same structure, then relative yields of the suspected $K^\pi = 0^+_2$ bands in both nuclei should be similar and less than the relative yields of the $\gamma$ bands. At the $10^+$ levels, the total relative yield $r_{^{180}\text{Hf}}$ = $Y_{\text{bandA}}/Y_{\gamma\text{-band}}$ is 0.7(2). The relative yield of the $10^+_2$ state in the Coulomb excitation of $^{180}$Hf is $r_{^{180}\text{Hf},^{136}\text{Xe}} = 0.33(3)$, calculated from the data of [39,40] and corrected for the greater population of the $10^+_2$ level relative to the GSB. Constraints on the energy sum and a smoothly varying moment of inertia suggest that the $6^+_2$ level currently assigned at 1731 keV [41] will be found closer to 1700 keV. An increase in the $K^\pi = 0^+_2$ band’s moment of inertia at spin $\gtrsim 6\hbar$ is predicted from the present data.

3. High-K bands

a. The $4^+$ band at 1514 keV. The tentative $7^+$ level [33] in the previously known $4^+$ band at 1514 keV was confirmed, and the band was extended from spin $7^+$ to spin $16^+$ with tentative $15^+$ and $18^+$ levels.

b. The $6^+$ isomer band at 1554 keV. A $14^+$ level was added to the $6^+$ isomer band, along with a tentative $15^+$ level. The moment of inertia exhibits an up-bend above spin 12h, which is not seen in the other isomer bands.

c. The $8^+$ bands at 1174 keV and 1479 keV. The $8^+_1$ (isomer) and $8^+_2$ (1479 keV) bands were observed up to spin $15^+$, but new levels were not found.

d. The $16^+$ band at 2446 keV. The $K^\pi = 16^+$ levels up to $20^+$ were seen in the prompt triple-$\gamma$ data of the Hf(Xe, Xe)Hf experiment. The intraband $\gamma$-decay yields were measured for the $19^+ \rightarrow 18^+$ transition, but the yield of the $20^+ \rightarrow 19^+$ transition was too small to measure in the $9^0/\text{lab}$ intervals.

4. Other isotopes

The high sensitivity of the CHICO+Gammasphere combination permitted observation of the GSB states (Fig. 9) up to $\approx 20h$ in all but the least abundant isotope, $^{174}$Hf. The
A. Isotopes

The ground state bands of Hf isotopes in the target foil and the GSB of $^{177}$Hf were observed to the previously known $39/2^-$ isomer band [43], and the GSB of $^{179}$Hf was extended from the $21/2^+$ level [43] to a tentative $39/2^+$ level (Fig. 9). Trace contaminants of $^{176}$Hf made the GSB visible up to the $18^+$ state. The GSB of $^{180}$Hf was extended from spin $12^+$ to $18^+$, using data from the $3^%$ $^{180}$Hf impurity. Ngijo-Yogo [39,40] independently placed the $18^+$/2 band intraband yields, which were selected by a gate on the $14^+$/0 isomer band. The yields of the GSB state in a prompt-delayed matrix (Fig. 11). The yields obtained from the prompt-delayed data were normalized to the relative yields from prompt $\gamma$-ray data using the $9^K→8^K$ transition.

In the $p,p′,\gamma$ singles spectrum, the yields of the GSB up to the $14^+$ level were recorded in 2° intervals over $20^°<\theta_{\text{c.m.}}<80^°$. The yields ($\geq10^{-4}$ relative to the GSB) of most bands were obtained from the $\gamma$-ray triples (e.g., Fig. 10) binned in 9° intervals with the notable exception of the $6^+$ isomer band intraband yields, which were selected by a gate on the delayed ($50\,\text{ns}<t<500\,\text{ns}$) isomer decays to the $6^+_\text{GSB}$ state in a prompt-delayed matrix (Fig. 11). The yields obtained from the prompt-delayed data were normalized to the relative yields from prompt $\gamma$-ray data using the $9^K→8^K$ transition.

In the Hf(Xe, Xe)Hf experiment, the $\gamma$-ray yields were normalized to the $8^G→6^G$ transition, which is the lowest-spin transition having good statistics in the gated triples data. Relative yields were corrected for $\gamma$-ray efficiency and for the branching ratios and internal conversion of the gate transitions.

In the strongly populated levels of the $\gamma$ band and the GSB, the uncertainty in the $\gamma$-ray detection efficiency introduced the majority of the error, generally $\approx5\%$ error for each $\gamma$ ray gated or measured.

B. Yield measurements

In the $p,p′,\gamma$ singles spectrum, the yields of the GSB up to the $14^+$ level were recorded in 2° intervals over $20^°<\theta_{\text{c.m.}}<80^°$. The yields ($\geq10^{-4}$ relative to the GSB) of most bands were obtained from the $\gamma$-ray triples (e.g., Fig. 10) binned in 9° intervals with the notable exception of the $6^+$ isomer band intraband yields, which were selected by a gate on the delayed ($50\,\text{ns}<t<500\,\text{ns}$) isomer decays to the $6^+_\text{GSB}$ state in a prompt-delayed matrix (Fig. 11). The yields obtained from the prompt-delayed data were normalized to the relative yields from prompt $\gamma$-ray data using the $9^K→8^K$ transition.

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In the strongly populated levels of the $\gamma$ band and the GSB, the uncertainty in the $\gamma$-ray detection efficiency introduced the majority of the error, generally $\approx5\%$ error for each $\gamma$ ray gated or measured.
Doppler-shift correction was inappropriately applied. (The broadened triples, although only three clean gates (12 measurement of the branching ratios of the 6+ yields and delayed 6+ appear to be the result of random coincidences between strong GSB/f/Δ1I band head [44], was measured indirectly in this work by A. B. HAYES et al. Kπ The highly converted 40-keV decay branch to the unmeasured strength of the 6 branch accounts for 19(1)% of the 6 branch (keV) This work Hague Hague, adj. Decay branch $E_{\gamma}$ (keV) $\alpha_{IC}$ Reduced transition probability (W.u.) This work Hague Hague, adj. $\Delta I = 1$ Transitions

2. The $K^* = 16^+$ band

The 16+ isomer band, previously known up to a tentative 23+ state [32], was unexpectedly populated at the $\sim 10^{-4}$ level up to spin 20+ in the Hf(Xe, Xe)Hf experiment. The low intensity and the 31-yr half-life required prompt triples gating, so the only measurable yields in 95 $\theta_{lab}$ intervals were observed for the 19+ $\rightarrow$ 18+ transition. More information was derived from the 178Hf activation experiment.

In determining the activation cross sections for the $K^* = 16^+$, $t = 31$ yr isomer, the measured doubles ($\gamma$-$\gamma$) rates of the 326- and 426-keV transitions are proportional to the product $\kappa \epsilon P$ for each $\gamma$ ray, where $\kappa$ is the correction for internal conversion, $\epsilon$ is the absolute detection efficiency, and $P$ is the peak-to-total ratio, calculated from the measured ratio of $\gamma$ doubles to $\gamma$ singles. It was found that $P = 0.74(7)$ and 0.64(6) for the 426- and 326-keV transitions, respectively, measured using target 1. It should be noted that the $\gamma$-$\gamma$ doubles and $\gamma$ singles activities are not independent for target 1, because the singles rate was used to measure the peak-to-total ratio. Summing (capture of two or more $\gamma$ rays in the same Ge crystal) and detection probabilities were both treated using calculated angular correlation functions. The second-order and fourth-order angular correlation terms gave $\approx 10%$ and $\approx 1%$ contributions, respectively, and the total angular correlation corrections were $< 10%$ in all cases. Corrections to the $\gamma$-$\gamma$ efficiency were made for the oblique trajectory through the Ge crystals. The measured activities of the targets are given in Table III for the two single-$\gamma$ transitions measured and the $\gamma \gamma$ measurement using the 326- and 426-keV transitions.

The cross sections provided in Table III are calculated for the average target thickness during the ablation of the target material. This introduces a $\approx 20%$ error in $\sigma_{16+}$, calculated from the change in the calculated cross section as the integration limits in projectile energy change over time.

### Table II. The measured reduced transition strengths $B(M1; 6^+_K \rightarrow I^+_f, K)$ for the decay branches of the 6+ isomer, compared with values tabulated in Ref. [51] from the data of Hague et al. [41]. The Weisskopf unit is taken in the downward direction. The adjusted values are explained in the text.

<table>
<thead>
<tr>
<th>Decay branch $E_\gamma$ (keV) $\alpha_{IC}$ Reduced transition probability (W.u.)</th>
<th>This work Hague Hague, adj.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6^+<em>K \rightarrow 6^+</em>\text{GBS}$ 922 0.00498 8.6(6) $\times 10^{-5}$ 1.10(7) $\times 10^{-4}$ 8.94 $\times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>$6^+<em>K \rightarrow 4^+</em>\text{GBS}$ 1247 0.00268 1.16(9) $\times 10^{-5}$ 1.25(6) $\times 10^{-5}$ 1.02 $\times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>$6^+<em>K \rightarrow 8^+</em>\text{K}=8$ 407 0.291 0.020(3) 0.0269(15) 0.0219</td>
<td></td>
</tr>
<tr>
<td>$6^+<em>K \rightarrow 4^+</em>\text{K}=4$ 40 216.0 1.03(7) -- --</td>
<td></td>
</tr>
<tr>
<td>$6^+<em>K \rightarrow 4^+</em>\text{K}=2$ 169 0.516 0.018(3) 0.0375(21) 0.0305</td>
<td></td>
</tr>
</tbody>
</table>
transitions well, whereas population by $K$ (SDM) model (Equation 4-95 in Ref. [7]) in several cases. Transitions was described well by the spin-dependent mixing least-squares fit at unsafe scattering angles beyond two bands was evaluated using the results of a correlated errors using the criterion loss of accuracy in the reproduced yields. Final adjustments of $K > \pi$ band were strongly coupled, so that their matrix elements could be eliminated if it could not reproduce a significant fraction in which case the intrinsic matrix element was measured as a pathway was either eliminated as a possibility or confirmed, the matrix elements which gave the minimum value of the Alaga rule was found to reproduce probabilities for the particular multipolarity and the change of the measured yields using reasonable reduced transition was eliminated if it could not reproduce a significant fraction in which case the intrinsic matrix element was measured as a.

3. The $K^\pi = 14^−$ band

The $K^\pi = 14^−$ intraband $\gamma$ decays were not visible in the data sets, but an upper limit of population strength was set at 50% of the $16^+ \pi^+$ band population for the $16^−$ level, based on a tentative peak in the garded doubles spectra of the Hf(Xe, Xe)Hf experiment. The $1-\mu s$ time window for $\gamma$-ray collection did not permit observation of the $14^−$ ($t_1/2 \approx 68 \mu s$) isomer decays.

C. Fit of matrix elements: The $K^\pi = 0^+, 2^+, 4^+$ and $8^−$ bands

The effectiveness of a particular excitation path between two bands was evaluated using the results of a $\chi^2$ minimization of the calculated $\gamma$-ray yields with the coupled-channel semiclassical Coulomb excitation code GOSIA [45]. A pathway was either eliminated as a possibility or confirmed, in which case the intrinsic matrix element was measured as a single parameter connecting the two bands. An excitation path was eliminated if it could not reproduce a significant fraction of the measured yields using reasonable reduced transition probabilities for the particular multipolarity and the change in single-particle structure. In cases where multiple excitation paths were discovered, the relative importance of each can be evaluated by its contribution to the calculated yields at the $\chi^2$ minimum. The Alaga rule was found to reproduce $K$-allowed transitions well, whereas population by $K$-forbidden transitions was described well by the spin-dependent mixing (SDM) model (Equation 4-95 in Ref. [7]) in several cases.

For the strongly coupled $K^\pi = 0^+, 2^+$, and $4^+$ bands, an iterative fit process was employed to deduce the reduced matrix elements. The quadrupole moment of the most strongly populated band, the GSB, was adjusted first. After finding the matrix elements which gave the minimum value of $\chi^2$, the band with the next highest population, the $\gamma$-band, was added to the system along with its measured $\gamma$-ray yields. This procedure was repeated, fitting all of the adjustable parameters (quadrupole moments and interband $E2$ and $M1$ matrix elements) at each step. The GSB, the $\gamma$ band and the $4^+$ band were strongly coupled, so that their matrix elements could not be fit independently without the iterative technique. The $K > 4$ bands were treated as small perturbations without any loss of accuracy in the reproduced yields. Final adjustments of each measured matrix element were made by measuring the correlated errors using the criterion $\chi^2(\tilde{x} \pm \sigma) = \chi^2(\tilde{x}) + 1$, where $\tilde{x}$ is the best value at $\chi^2_{\text{min}}$ [46] (Table IV). For the least-squares fit at unsafe scattering angles beyond $\theta_{\text{lab}}^\text{min} = 52^\circ$, the weight of the data was reduced to $\frac{1}{2}$ ($\frac{1}{4}$, in terms of $\chi^2$) relative to the data from the safe region, so that nuclear interference effects in the data would not greatly influence the purely electromagnetic analysis.

D. Fit of matrix elements: The $K^\pi = 16^−$ band

Because the first experiment showed that the excitation function of the GSB levels can be described accurately assuming a rigid rotor, the $16^−$ band’s excitation function can in principle be used to extract a large subset of the $\langle I_K=16||E2||I_{GBS} \rangle$ matrix elements. As the beam energy in the target is increased, higher GSB levels become populated with sufficient strength to contribute to the $16^−$ isomer activation, and matrix elements coupling higher-spin levels become important. Ideally, one matrix element for each GSB level would be far more effective than the others, either because of the change $\Delta I$ in spin or because of the $E5$ factor in the $\gamma$ decay width in cases where feeding dominates.

The $16^+$ isomer activity data and the prompt $19_{K=16}^+ \gamma$-ray yields of the first experiment (Fig. 12) were combined in an effort to find a single set of matrix elements that would reproduce the data of both experiments. An unsuccessful attempt was made to reduce the number of fit parameters using the SDM model, which was found to fail for this high-$K$ band. Ultimately, the GSB→$K^\pi = 16^−$ matrix elements were adjusted individually to reproduce the data, observing the

<table>
<thead>
<tr>
<th>$E_{\text{beam}}$ (% barrier)</th>
<th>Count time (h)</th>
<th>$\gamma$ Singles rate ($h^{-1}$)</th>
<th>Doubles $\sigma^{16−}$ (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>86 (Targ. 1)</td>
<td>16.5</td>
<td>173.10</td>
<td>123.9</td>
</tr>
<tr>
<td>80 (Targ. 3)</td>
<td>17.5</td>
<td>80.6</td>
<td>73.7</td>
</tr>
<tr>
<td>76 (Targ. 4)</td>
<td>105.6</td>
<td>24.2</td>
<td>17.315</td>
</tr>
<tr>
<td>73 (Targ. 5)</td>
<td>237</td>
<td>6.916</td>
<td>10.914</td>
</tr>
</tbody>
</table>

FIG. 12. Experimental and calculated in-band $\gamma$-ray yields calculated using the matrix elements from the data of the Hf(Xe, Xe)Hf and Ta(Hf,Hf)Ta experiments.
TABLE IV. The intrinsic matrix elements \( m \equiv \langle K_f | M \lambda | K_i \rangle = m_0 + \Delta \tilde{T} m_1 \). Errors for the \( 6^+ \) band intrinsic matrix elements were propagated from measured isomer branching ratios. Correlated errors are given for the other bands.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( v )</th>
<th>( m_0 )</th>
<th>( m_1 )</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2^+</td>
<td>E2</td>
<td>0^+ )</td>
<td>0</td>
<td>0.266(12) eb</td>
</tr>
<tr>
<td>( 4^+</td>
<td>E2</td>
<td>2^+ )</td>
<td>0</td>
<td>0.45(2) eb</td>
</tr>
<tr>
<td>( 4^+</td>
<td>E2</td>
<td>0^+ )</td>
<td>2</td>
<td>( 9.1 \times 10^{-4} ) eb</td>
</tr>
<tr>
<td>( 4^+</td>
<td>M1</td>
<td>0^+ )</td>
<td>3</td>
<td>( 6.3 \times 10^{-3} ) ( \mu_N )</td>
</tr>
<tr>
<td>( 6^+</td>
<td>E2</td>
<td>4^+ )</td>
<td>0</td>
<td>0.094(3) eb</td>
</tr>
<tr>
<td>( 6^+</td>
<td>E2</td>
<td>2^+ )</td>
<td>2</td>
<td>0.00116(10) eb</td>
</tr>
<tr>
<td>( 6^+</td>
<td>E2</td>
<td>0^+ )</td>
<td>4</td>
<td>( 1.57 \times 10^{-6} ) eb</td>
</tr>
<tr>
<td>( 6^+</td>
<td>M2</td>
<td>8^+ )</td>
<td>0</td>
<td>0.102(9) ( \mu_N b^{1/2} )</td>
</tr>
<tr>
<td>( 8^-</td>
<td>E3</td>
<td>2^+ )</td>
<td>3</td>
<td>( 0.36^{+0.07}_{-0.01} ) ( \text{eb}^{3/2} )</td>
</tr>
<tr>
<td>( 8^-</td>
<td>E3</td>
<td>0^+ )</td>
<td>5</td>
<td>( 0.37^{+0.07}_{-0.01} ) ( \text{eb}^{3/2} )</td>
</tr>
</tbody>
</table>

\(^a\)To first order in \( \Delta \tilde{T}^2 \).
\(^b\)For \( I_{\text{GSB}} \gamma > 9 \) only; reduced matrix elements were attenuated for \( I < 8 \).

It was found during the adjustment of the matrix elements that the majority (\( \gtrsim 75\% \)–\( 80\% \)) of the isomer activation comes directly from connections between the GSB and the \( 17^+ \) and \( 16^+ \) isomer band levels (e.g., Fig. 13), as long as the intrinsic matrix element coupling the GSB and the isomer band does not decrease with increasing spin and reasonable upper limits are imposed on the \( B(E2) \) strengths. This means that the activation function (Fig. 14) of the Ta(Hf,Hf)Ta experiment and the \( 19^+ \) yield calculated for the Hf(Xe, Xe)Hf experiment could be fit almost independently, an obvious constraint being that a smooth, monotonic trend in the magnitudes of the matrix elements is maintained. A second constraint on the fit is that the matrix elements do not introduce \( \gamma \)-decay feeding strengths greater than the measured upper bounds (\( \approx 10^{-4} \) relative to the \( 8^+_\text{GSB} \rightarrow 6^+_\text{GSB} \) yield) on feeding in the Hf(Xe, Xe)Hf experiment.

There was insufficient sensitivity to determine the complete set of matrix elements with correlated errors, but a coherent set of matrix elements with upper limits, some lower limits and diagonal (uncorrelated) errors was found that meets the physical constraints described above. Feeding from the GSB is significant (Fig. 13), but the matrix elements which reproduce the yields are consistent with nonobservation of feeding. In particular, the implied 1\% \( 20^+_{\text{GSB}} \rightarrow 20^+_{K=16} \gamma \)-decay branch is three times smaller than the observable lower limit. The energetically favored high \( E\gamma \) GSB\( \rightarrow K = 16 \) transitions could not be observed, due to the lack of available double-\( \gamma \) gates and clean single-\( \gamma \) gates.

For the phases of the matrix elements, the arbitrary choice was made to use the signs of the Clebsch-Gordan coefficients \( \langle I_{\text{GSB}} | -\lambda \lambda \lambda | I_K = 16 \rangle \), borrowed from the SDM model, but the goodness of the fit was found to be insensitive to the relative phases, because feeding is more important than direct

FIG. 13. Population modes of the \( 16^+ \) isomer band based on the best fit matrix elements. The fractions of the population of each level attributed to direct Coulomb excitation and \( \gamma \)-decay feeding are given for Hf(Xe, Xe)Hf scattering over \( 52^\circ \)–\( 78^\circ \).

FIG. 14. Measured \( 16^+ \) isomer activity versus bombarding energy (points) and predicted activity (line) from the direct fit of the GSB\( \rightarrow K^\pi = 16^+ \) matrix elements with \( \chi^2 = 3.5 \). Points are labeled with measured \( \sigma_{16^+} \) (measured) and \( \sigma_{\text{Rutherford}} \) (calculated for the mean projectile energy in the target). The gray bars indicate the projectile energy range in the targets.
shown in Figs. 16 and 17. The overall agreement between the measured and calculated yields for a wide range of spin, (lines) from the best fit matrix elements.

population in the case of the $16^+$ isomer activation and the $19^+_K=16$ level yield.

IV. RESULTS

A. The ground-state band

The population of the GSB via heavy-ion induced Coulomb excitation is insensitive to the quadrupole moments of the lowest-spin levels, so only $E2$ matrix elements connecting the $6^+$ through $18^+$ states were adjusted individually. Allowing the $E2$ matrix element connecting each pair of GSB levels, coupled to the static moment of the upper level, to vary independently resulted in a seven-parameter fit of the intrinsic quadrupole moments for the $6^+$ through $18^+$ states. The intrinsic quadrupole moment for transitions between GSB states $\sqrt{\frac{2e}{I+1}} q_0$ is nearly constant at 2.164(10) eb, with deviations of $\leq 1.5\%$ between the ground state and the $18^+$ state. Calculated GSB yields reproduced by the fit are presented in Fig. 15. Up to $I \approx 16^+$, the agreement between the measured and calculated yields is good, but the calculated yields are higher than the measurements for higher spin. The deviation occurs at the highest states populated, where the only observable yields are at unsafe scattering angles, where destructive nuclear interference could be a significant factor. Regardless of the cause, the deviations at the top of the GSB are not detrimental to the rest of the analysis. The results shown in Fig. 15 are a small subset of the GSB, $\gamma$ band, and $4^+$ band yields used in the fit and do not represent the overall systematic deviation in the GSB yields at the $\chi^2$ minimum.

B. The $K^\pi=2^+ \gamma$ band

The lowest levels of the $\gamma$ band were populated nearly to the $10\%$ level in the $\text{Hf}(\text{Xe, Xe})\text{Hf}$ experiment, relative to the $8^+_\text{GSB} \rightarrow 6^+_\text{GSB}$ yield. Typical results from the $\gamma$-band fit are shown in Figs. 16 and 17. The overall agreement between the measured and calculated yields for a wide range of spin, multipolarity, and scattering angles is remarkable, considering that the $\gamma$-band data were fit with only one parameter to describe the connection between the GS and $\gamma$ bands using the Alaga rule with the a first-order Mikhailov term. The only significant disagreements (by factors of $\approx 2$–5) are in the intraband transition yields, where errors in the branching ratios and the difficulty in measuring branching ratios for higher spin may have led to an inaccurate extrapolation of the Mikhailov term in the matrix element.

C. The $K=4^+$ band

The SDM model of Bohr and Mottelson [Eq. (1)] for $K$-forbidden transitions was used to reduce the number of parameters in fitting the $E2$ and $M1$ matrix elements coupling the GSB ($K=0^+$) and the $K=4^+$ band (Fig. 18). The matrix elements are given by

$$
\langle K_f I_f | M(\lambda) | K_i I_i \rangle = N \sqrt{2I_i + 1} |I_i K_f - \lambda \lambda \lambda |I_f K_f\rangle \times \sqrt{(|I_i - K_f| + \nu)! \times (I_i - K_f - \nu)!} \times K_f |m_{\Delta K=\lambda,-\nu,\mu=\lambda} |K_i\rangle,
$$

where the normalization $N$ is $\sqrt{2}$ for $K_f = 0$ and 1 for $K_i \neq 0$. (The Mikhailov term makes $\langle K_f |m_{\Delta K=\lambda,-\nu,\mu=\lambda} |K_i\rangle$ non-unique.)
Four quantities were adjusted in the linear fit of A. B. Hayes et al. of the K for either choice of phase, owing to the relative phases of the backward angles (phase “D”) and ratios as a single parameter in the fit. The value of \( \chi^2 \) was minimized for two relative phases of the \( K = 2^+ \rightarrow E2 \rightarrow K = 4^+ \) and \( K = 0^+ \rightarrow E2 \rightarrow M1 \) \( K = 4^+ \) excitations, giving \( \chi^2 = 0.83 \) for the more destructive choice in the backward angles (phase “D”) and \( \chi^2 = 1.09 \) for the more constructive choice (phase “C”), shown in Figs. 19 and 20. The interference is highly destructive (\( \approx 75\% \)) at the \( \chi^2 \) minimum for either choice of phase, owing to the relative phases of the Clebsch-Gordan coefficients in the SDM and Alaga systems. The Alaga rule (for K-allowed transitions) and the SDM model (for K-forbidden transitions) simultaneously reproduced the many features of the 4\(^+\) band yield data—using reasonable reduced transition probabilities (\( \lesssim 10 \) W.u.), even for the high-spin states.

An earlier analysis of the present fit results [4] concluded that the 4\(^+\) band was populated predominantly by a two-step process from the GSB, through the \( \gamma \) band. A more systematic analysis of the effect of each intrinsic matrix element indicates that strong interference effects actually make the GSB \( \rightarrow \gamma \)-band \( \rightarrow 4^+ \) paths approximately equal in importance. The less destructive sign choice results in a smaller value of \( \langle 4^+ | E2 | 2^+ \rangle \) at the \( \chi^2 \) minimum, a corresponding \( \approx 30\% \) reduction in the \( B(E2; K = 2 \rightarrow K = 4) \) strength and an intrinsic moment ratio of \( \langle 4^+ | E2 | 2^+ \rangle / \langle 2^+ | E2 | 0^+ \rangle = 1.44(9) \), where the harmonic limit is \( \sqrt{2} \). However, this choice of phase is less accurate in its overall reproduction of the behavior of the \( \gamma \)-ray yields as a function of scattering angle (Figs. 19 and 20). In addition, this choice of phase

FIG. 19. Measured and calculated \( K^+ = 4^+ \) intraband yields showing interference effects. Phases “D” and “C” of the \( K = 2^+ \rightarrow K = 4^+ \) and \( K = 0^+ \rightarrow K = 4^+ \) intrinsic matrix elements are given by the solid and dashed lines, respectively (see text).

FIG. 20. Measured and calculated interband yields for the 4\(^+\) band. Phases “D” and “C” of the \( K = 2^+ \rightarrow K = 4^+ \) and \( K = 0^+ \rightarrow K = 4^+ \) intrinsic matrix elements are given by the solid and dashed lines, respectively (see text).
gives a ≈15% greater, though not physically unreasonable, \( \langle 4^+|E2|\text{GSB} \rangle \) matrix element at the \( \chi^2 \) minimum.

The more destructive relative phase of the \( \langle 4^+|E2|\gamma \rangle \) matrix element with respect to \( \langle 4^+|E2|\text{GSB} \rangle \) gives the best overall agreement with the intraband and interband yield data. Some calculated yields rise above the measured data, but the more destructive phase reproduces the complicated yield vs. \( \theta_{\text{scatter}} \) slope features remarkably well (Figs. 19 and 20).

The matrix elements for \( K \)-allowed transitions between the GSB, the \( \gamma \) band and the \( 4^+ \) band reproduce the data well, verifying that the Alaga rule, with an added Mikhailov coupling term where necessary, is sufficiently accurate for describing the \( K \)-allowed matrix elements coupling to the high-\( K \) bands, i.e., the \( 6^+ \) band. The SDM matrix elements were successful in the Coulomb excitation calculations for the population of the \( 4^+ \) band from the GSB, showing that the model is useful for low \( K \) and low spin. Although the 
\[ \frac{V_{4^+\gamma}}{(I_{\text{GSB}}-K_{\gamma})(I_{\text{GSB}}+K_{\gamma})} \]
term in Eq. (1) will behave pathologically for high spin, where the perturbation breaks down, the SDM description predicts physically reasonable matrix elements as long as it is applied to levels having spin \( I \) not large compared to \( K \).

D. The \( K^* = 8^-_1, 8^-_2 \) bands

Coulomb excitation calculations showed that only \( E3 \) matrix elements coupling the GSB and the \( K^* = 2^+ \) \( \gamma \) band to the \( 8^- \) bands (Fig. 18) can reproduce the measured population strengths of the \( K^* = 8^- \) bands with smoothly varying, reasonable \( B(E3) \) values \( \lesssim 1 \) W.u. Intermediate steps through bands populated below the \( 10^{-2} \) level were ineffective, leaving the GSB and the \( \gamma \) band as the only viable candidates to populate the \( 8^- \) bands. The \( 8^- \) isomer can decay to the \( 8^-_{\text{GSB}} \) level via an \( E1 \) or \( E3 \) transition and to the \( 6^-_{\text{GSB}} \) level via an \( E3 \) transition, so that its long lifetime \( (t_{1/2} \approx 4 \) s) puts a very small upper limit on the \( E1 \) and \( E3 \) matrix elements that can populate the \( 8^-_1 \) isomer state from the GSB. Because it is not possible to populate the \( 8^- \) bands to the measured strength without a strong GSB contribution, the GSB→\( 8^-_1 \) matrix elements must increase rapidly with increasing spin. For this reason, the \( \langle 8^-|E3|\text{GSB} \rangle \) matrix elements were attenuated so that \( B(E3; K = 0 \rightarrow K = 8) \) decreased by approximately an order of magnitude per unit of spin as \( I_{\text{GSB}} \) decreased from 10 to 6 W.u. (Fig. 21). A similar attenuation was applied to the \( \langle 8^-|E3|K^* \rangle = 2^+ \) matrix elements.

Calculations using a single \( K \) admixture in the GSB and in the \( \gamma \) band gave the best agreement in terms of intraband \( \gamma \)-ray yields, isomer cross section, and reasonable \( B(E3) \) strengths. Fitting the measured intraband yields with the SDM model more accurately reproduced the absence of population staggering between odd and even spin levels in the isomer band data, whereas fits of an intrinsic matrix element in the Alaga rule produced more staggering, regardless of the particular admixture used. However, the SDM model fails in the case of the \( 8^- \) band, because the high degree of forbiddenness causes some matrix elements to take unphysically large values at high spin, some ineffective transitions taking values of \( \sim 100 \) W.u. at the \( \chi^2 \) minimum. The slope of calculated yield versus scattering angle was overpredicted by the SDM model by about a factor of 3 with respect to the Alaga rule calculations.

Intraband \( M1 \) moments and interband \( E2 \) and \( M1 \) intrinsic matrix elements connecting the two \( 8^- \) bands were derived from the data of Smith [47] and Mullins [32,42], which gave \( g_K - g_R = 0.51(5) \) for the \( 8^-_1 \) band, assuming the same quadrupole moment as that of the GSB (Sec. V A). A value of \( g_K - g_R = 0.32(4) \) was calculated for the second \( 8^- \) band from branching ratios of Mullins [42]. The interband intrinsic matrix element \( \langle K^* = 8^- | E2 | K^* = 8^- \rangle = 0.17(3) \) eb was calculated from the only known ratio of intraband/interband \( E2 \) \( \gamma \) intensities for the \( 8^- \) bands [47]. This gave \( \langle K^* = 8^- | M1 | K^* = 8^- \rangle = 1.10(6) \) \( \mu_N \). The limited data on the \( 8^-_2 \) band necessitated locking \( \langle 8^-_1 | E3 | \text{GSB} \rangle = \langle 8^-_2 | E3 | \text{GSB} \rangle \) as one parameter and \( \langle 8^-_1 | E3 | \gamma \rangle = \langle 8^-_2 | E3 | \gamma \rangle \) as a second parameter in the \( \chi^2 \) minimization. Both \( 8^- \) bands were populated to approximately equal strengths, so within the precision of the measured yields of the \( 8^- \) bands, this is a good approximation. The gyromagnetic ratios and the \( \langle 8^-_1 | M4 | 8^-_1 \rangle \) matrix elements were held constant.

When the second \( 8^- \) band at 1479 keV was included in the calculations, \( \gamma \) decay feeding from the \( 8^-_2 \) band to the \( 8^- \) isomer band reduced the staggering in the calculated yields, so that the Alaga rule fits, using \( K = 5^+ \) admixtures in the low-\( K \) bands, gave good results with \( \langle 8^-|E3|\gamma \rangle = 0.36^{+0.00}_{-0.06} \) eb\(^{3/2}\) and \( \langle 8^-_1 | E3 | \text{GSB} \rangle = 0.37^{+0.07}_{-0.01} \) eb\(^{3/2}\). From these parameters and the forced attenuation at low spin, the Alaga rule gave matrix elements corresponding to \( B(E3; K_i \rightarrow K_f = 8) \) values that range from \( 10^{-4} \) to 4 W.u. for the GSB connections, except for the very small \( \langle I_{K=8}|E3|I_{K=0} \rangle \) and \( \langle I_{K=8}|E3|I_{K=8} \rangle \) matrix elements, which were effectively set to zero. 

**FIG. 21.** Values of the intrinsic matrix elements \( \langle K^* = 8^-_1 | E3 | K^* = 0^+ \rangle \) (solid line) and \( \langle K^* = 8^-_1 | E3 | K^* = 2^+ \rangle \) (dashed line) vs. spin in the \( K = 0 \) 2 bands. Both \( \langle K^* = 8^-_1 | E3 | K^* = 0^+ \rangle \) and \( \langle K^* = 8^-_1 | E3 | K^* = 2^+ \rangle \) were fit as a single parameter. The intrinsic matrix elements for \( I_{K=0} \geq 12 \) were held to a single value. The plotted values correspond to \( B(E3; 12^-_{K=0} \rightarrow 15^-_{K=9}) = 1.7 \) W.u. and \( B(E3; 9^-_{K=3} \rightarrow 12^-_{K=8}) = 2.9 \) W.u. using the Alaga rule as described in the text. The Weisskopf unit taken in the upward direction is \( B(E3; 0^+ \rightarrow 3^-)_{\gamma} = 0.0132 \) eb\(^2\).
values for the \( \gamma \)-band\(\rightarrow 8^-\) band transitions ranged from \(7 \times 10^{-4}\), as a result of the attenuation applied at low spin in the \( \gamma \) band, to 4 W.u. at high spin. [In the present discussion, the Weisskopf unit is taken in the upward or excitation direction as \(B(E3; 0^+ \rightarrow 3^-) = 0.0132 \, \text{e}^2 \text{b}^2\).]

Nearby nuclei with \(N \approx 100\) are characterized by typical octupole strengths closer to 1 W.u. for the \(3^- \rightarrow 0^+\) transitions, but the strength of the \(3^+_K \rightarrow 0^+_\text{GSB}\) transition in \(^{178}\)Hf has been measured to be 4 W.u. \(^{[48]}\), so the maximum values here of \(\leq 4\) W.u. are not unrealistically large. Because the calculated yields are not extremely sensitive to reduction of the few matrix elements with \(B(E3) \approx 4\) W.u., additional data might in fact show that the largest matrix elements are actually smaller than those given by the Alaga rule. Figures 22 and 23 display the agreement between the measured and calculated \(8^-\) bands yields. The calculated yields for the \(K = 5\) admixture put an upper limit on the population of the \(8^-\) bands by \(\gamma\)-decay feeding from the low-\(K\) bands of \(\leq 10^{-3}\) of the total yield. Admixtures other than \(K = 5\) in the low-\(K\) bands give very similar results, so the same general conclusions would be drawn from any postulated admixture, but in the absence of any contradictory information, it was assumed that the lowest \(K\) admixtures enter the GSB wave function first and dominate the GSB\(\rightarrow 8^-\) transitions.

From the \(\chi^2\) fit, the population was calculated to be \(\approx 50\text{–}60\%\) from the \(\gamma\) band, depending on the scattering angle range. Coulomb excitation calculations showed that the \(10^3_\text{GSB}\) level is responsible for the largest fraction of the population from the GSB, regardless of the particular model chosen, and is mostly determined by the experimental parameters (beam energy, etc.). The \(10^3_\text{GSB} \rightarrow 12^+_K\) and \(10^3_\text{GSB} \rightarrow 13^+_K\) excitations to the isomer band are the most effective, with the excitation of the unnatural parity \(12^-\) state competing because of the 4 times greater \(B(E3)\) value of the Alaga rule systematics for \(K = 5\) to \(K = 8\) coupling.

Hamilton \(^{[8]}\) and Xie \(^{[9]}\) have measured the probability of exciting the \(8^-\) isomer from the ground state by Coulomb excitation. Xie \textit{et al.} measured a lower cross section than Hamilton \textit{et al.}, and calculations of the cross sections for these two experiments based on the matrix elements from the present work lie between the measurements of Hamilton and Xie (Table V).

Hamilton \textit{et al.} achieved the first Coulomb excitation of the \(K = 8^-\) isomer in a 35-mg/cm\(^2\)\(^{178}\)Hf target with 594 MeV \(^{136}\)Xe ions. By comparing the beam-off \(\gamma\)-ray yield of the GSB \(8^- \rightarrow 6^-\) transition to the beam-on yield, the isomer population was measured relative to the \(8^+_\text{GSB}\) yield at \(\approx 0.9\%\) (Table V). The present results were used to simulate the Hamilton experiment, integrating over \(4\pi\) sr scattering, giving a calculated isomer decay feed of 0.5\%, i.e., within a factor of 2 of the Hamilton result.

Xie \textit{et al.} Coulomb excited a 0.5-mg/cm\(^2\) \(^{178}\)Hf target with \(^{130}\)Te beams at 560, 590, and 620 MeV and counted delayed GSB \(\gamma\)-ray yields. A simulation of the Xie experiment based on the present results predicts an absolute cross section \(\sigma_{8^-}\) five to six times larger than that reported by Xie \textit{et al.}. The present set of model-dependent matrix elements reproduced the measured prompt \(8^-\) isomer band yields with \(\chi^2 \approx 1.6\).

![FIG. 22. Intraband yields of the \(8^-\) isomer band and calculated yields from the best fit: total calculation (solid line), calculation for the \(\gamma\)-band\(\rightarrow 8^-\) path only (dotted line) and calculation for the GSB\(\rightarrow 8^-\) path only (dashed line).](image)

![FIG. 23. Interband yields for \(\gamma\) decays from the upper \(8^-\) band at 1479 keV to the \(8^+\) isomer band and calculated yields from the best fit (solid line).](image)

**TABLE V.** Comparison of the measurements of Xie \textit{et al.} \(^{[9]}\) \(^{[178]}\)Hf\(^{130}\)Te, \(^{[178]}\)Hf, \(^{[130]}\)Te\(^{178}\)Hf, thin target\) and Hamilton \textit{et al.} \(^{[8]}\) \(^{[178]}\)Hf\(^{136}\)Xe, \(^{136}\)Xe\(^{178}\)Hf, thick target\) to predictions based on the present best fit matrix elements. The intensity ratio \(r\) is defined as \(\frac{\sigma_\text{GSB}}{\sigma_\text{GSB, delayed}}/\frac{\sigma_\text{GSB}}{\sigma_\text{GSB, total}}\). The experiments are described in the text.

<table>
<thead>
<tr>
<th></th>
<th>560 MeV</th>
<th>Xie \textit{et al.}</th>
<th>Hamilton \textit{et al.}</th>
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<td></td>
<td>cross sections (mb)</td>
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<td>590 MeV</td>
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<td></td>
<td>2.7 \pm 1.9</td>
<td>4.3 \pm 1.4</td>
<td>7.5 \pm 1.2</td>
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<tr>
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<td>15.7</td>
<td>25.3</td>
<td>37.6</td>
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<tr>
<td>Calc.</td>
<td>5.8</td>
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<td>5.0</td>
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and a $\sigma_6$ value lying between those of the Xie and Hamilton experiments.

Coulomb excitation calculations were used to rule out significant $E1$ and $E5$ contributions to the population of the $K = 8^-$ bands. Electric dipole transitions between the low-$K$ and $K = 8$ bands would be dominated by $>90\%$ $\gamma$-decay feeding, even where feeding transitions into the $8^-$ band have $E_{\gamma} \approx 300$ keV. $E1$ transitions with reduced transition strengths of the order of $10^{-5}$ W.u. (comparable to known values in $^{176}$Hf [33,49]) could populate the $8^-$ bands to the measured levels from the GSB or the $\gamma$ band, but the measured feeding upper limits restrict the $E1$ contribution to $\sim 5\%$ of the measured yields. Assuming a set of extremely large $E5$ matrix elements, 1 W.u. for the $9_{K=8} \rightarrow 4_{GSB}^\pi$ transition, results in predicted $8^-$ band yields that were more than three orders of magnitude too small, ruling out a significant $E5$ contribution.

E. The $K^* = 6^+$ isomer band

Rather than using a $\chi^2$ fit to determine the $6^+$ band matrix elements, measured $B(E2)$ values of the $6^+$ isomer decays (Table II) were used to extrapolate the matrix elements in the Alaga, Mikhailov, and SDM systematics as appropriate. Coulomb excitation calculations with these matrix elements reproduced the measured in-band yields of the $6^+$ band. The matrix element $\langle 6^+|E2|4^+ \rangle = 0.094(3)$ eb has been determined from the measured $B(E2)$ value, because the $K$-allowed transitions between the $4^+$ and $6^+$ bands should be well-described by the Alaga rule. The $K = 4^+ \rightarrow K = 6^+$ path (Fig. 18) was found to be important in the Coulomb excitation of the $6^+$ isomer band (Fig. 24). Measured errors in the reduced transition probabilities for the $6^+$ isomer decay branches were used to directly calculate errors in the intrinsic matrix elements (Table IV). Each quoted error represents the sum of several contributions, primarily from the measured $\gamma$-ray yields ($\leq 10\%$) and the relative $\gamma$-ray efficiency ($\approx 5\%$).

The calculated $\gamma$-ray yields are in agreement with the measured $\gamma$-ray triples yields for the $6^+$ band for safe scattering angles forward of $60^\circ$, as shown in Figs. 25 and 26. The contributions of the GSB and the $\gamma$ band have been determined, insofar as the SDM model [Eq. (1)] correctly describes the $K$-forbidden transitions, along with the relative phases of the intrinsic matrix elements $\langle K^* = 6^+|E2|K^* = 2^+ \rangle$ and $\langle K^* = 6^+|E2|K^* = 4^+ \rangle$, indicating that constructive interference is required to reproduce the yields. It appears that the $\gamma$- and $4^+$ band connections together are responsible for $>50\%$ of the $6^+$ band population at safe scattering angles. Connections with the GSB are responsible for $\leq 25\%$ at forward angles, but may be responsible for the majority of the $6^+$ band excitation at unsafe scattering angles. The correct phase of the $\langle 6^+|E2|\text{GSB} \rangle$ intrinsic matrix element, relative to the other two population paths, cannot be determined from the present data. The relative significance attributed to connections with the GSB and the $\gamma$ band depend on the model used.
because the Alaga rule does not describe these $K$-forbidden transitions.

**F. The $K^* = 16^+$ band**

The best agreement with the slope of the activity vs. the bombarding energy was obtained by setting $(16^+_{K=16}∥E2∥14^+_{GSB}) = (16^+_{K=16}∥E2∥16^+_{GSB}) = (16^+_{K=16}∥E2∥18^+_{GSB})$. Matrix elements coupled to the $18^+_{K=16}$ state were increased to the feeding limit in the fit. The overall magnitudes of the $17^+_{K=16}$ matrix elements were roughly interpolated between the values of the $16^+_{K=16}$ and $18^+_{K=16}$ ones, continuing the trend of increasing $K$ mixing with increasing spin.

The matrix elements for the $16^+_{K=16}$, $17^+_{K=16}$, and $18^+_{K=16}$ levels given in Fig. 27 were found to be ineffective in populating the $19^+_{K=16}$ level, whose yield was measured in the Hf(Xe,Xe)Hf experiment, so the measured $19^+ \rightarrow 18^+$ yield could be fit independently by adjusting the matrix elements, connecting the GSB to the $19^+_{K=16}$, $20^+_{K=16}$, and $21^+_{K=16}$ levels. Because most of the $19^+$ population comes from the $(19^+_{K=16}∥E2∥18^+_{GSB})$ and $(20^+_{K=16}∥E2∥18^+_{GSB})$ matrix elements, ranges for the other five matrix elements populating the $19^+$ level could not be determined, except for upper limits based on feeding. The absolute upper limit of $(20^+_{K=16}∥E2∥18^+_{GSB})$ was estimated by setting $(19^+_{K=16}∥E2∥18^+_{GSB})$ to zero and increasing $(20^+_{K=16}∥E2∥18^+_{GSB})$ until $\chi^2 = \chi^2_{\text{min}} + 1$. Correlations with the other five matrix elements were found to be small. Correlated error calculations between the matrix elements $(19^+_{K=16}∥E2∥18^+_{GSB})$ and $(20^+_{K=16}∥E2∥18^+_{GSB})$ could not define any lower limit or further restrict the upper limit for $(19^+_{K=16}∥E2∥18^+_{GSB})$, because the $(20^+_{K=16}∥E2∥18^+_{GSB})$ matrix element has a higher upper limit and can independently reproduce the measured $19^+ \rightarrow 18^+$ yield with $\chi^2 < \chi^2_{\text{min}} + 1$. One lower limit on $(20^+_{K=16}∥E2∥18^+_{GSB})$ was obtained.

The diagonal (uncorrelated) errors (Fig. 27) were measured in the six matrix elements that provided enough sensitivity for the calculation. The matrix elements for the $16^+_{K=16}$ level were coupled as one parameter and varied until $\chi^2 = \chi^2_{\text{min}} + 1$, giving a range of 0.20–0.26 e\(b\). The matrix elements of the $17^+_{K=16}$ level were coupled in an identical error calculation, giving ranges of 0.27 e\(b\) $\leq (17^+_{K=16}∥E2∥16^+_{GSB}) \leq 0.44 e\(b\)$ and 0.25 e\(b\) $\leq (17^+_{K=16}∥E2∥18^+_{GSB}) \leq 0.40 e\(b\)$.

More restrictive upper limits (below the feeding limits) for six matrix elements were obtained, based on the measured activities from the second experiment and the prompt $19^+ \rightarrow 18^+$ yields from the first experiment. These are the six points in Fig. 27 with solid black bars. Since each of the three GSB connections to the $16^+$ level contributes approximately the same fraction of the total level population, reducing any two to zero would require a factor of $\approx 3$ increase in the $B(E2)$ value ($\sqrt{3}$ in the matrix element) of the third to maintain the observed yield.

The $17^+$ level connections contribute about $25\%$ of the total isomer yield (Figure 13). If the $16^+$ level connections were reduced to an ineffective level, the $17^+$ matrix elements would have to be increased by a factor of about $\sqrt{3}$. These considerations lead to the upper limits for the first five matrix elements in Table VI and Fig. 27.

Population of the $16^+$ isomer due to feeding from the $t_{1/2} = 68$ $\mu$s, $14^-$ isomer at 2573 keV could make a significant contribution to the $16^+$ activation, if the $14^-$ isomer were populated to a strength comparable to the direct excitation of the $16^+$ band. The known decay branch to the $16^+$ isomer from the $14^-$ isomer is shown with the known relative intensities in Fig. 28. None of the data sets provided an unambiguous yield from the $14^-$ isomer band, but a gate on the $15^+ \rightarrow 14^-$, 337-keV transition in a prompt-prompt matrix above the safe angle $(52^–78^\circ)$ yielded a peak possibly belonging to the $16^+ \rightarrow 15^-$, 355-keV transition, lying in a rich background of coincident peaks. The $18_{K=16} \rightarrow 17_{K=16}$ yield was measured in the same matrix, although the identities of both peaks were
not certain at the $10^{-3}$ level in the two-dimensional data. After correcting for the relative efficiency of the transition energies and internal conversion, the ratio of the populations of the second level in the $16^+$ and $14^+$ isomer bands was found to be $P_{14^+} / P_{16^+} = 0.47$.

Assuming that the 437-keV, $1_{K=14} \rightarrow 12_{K=8}^-$ transition is purely $E2$ in character gives the largest estimate of the $14_{K=14}^- \rightarrow 16_{K=16}^- 12_{K=8}^-$ partial width calculated from the known $68 \mu s$ half-life of the $14^-$ isomer. Then, the following estimate can be made of the $14_{K=14}^-$ feeding contribution to the $16^+$ isomer state. The known $\gamma$-ray intensities and internal conversion coefficients would indicate reduced transition strengths of $B(E3) = 510 \text{ W.u.}$, $B(M2) = 0.014 \text{ W.u.}$, and $B(M4) = 1.5 \times 10^{11} \text{ W.u.}$ The absurdly high values for the $E3$ and $M4$ possibilities leave $M2$ as the only realistic multipole character. If the observed 355-keV peak belongs to the $16_{K=14}^- \rightarrow 15_{K=14}^- 12_{K=8}^- 10_{K=10}^-$ transition (Fig. 28), then a >10% decrease in the matrix elements of Table VI would make up for the feeding contribution from the $14^-$ band.

G. The question of transfer reaction contributions

It can be argued that the isomer bands in question might have been populated in the Hf(Xe, Xe)Hf experiment through transfer reactions involving the $^{177,179}$Hf contaminants (4% and 3%, respectively) in the target. The most straightforward test to address this possibility would be to select events containing transitions in $^{178}$Hf and compare the relative amounts of Xe isotopes in coincidence. Although this shows no indication of transfer reactions, the background of Doppler-broadened lines of hafnium in the resulting Xe spectra is large and does not allow the observation of small amounts of $^{135}$Xe and $^{137}$Xe that would result from potentially significant transfer reactions (Fig. 29). An upper limit on the rate of the transfer reaction $^{178}$Hf($^{136}$Xe, $^{135}$Xe)$^{179}$Hf was set experimentally and then used to estimate the rate of $^{177}$Hf($^{136}$Xe, $^{135}$Xe)$^{178}$Hf transfer.

The spectra of Fig. 29 for safe scattering angles $25^\circ < \theta_{\text{lab}} < 52^\circ$ contain no transitions of Xe isotopes other than $^{136}$Xe, when gated on “random coincidences” [Fig. 29(a)] with two arbitrary energies between the known $^{178}$Hf peaks, when gated on two known GSB transitions [Fig. 29(b)] and on the known transitions of the $^{179}$Hf GSB [Fig. 29(c)]. An upper limit on $^{178}$Hf($^{136}$Xe, $^{135}$Xe)$^{179}$Hf transfer reactions was set using the only possible $^{135}$Xe transition (288 keV) observed in coincidence with a double gate on $^{178}$Hf GSB transitions [Figs. 29(c) and 30(c)]. In the safe Coulomb excitation region, $25^\circ < \theta_{\text{lab}} < 52^\circ$, where significant populations of the $K^\pi = 6^+_\text{isom}, 8^-_{\text{isom}}$ bands are already seen, an upper limit on $^{177}$Hf($^{136}$Xe, $^{135}$Xe)$^{178}$Hf transfer was set at $10^{-5}$ of the $^{178}$Hf GSB excitation. The neutron transfer cross sections reach a maximum at $Q = 0$, and the $Q$ values of the $^{177}$Hf($^{136}$Xe, $^{135}$Xe)$^{178}$Hf ($Q = -0.4 \text{ MeV}$) and $^{178}$Hf($^{136}$Xe, $^{135}$Xe)$^{179}$Hf ($Q = -1.9 \text{ MeV}$) reactions differ by only 1.5 MeV, so the two transfer cross sections are expected to be comparable [50]. Assuming that they are approximately equal, the upper limit on $^{177}$Hf($^{136}$Xe, $^{135}$Xe)$^{178}$Hf reactions in the 4% $^{177}$Hf impurity is $\approx 1/10$ of the observed $16^+$ isomer

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isomers by transfer reactions is in the results of the $^{178}$Hf beam activation experiment, where the $16^+$ isomer was activated with significant cross sections 27% below the Coulomb barrier, consistent with the Coulomb excitation function (Fig. 14).

H. Summary of results

Three distinctly different paths have been determined that Coulomb excite the $K^\pi = 6^+, 8^-$, and $16^+$ isomer bands. Multiple-step excitations populate the $6^+$ band via the $\gamma$ and $4^+$ bands primarily, using both $K$-allowed and $K$-forbidden transitions. The $8^-$ bands are excited directly from the $\gamma$ band and the GSB by highly $K$-forbidden $E3$ transitions. The Hf(Xe, Xe)Hf data are consistent with direct $\gamma$-decay feeding into the $16^+$ band; however, the additional data provided by the Ta(Hf, Hf)Ta activation experiment leads to a departure from the SDM model, and direct excitation is not insignificant in the population of all of the $16^+$ isomer band states. These three results, as well as those for the $\gamma$ and $4^+$ bands have shown that the SDM model is a useful approximation for low spin and low forbiddenness, $\nu$.

The SDM model could not simultaneously reproduce the $16^+$ data of the first and second experiments, even with unrealistically large matrix elements. The population of the isomer band head was found to proceed primarily, $\approx 75\%$, by $\gamma$-decay feeding from the GSB to the $16^+$ and $17^+$ levels. A nearly constant magnitude of the matrix elements was required to reproduce the entire data set from both experiments. Figure 14 shows the agreement between the calculated and measured isomer activity from the Ta(Hf, Hf)Ta experiment with $\chi^2 = 3.5$. The slope and magnitude are reproduced, the largest $\chi^2$ contribution coming from the third target. The agreement with the measured $19^+ \rightarrow 18^+ \gamma$-ray yield ($\chi^2 = 1.9$) of the Hf(Xe, Xe)Hf experiment is shown in Fig. 12. The present set of matrix elements simultaneously reproduced both the measured Ta(Hf, Hf)Ta activation and the $19^+ \rightarrow 18^+$ yield.

V. DISCUSSION

A. Electromagnetic properties

The principal parameters relevant to intraband and interband EM transition probabilities are presented below for the $K^\pi = 0^+, 2^+, 4^+, 6^+, 8^-, 14^-$, and $16^+$ bands studied. Measurements of the two-quasiparticle configurations in the $6^+$ and $8^-$ bands are discussed. The interband intrinsic and reduced matrix elements and a comparison of isomer cross sections are given in Tables IV, VI, and VII, respectively.

1. The GSB

The intrinsic quadrupole moment of the GSB was measured for $I \rightarrow I \pm 2$ transitions, assuming static moments corresponding to a prolate deformation, giving $\sqrt{\frac{\hbar}{\mu_0}} eQ_0 = 2.164(10)$ eb, constant within 1.5% between levels up to spin $18^+$, despite the strong up-bend observed in the moment of inertia. The present value is in agreement with the results of
The intrinsic quadrupole moment of the $\gamma$ band was measured, giving $\sqrt{\frac{1}{160}eQ_0} = 2.21(8)$ eb, in agreement with the GSB value. A linear dependence of the interband intrinsic matrix element $\langle \gamma | E2 | \text{GSB} \rangle$ on $\Delta E^2$ was included, adjusted to simultaneously fit several previous measurements of $B(E2; I_g \rightarrow I_{\text{GSB}})$ [34,43] and the present measured branching ratios. The overall effect of the rotational-vibrational coupling is a $\approx 10\%$ increase in the interband $B(E2; \text{GSB} \rightarrow K = 2)$ strength between the 2$^+$ and 16$^+$ states of the $\gamma$ band, over the Alaga-rule systematics. The $B(E2; \text{GSB} \rightarrow \gamma)$ values resulting from the fit reached a maximum of 4 W.u. The present measurement of the $\langle \gamma | E2 | \text{GSB} \rangle$ matrix element (Table IV) is comparable to values for other nuclei in the $A \approx 180$ mass region [53]. The population of the $\gamma$ band was found to be insensitive to both the intraband and interband $M1$ intrinsic matrix elements, within the experimental error.

previous lifetime measurements of the 2$^+$ state, $\sqrt{\frac{1}{160}eQ_0} = 2.17(3)$ eb [51], and indicates a quadrupole deformation parameter of $\beta_2 \approx 0.25$ [7,52].

2. The $\gamma$ band at 1175 keV

The intrinsic quadrupole moment of the $\gamma$ band was measured, giving $\sqrt{\frac{1}{160}eQ_0} = 2.21(8)$ eb, in agreement with the GSB value. A linear dependence of the interband intrinsic matrix element $\langle \gamma | E2 | \text{GSB} \rangle$ on $\Delta E^2$ was included, adjusted to simultaneously fit several previous measurements of $B(E2; I_g \rightarrow I_{\text{GSB}})$ [34,43] and the present measured branching ratios. The overall effect of the rotational-vibrational coupling is a $\approx 10\%$ increase in the interband $B(E2; \text{GSB} \rightarrow K = 2)$ strength between the 2$^+$ and 16$^+$ states of the $\gamma$ band, over the Alaga-rule systematics. The $B(E2; \text{GSB} \rightarrow \gamma)$ values resulting from the fit reached a maximum of 4 W.u. The present measurement of the $\langle \gamma | E2 | \text{GSB} \rangle$ matrix element (Table IV) is comparable to values for other nuclei in the $A \approx 180$ mass region [53]. The population of the $\gamma$ band was found to be insensitive to both the intraband and interband $M1$ intrinsic matrix elements, within the experimental error.

3. The 4$^+$ band at 1514 keV

As in the $K^\pi = 0^+, 2^+$ bands, a prolate rigid rotor was assumed in fitting the intrinsic transition quadrupole moment of the 4$^+$ band, giving a value of $\sqrt{\frac{1}{160}eQ_0} = 2.07(10)$ eb, consistent with the GSB and the $\gamma$ band. The 4$^+$ band has been tentatively identified as the two-phonon $\gamma$-vibrational band [54,55], based largely on the similarity in the dynamic moments of inertia of the $K^\pi = 2^+$ and $K^\pi = 4^+$ bands. For an isolated harmonic vibrator system, the ratio of the level energies $E_{\text{lash}}/E_{\text{ground}} \approx 2$ is expected, compared to 1.29 in the present case. The Alaga rule fit gave a ratio of intrinsic matrix elements of $\frac{E_{\text{lash}}}{E_{\text{ground}}} = 1.77(11)$, compared to the expectation of $\sqrt{2}$ for pure harmonic vibrators [53]. The energy of the 2$^+$ state is in line with the expectation of $\approx 1$ MeV for a harmonic oscillator with $A = 178$, and the intrinsic matrix element $\langle K = 2^+ | E2 | K = 0^+ \rangle = 0.252(11)$ is similar to the values measured in $^{156}\text{Gd}$, $^{160}\text{Dy}$, and $^{168}\text{Er}$ [53], whereas $\langle K = 4^+ | E2 | K = 2^+ \rangle = 0.45(2)$ eb is closer to values for heavier oxygen nuclei [53]. Assuming that the 4$^+$ state is indeed a two-phonon $\gamma$-vibration, then there is significant anharmonicity, implied by both the energy ratio and the ratio of intrinsic moments.

Previous attempts to measure the lifetime of the 4$^+_K = 4^+_K$ state have succeeded in setting a lower limit of 0.94 ps [37], and the intensity ratios of its $K$-forbidden and $K$-allowed $\gamma$ decays have been measured, along with a single $E2/M1$ mixing ratio [41]. The present Xe beam experiment provided an upper limit on the $4^+_K = 4^+_K$ lifetime of $\approx 4$ ns from the width $\Delta E$ of the Doppler-corrected 1207 keV $4^+_K = 4^+_K \rightarrow 4^+_K E_\gamma$ peak and particle time-of-flight considerations (not to be confused with a Doppler lineshape measurement). For states with lifetimes greater than 1 ns and recoil velocities of $\beta > 0.05$, the measured particle-$\gamma$ opening angle $\theta_{\text{lab}}$ deviates from the true laboratory angle because of the large recoil distance when the $\gamma$ ray is emitted, and the resulting increase in $\frac{\Delta E}{E}$. In the limit as the lifetime $\tau \rightarrow 0$, the resolution is $\frac{\Delta E}{E} \approx 0.5\%$. The measured FWHM of the 1207 keV peak, $\frac{\Delta E}{E} = 0.8\%$, leads to an upper limit on the lifetime of the 4$^+$ state of $\approx 4$ ns. The limits 0.94 ps $< \tau_4^+ < 4$ ns suggest approximate bounds for the $\nu = 2$ $K$-forbidden matrix elements connecting the GSB and 4$^+$ bands, corresponding to reduced hindrance values of approximately $1 < \nu_4 < 90$. The $4^+_K$ lifetime calculated from the present Coulomb excitation data is 90 ps, and the calculated $E2$ fraction is 85(14)% for the $4^+_K = 4^+_K \rightarrow 4^+_K E_\gamma$ decay, satisfying both the present $\gamma$-ray yield data and the previous measurement of 82(10)% $E2$ [41].

4. The 6$^+$ isomer band at 1554 keV

The gyromagnetic ratio for a state of spin $I$ is given by [7]

$$g(I) = g_K + (g_K - g_R) \frac{K^2}{I(I+1)},$$

where $g_K$ and $g_R$ are characteristic of the single-particle and rotational motions, respectively. The two factors $g_K$ and $g_R$ are independent to a first approximation, but are not completely
decoupled \cite{56}.) If the gyromagnetic moments $\mu_1 = g_1 I$ and $\mu_2 = g_2 I$ are known for two particles or quasiparticles, the net moment can be calculated according to the additivity relation \cite{57},
\begin{equation}
g(I) = \frac{1}{2} (g_1 - g_2) + \frac{j_1(j_1 + 1) - j_2(j_2 + 1)}{2I(I + 1)} (g_1 - g_2),
\end{equation}
for two particles with angular momenta $j_1$ and $j_2$ in a state of total angular momentum $I$.

In-band intensity ratios $\Gamma_{\Delta I=2}/\Gamma_{\Delta I=1}$ measured in the Xe beam experiment were used to determine the value of $g_K - g_R$, giving a mean value for the $8^+_K$ and $13^+_K$ states (Fig. 31) of $g_K - g_R = 0.56(2)$. The value $g_K = 0.48(2)$ was obtained using Eq. (2) and a previous measurement of $g = 0.959(8)$ \cite{58} for the $6^+$ state. This leads to a value of $g_K = 1.04(3)$ for the $6^+$ band, in agreement with Mullins’ measurement of $1.06(7)$ \cite{32}, calculated with the same $g$ measurement.

Khoo has proposed a $69\% \pi^+_2 [404], \pi^+_2 [402]$ plus $31\% \nu^+_2 [514], \nu^+ [512]$ structure for the $6^+$ band \cite{44}. The expected values of $g_K^2$ and $g_K^3$ are 0.222 and 0.768, respectively, using Eq. (3) and measured values of the single-particle contributions from even-odd neighbors \cite{59–61}. The $31\% \nu^3$ component seems unlikely, because the present value of $g_K$ is not in agreement with a $\nu^3$ admixture, and the $\pi^+_2$ \cite{512} Nilsson orbital should be occupied in the ground state. It could be argued that there may be other proton admixtures (e.g., the next $\pi^+_2$ \cite{642} orbital in the $f_{7/2}$ subshell) that could contribute to a large value of $g_K$, but this $f_{7/2}$ orbital, combined with the $g_{7/2}$ Proton, would be expected to give $g_K = 0.84$. No other two-proton Nilsson state appears to have the correct parity, $K$ value, and band head energy for the $6^+$ state. The present measurement agrees with Mullins’ conclusion that the $6^+$ band is purely or predominantly $\pi^3$ in character.

The precise measurements of $B(M3)$ values of the $6^+$ isomer decay branches, combined with the Alaga rule (for $K$-allowed transitions) and the SDM model (for $K$-forbidden transitions) yielded a strikingly accurate reproduction of the $6^+$ isomer band Coulomb excitation yields with no adjustable parameters. Within the confines of the Alaga and SDM systematics, the intrinsic matrix elements coupling the $K^T = 0^+, 2^+, 4^+$, and $6^+$ bands were derived from the measured $B(M3)$ values (Table IV).

The correct choice of relative phases of $\langle 6^+|E2|\gamma \rangle$ and $\langle 6^+|E2|4^+ \rangle$ was more apparent than the analogous problem in the $4^+$ band. Because the $\langle 6^+|E2|GSB \rangle$ matrix element is much less effective than the other two, the problem could be treated as a two-path interference problem, showing that the only effective choice of phases is that of constructive interference between $\langle 6^+|E2|\gamma \rangle$ and $\langle 6^+|E2|4^+ \rangle$ (Figs. 25 and 26). At large $\theta_{\text{coul}}$ angles, the $\langle 6^+|E2|GSB \rangle$ term becomes more important. The overprediction of the yields for large $\theta_{\text{coul}} > \theta_{\text{safe}}$ can be explained by two conclusions: (1) The SDM model overpredicts $B(E2)$ strengths at high spin or (2) Coulomb-nuclear interference is becoming important above the $\theta_{\text{safe}} = 53^\circ$ safe angle. The decreased success of the SDM model for higher-$K$ bands indicates the former.

Coulomb excitation calculations, using the final set of matrix elements, indicate that the $I \rightarrow I + 2$ excitations are most effective in populating the $6^+$ band and that the excitation generally follows the path of the fewest possible steps. The three paths to the excitation of the $6^+$ band are illustrated in Fig. 18.

5. The $8^-$ isomer band at 1147 keV

The intrinsic quadrupole moment of the $8^-$ band could not be determined from the intraband $\gamma$-ray yields, because of the high sensitivity of the calculated yields to the interband $E3$ matrix elements. The authors of Refs. \cite{32,56,62} have used the value $eQ_0 = 6.95$ eb ($\sqrt{\frac{1}{2}} = 2.19$ eb), apparently originating from an early measurement of the $B(E2; 2^+_K \rightarrow 0^+_K)$ strength \cite{56}, but direct measurements of the quadrupole moment of the $8^-$ isomer band have not been made. Measurements for the $16^+$ isomer state of $Q_0 = 7.2(1)$ b \cite{63} and $Q_0 = 8.2(11)$ b \cite{64} suggest that the $8^-$ bands would have very similar quadrupole moments to both the $16^+$ band and the GSB, because the $16^+$ isomer is believed to be the product of the two-quasiparticle wave functions of the two $8^-$ bands.

The present analysis of the $8^-$ isomer band population has shown that the coupling (and therefore the mixing) between the two $8^-$ bands is important in reproducing the Coulomb excitation yields of both bands. Previous experiments have led to measurements of the mixing amplitudes between the $8^-$ and $8^+_2$ bands \cite{32,47,56,62,65,66}. Most results indicate that the $8^-_1$ (isomer) band is predominantly a $\nu^3$ band and that mixing is strong, with interaction potentials of $\geq 100$ keV. The present and past measurements of the mixing of the $8^-$ bands can be understood in terms of simple two-state mixing \cite{67}. The mixed-state (perturbed) wave functions $\psi_A$ and $\psi_B$ can be written in terms of the unperturbed (pure) basis states $\phi_A$ and $\phi_B$ as
\begin{equation}
\psi_A = \alpha \phi_A + \beta \phi_B,
\end{equation}
\begin{equation}
\psi_B = -\beta \phi_A + \alpha \phi_B.
\end{equation}
from previous measurements [Fig. 32 (top), error from the

\[ V = \langle \phi_a | \hat{V} | \phi_b \rangle \] (5)

\[ V = \frac{\Delta E_f}{\sqrt{1 + x^2} + 4} \] (6)

terms of the measured separation energy \( \Delta E_f \) between
the mixed states and the mixing fraction \( \beta^2 \) [68], where
\( x \equiv \frac{\pi}{\beta} - 2 \). The observed gyromagnetic ratios \( \tilde{g}_K \) of the
mixed states are given by

\[ \tilde{g}_K^a = (1 - \beta^2)g_K^a + \beta^2 g_K^b \]

\[ \tilde{g}_K^b = \beta^2 g_K^a + (1 - \beta^2)g_K^b \] (7)

where \( g_K^a \) and \( g_K^b \) are the gyromagnetic ratios of the
unperturbed basis states. Then, for the \( ^8_{\tilde{g}} \) system,
composed as \( \psi_{\tilde{g}} = a\phi_1 + \beta\phi_2 \), the observed gyromagnetic
ratio \( \tilde{g}_K = (1 - \beta^2)g_K^a + \beta^2 g_K^b \) can be used to measure \( \beta^2 \).
For any two-state mixing system an interaction energy of \( V \approx \frac{\Delta E_f}{2} \) (166 keV for the \( ^8_{\tilde{g}} \) and \( ^8_{\tilde{s}} \) system) indicates that two
nearly degenerate basis states are almost completely mixed.

Helmer and Reich [65] proposed that the \( ^8_{\tilde{s}} \) isomer band is a mixed
two-quasiproton (\( \pi^0_9 \) [514]\pi^+_7 [404]) and
two-quasineutron (\( \nu^0_7 \) [514]\nu^+_8 [624]) configuration, approx-
imately one third \( \pi^2 \) from their measured \( \gamma \)-ray intensities.
This proposed \( \pi^0_7 \pi^+_7 \), \( \nu^0_7 \nu^+_8 \) configuration is also consistent with
succeeding measurements of the gyromagnetic moments.

Two intraband branching ratios were measured in the
Hf(Xe, Xe)Hf data for the \( 1^{-} \) and \( 10^{-} \) levels. Assuming the
same quadrupole moment as in the GSB, the branching ratios
yield \( |g_K - g_\beta| = 0.12(7) \) for the \( 1^{-} \) state. (The branching
ratio of the \( 10^{-} \) state does not give a real solution for \( g_K - g_\beta \),
and Eqs. (7) imply that the \( 10^{-} \rightarrow 9^{-} \) decay is \( \approx 100\% \) \( E2 \), but the measured \( \gamma \)-ray intensity ratio was \( I_{\gamma}^7/\gamma_{\gamma}^8 = 2.2(3) \),
compared to previous measurements of 1.67(10) [42] and
1.8(5) [47].)

Using \( g_\beta = 0.48(2) \) from the present \( 6^{-} \) band measure-
ments and previous calculated \( g_K \) values [56] for the neutron
and proton configurations of the \( 8^{-} \) band, the present measure-
ments of branching ratios for the \( 11^{-} \) state indicate a \( 58(7)\% \)
\( \pi^2 \) component, about twice the result of Tlustý et al. [62]
(Fig. 32, bottom panel). However, \( V \) differs by at most 15% from
previous measurements [Fig. 32 (top), error from the
branching ratio]. For \( g_\beta \approx 0.3 \) (a commonly assumed value for
\( ^{178} \)Hf), which can be obtained from an unweighted average of experimental \( g_\beta \) values for the quasicomposite configurations of the two \( 8^{-} \) states [69], the present data would give a \( 41(6)\% \)
\( \pi^2 \) component in the \( 8^{-} \) band.

Table VIII gives \( \pi^2 \) admixture fractions in the \( 8^{-} \) states (\( \beta^2 \))
and their corresponding interaction potentials \( V \) from previous
and present measurements. In contrast, in lighter isotopes (e.g.,
\( ^{172} \)Hf [58], \( ^{174} \)Hf [70], \( ^{176} \)Hf [71]) the configuration of the
lower of the two \( 8^{-} \) band heads (the only one observed in
\( ^{172} \)Hf) has been identified as a nearly pure \( \pi^2 \) state, whereas
all previous measurements for \( ^{178} \)Hf indicate a strongly mixed
system, with a dominant \( \nu^2 \) admixture in the isomer band.
However, the energies of the \( 8^{-} \) isomers in the lighter isotopes
are 400–850 keV higher than \( ^{178m} \)Hf, so that a difference might be
expected in the strength of the mixing of the \( 8^{-} \) states in
\( ^{178} \)Hf.

The proton contributions to \( \tilde{g}_K \) are dominant, and with \( g_K \approx 0.3 \) for \( ^{178} \)Hf, a maximum \( 70\% \) \( \nu^2 \) component is predicted in the
limit \( g_K - g_\beta \rightarrow 0 \), using the values \( g_K = 1.03(11) \) for the
\( \pi^0_9 \pi^+_7 \) state and \( g_K = -0.013(3) \) for the \( \nu^0_7 \nu^+_8 \) state
[56]. Naturally, measurements of relative intensities that would
give \( (g_K - g_\beta)^2 < 0 \) after subtraction of the \( E2 \) component
give no meaningful result, so that for estimates of \( g_\beta \approx 0.3 \),
the lower bound on measurements of the \( \pi^2 \) fraction is 30%,
not far from many published measurements (Table VIII and
Figure 32). However, the mixing potential \( V \) varies slowly with
\( \beta^2 \) for strongly mixed states, such as in the present case,
and can be measured to good precision within the two-state
mixing model. (For the two mixed \( \tilde{g}_K \) states separated by
\( \Delta E = 332 \) keV, \( 0.25 \leq \beta^2 \leq 0.35 \Rightarrow 144 \leq V \leq 158 \) keV.)

6. The \( 14^{-} \) isomer band at 2574 keV

It was not possible to find an unambiguous indication of
population of the \( 14^{-} \) isomer band, but an upper limit on the
cross section was calculated from the \( \gamma \)-ray doubles data.
\( \Lambda (K = 14 | E3 | K = 0) \) matrix element was fit to the upper
limit, assuming that the \( 14^{-} \) band is populated directly from
the GSB by matrix elements \( \langle f, K = 14 | E3 | i, K = 0 \rangle \approx 1.3 \) eb\(^{3/2}\), all of approximately equal magnitude. The upper
limit on the isomer cross section was calculated for the Xe beam experiment based on this simple model, giving \( \sigma_{14^{-}} < 2 \) mb, considerably smaller than those of the other isomers
populated in this work (Table VII).
TABLE VIII. A survey of measurements of V and β² values of the 8⁺ band states. Errors in V were calculated from the errors in the authors’ measured quantities. Systematic uncertainties in estimates of gK and gR values are not included.

<table>
<thead>
<tr>
<th>I</th>
<th>gK − gR (or δ)</th>
<th>β²</th>
<th>V (keV)</th>
<th>ν−</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.37(2)³</td>
<td>0.36(2)³</td>
<td>(Ref. [56]) 160(2)</td>
<td>0.48(4)</td>
</tr>
<tr>
<td>8</td>
<td>0.36(2)³</td>
<td>0.36(2)³</td>
<td>(Ref. [62]) 159(2)</td>
<td>0.48(4)</td>
</tr>
<tr>
<td>9</td>
<td>0.36(1)</td>
<td>0.36(1)</td>
<td>(Ref. [79]) 162(2)</td>
<td>0.47(7)</td>
</tr>
<tr>
<td>10</td>
<td>0.31(1)</td>
<td>0.31(1)</td>
<td>(Ref. [62]) 150(4)</td>
<td>0.43(11)</td>
</tr>
<tr>
<td>11</td>
<td>0.22(1)</td>
<td>0.22(1)</td>
<td>(Ref. [42]) 146(5)</td>
<td>0.41(11)</td>
</tr>
<tr>
<td>12</td>
<td>0.19(1)</td>
<td>0.19(1)</td>
<td>(Ref. [47]) 137(3)</td>
<td>0.39(3)</td>
</tr>
<tr>
<td>13</td>
<td>0.18(1)</td>
<td>0.18(1)</td>
<td>(Ref. [62]) 136(6)</td>
<td>0.38(17)</td>
</tr>
<tr>
<td>13</td>
<td>0.34(2)³</td>
<td>0.34(2)³</td>
<td>(Ref. [47]) 167(2)</td>
<td>0.47(7)</td>
</tr>
<tr>
<td>13</td>
<td>0.16(1)</td>
<td>0.16(1)</td>
<td>(Ref. [47]) 131(3)</td>
<td>0.37(1)</td>
</tr>
</tbody>
</table>

*From β-decay experiments.
*From a simultaneous fit of gK − gR to a quadratic function of l². The author’s largest error estimate was used.
*V calculated in Ref. [47], β² calculated from V.
*Reference [56] used gK = 0.262(14).
*From δ, using gK − gR from Ref. [56].
*Calculated in Ref. [62] from the data of Ref. [79].
*Calculated from the intraband intensity ratios of Ref. [42] using gK = 0.3.
*Calculated from the intraband intensity ratios of Ref. [47] using gK = 0.3.
*Present work.

7. The 16⁺ isomer band at 2446 keV

The 16⁺ band appears to be populated directly from the GSB (Fig. 18). In the Xe beam experiment, Coulomb excitation calculations based on the fitted matrix elements showed that the principle mode is by γ-decay feeding from the GSB (≈10% by direct excitation). The E2 strength deduced from the combined data of the Xe-beam and Ta-target experiments is ≈1.4 W.u. (Table VI), and the modest 0.05–0.14 W.u. transitions to the 16⁺ and 17⁺ states are responsible for ≈80% of the isomer population in the Xe experiment, as a consequence of the high energies of the γ-ray feeding transitions. The measured (model-independent) 16⁺ isomer cross sections from the 178Hf beam activation experiment (Table VII) gave an excitation probability of 14(5) × 10⁻⁴ at 80% of the Coulomb barrier, whereas the isomer excitation probability for the 25 ≤ θlab ≤ 78° scattering range in the 136Xe beam experiment was 4(2) × 10⁻⁴ (calculated from the fitted matrix elements).

A direct measurement of the quadrupole moment of the 16⁺ band using laser spectroscopy on 178m2Hf nuclei gave Q₀ = 7.2(1) b [63], whereas Coulomb excitation of a 16⁺ isomer-enriched target gave Q₀ = 8.2(11) b [64]. The first measurement is within 5% of the Q₀ moment of the GSB, and the second is within ≈1σ (19% greater), but the 19⁺ → 18⁺ yields were not sensitive enough to the quadrupole moment to allow a measurement using the present data sets.

B. K mixing

The sets of EM reduced matrix elements for K-forbidden transitions to the high-K bands, or, equivalently, the hindrance values for the transitions, reveal the systematics of K mixing and identify the bands where the mixing occurs. Decreasing hindrance of the transitions between two bands can be understood in terms of increasing K mixing in the wave functions of one or both bands.

Reduced hindrance values range from fᵥ = 9.6 to 23 for the K-forbidden E2 decay branches of the 4⁺ band head to the GSB and the γ-band—within the limits set by the measured bounds on the lifetime, but well below the value of 100 of the Ruskov rule frequently suggested for K-forbidden transitions [21, 72]. The GSB → K = 4⁺ coupling of the SDM model fit
The intrinsic matrix element and the \( B(E2; \text{GSB} \rightarrow K = 16) \) values are nearly saturated at \( \approx 1 \) W.u. (the approximate upper bound expected for noncollective transitions) by \( I_{\text{GSB}} = 14 \), the lowest GSB state that populates the isomer band. Hence, there is no hindrance of the GSB\( \rightarrow K = 16 \) transitions due to \( K \) forbiddenness at \( I_{\text{GSB}} = 14 \). In contrast, the \( K \)-forbidden matrix elements populating the \( K = 4 \)–8 bands take values \( \geq 4 \) orders of magnitude below their saturation values at the lowest \( I_{\text{GSB}} \) and \( I_\gamma \) connections (Figs. 33 and 34). Even with transition probabilities \( <0.1 \) e\(^2\)b\(^2\) feeding the \( 16^+ \) band, the reduced hindrance values \( f_\nu \) range from 0.87 to 1.10, a factor of 100 smaller than the prediction of Rusinov’s rule.

The \( 16_{\text{isom}} \rightarrow K^\pi = 8^- \) and \( 14_{\text{isom}} \rightarrow K^\pi = 8^- \) decays are all highly hindered \([34,47]\), showing that the onset of significant high-\( K \) admixtures does not occur at sufficiently low spin \( (I^\pi = 12^-, 13^-) \) in the \( 8^+ \) band to account for the observed \( 8^- \) isomer band yields. Hence, mixing in the high-\( K \) bands is unable to explain the Coulomb excitation of all of the high-\( K \) bands, whereas the measured matrix elements are consistent with increasing mixing in the low-\( K \) bands in every instance.

The systematic decrease with increasing spin of the hindrance of \( K \)-forbidden transitions is apparent from Figs. 33 and 34 and Table IX. For each of the high-\( K \) isomer bands observed, reproduction of the measured yields requires that the interband \( B(E\lambda) \) values increase with increasing spin and saturate at \( \approx 1 \) W.u. for \( I_\gamma \approx 12 \) in the GSB and the \( \gamma \) band. This saturation point represents the maximum mixing

\[ B(E2) (e_b^2) = 0.0297 e_b^2, \]
\[ B(E3) (e_b^3) = 0.0132 e_b^3. \]
of $K$. For $I \geq 12$, reduced hindrance values of $K$-forbidden transitions from low-$K$ to high-$K$ bands are as low as $f_\nu \sim 1$, indicating that for these transitions the $K$-selection rule has little predictive power at high spin, i.e., highly $K$-forbidden transitions have strengths similar to allowed interband transitions. A notable exception is the unobserved $3^+ \rightarrow 2^+$ transition for which the $K=2$ admixture in the GSB is insignificant. For $I \leq 14$, the $8^+_{\text{GSB}} \rightarrow 8^+_{K=8}$ transitions ($f_\nu > 9$) and $6^+_{\text{GSB}} \rightarrow 8^+_{K=8}$ ($f_\nu > 70$) transitions [73] suggest that the Alaga rule may not describe well all of the $K$-forbidden couplings. Compared to the Alaga rule, a more rapid increase in the magnitudes of the EM matrix elements was found in the low-$K$ to $K=8$ transitions.

TABLE IX. Values of the reduced hindrance $f_\nu$ given in the direction $I_i \rightarrow I_f$ for selected $K$-forbidden transitions in $^{176}$Hf. Weisskopf estimates $B(M\lambda \downarrow_{\text{w.a.}})$ are 0.020 e$^2$b ($E1$), $6.0 \times 10^{-3}$ e$^2$b ($E2$), $2.0 \times 10^{-3}$ e$^2$b ($E3$), $2.2 \times 10^{-4}$ e$^2$b ($E5$), 1.8 $\mu^2_{\text{B}}$ (M1), 0.52 $\mu^2_{\text{B}}$ (M2), 0.055 $\mu^2_{\text{B}}$ (M4). Defined $B(M\lambda \downarrow_{\text{w.a.}}) = (2\lambda + 1) B(M\lambda \downarrow_{\text{w.a.}})$.

<table>
<thead>
<tr>
<th>Bands</th>
<th>$I_i$</th>
<th>$I_f$</th>
<th>$M\lambda$</th>
<th>$\nu$</th>
<th>$f_\nu$</th>
</tr>
</thead>
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<td>4</td>
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<td>6</td>
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<td>12</td>
<td>14</td>
<td>$E2$</td>
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</tr>
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<td>12</td>
<td>14</td>
<td>$E2$</td>
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<td>5.</td>
</tr>
<tr>
<td>GSB $\rightarrow K^\pi = 8^+$</td>
<td>8</td>
<td>8</td>
<td>$E1$</td>
<td>7</td>
<td>67(1)$^a$</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>8</td>
<td>$M2$</td>
<td>6</td>
<td>&gt;130$^a$</td>
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<td>11</td>
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<tr>
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<td>16</td>
<td>$E4$</td>
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<td>$E2$</td>
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<tr>
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<td>11</td>
<td>$E5$</td>
<td>3</td>
<td>165(5)$^b$</td>
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<tr>
<td>(isomer decays)</td>
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<td>12</td>
<td>$M4$</td>
<td>4</td>
<td>72(2)$^b$</td>
</tr>
<tr>
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<td>16</td>
<td>13</td>
<td>$E3$</td>
<td>5</td>
<td>66(1)$^b$</td>
</tr>
<tr>
<td>$K^\pi = 14^+ \rightarrow K^\pi = 8^+$</td>
<td>14</td>
<td>13</td>
<td>$M1$</td>
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<td>90$^c$</td>
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<td>14</td>
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<td>4</td>
<td>33$^c$</td>
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</table>

$^a$Calculated from Ref. [47].
$^b$Reference [47].
$^c$Reference [34].

Band interactions are reflected in the measured moments of inertia by a increase in slope of the moment of inertia $I(\omega)$, seen at $I \approx 6$ and $I \approx 10$ in the $\gamma$ and GS bands, respectively (Fig. 35). As described in Sec. IV D, the $B(E\lambda)$ values saturate at $\sim 1$ W.u. as low as $\omega \approx 8$ and $I \approx 10$ for transitions from the $\gamma$-band and the GSB, respectively, to reproduce the measured $\gamma$-ray yields in the $K^\pi = 4^+, 6^+, 8^-$, and $16^+$ bands. Moreover, Coriolis alignment is expected to happen at much lower spin in low-$K$ bands than in high-$K$ bands [74], which are strongly deformation coupled. The moments of inertia of the high-$K$ bands are relatively constant in slope, with the exception of the $6^+$ band at $I \approx 12$, consistent with purity of the high-$K$ bands up to much higher spin, compared to the low-$K$ bands. The $16^+$ band has a remarkably constant moment of inertia [32] up to $I = 22$, consistent with, but not necessarily indicative of, the nearly constant $B(E2; \text{GSB} \rightarrow K = 16)$ values with $f_\nu \approx 1$. (Mixing in high-$K$ bands can also produce a fairly constant moment of inertia [75].)

The $16^+_{\text{isom}} \rightarrow K^\pi = 8^-$ and $14^+_{\text{isom}} \rightarrow K^\pi = 8^-$ $\gamma$ decays are strongly hindered with $33 \leq f_\nu \leq 165(5)$ in all of the five known branches, indicating that the onset of significant high-$K$ admixtures in the $8^-$ band must occur at $I > 13$, if at all, whereas less hindered $f_\nu \sim 1$ transitions from the $\gamma$ and GS bands are required to reproduce the present measured yields. That is, the strongly hindered decays of the $16^+$ and $14^+$ isomers to the $11 \leq I_{K=8} \leq 13$ states are consistent with $K$ being a good quantum number for the high-$K$ bands, suggesting that mixing in the low-$K$ bands is primarily responsible for the $K$-selection violations and that the EM matrix elements coupling to the high-$K$ bands are sensitive probes of the $K$ distributions in the low-$K$ bands. Coulomb excitation of a band with projection $K$, assuming that it is reasonably pure, would require admixtures $K'$ in the low-$K$ (nominally $K)$ bands of $K - \lambda < K' \leq K + \lambda$. Hence, the mixing fractions of the $2 \leq K' \leq 6$ components are depicted in Figs. 33 and 34 as a function of spin by the
$B(E2; K_i \rightarrow K = 4)$ values, the $4 \leq K' \leq 8$ components by the $B(E2; K_i \rightarrow K = 6)$ values, the $5 \leq K' \leq 11$ components by the $B(E3; K_i \rightarrow K = 8)$ values, and the $14 \leq K' \leq 18$ components by the $B(E2; K_i \rightarrow K = 16)$ values. In the absence of any contradictory information, it might be assumed that the lowest $K$ value in each range enters the wave function first. Hence, the EM matrix elements coupling to the high-$K$ bands are sensitive probes of the $K$ distributions in the low-$K$ bands.

Two effects have been postulated to explain the mixing of high- and low-$K$ states: alignment of the single-particle or quasiparticle angular momenta due to collective rotations, and triaxial deformation effects such as softness to $γ$ deformation and $γ$-barrier tunneling. The $γ$-barrier penetration hypothesis has had success in explaining $K$ violations in more $γ$-soft nuclei such as $^{182}\text{W}$ and $^{181,182,184}\text{Os}$. However, the same treatment does not reproduce the measured hindrance values in more $γ$-rigid Hf nuclei $^{[13,16]}$. The $γ$-barrier penetration calculations make an absolute prediction of $K$-forbidden transition probabilities, but do not make predictions above the band head, high in the rotational bands where the present work predicts significant transition probabilities. $K$ mixing in the band head is a necessary feature of the $γ$-tunneling hypothesis, but this is strongly disputed by the high measured hindrance values of isomer decay branches. Predictions by Narimatsu et al. $^{[13]}$ of the hindrance of isomer decay branches are in agreement with measured hindrance values for $γ$-soft nuclei but differ by several orders of magnitude in the Hf nuclei, $^{178}\text{Hf}$ for instance.

The present work has resulted in measurements that are qualitatively similar to the SDM model in the most deviant cases ($8^- , 16^+$)—a rapid increase in mixing with increasing spin and a pure high-$K$ band head—and agree quantitatively with the SDM model (with two adjustable parameters) in the $K \leq 6$ data. In the case of the $K^\pi = 8^-$ band, the sets of matrix elements derived from the data qualitatively follow the decrease in hindrance with increasing spin predicted by the SDM model and are consistent with the Coriolis mixing hypothesis. Perhaps, with higher-order corrections, the SDM model would be useful for higher-$K$ bands.

The predictions of the projected shell model (PSM) are not limited to the high-$K$ band heads or to very small rotations $R$ as is the case in the $γ$-tunneling models. Rather, a softness to $γ$ deformations is predicted in $^{178}\text{Hf}$ for all states, including low-lying GSB states $^{[17]}$. This would naturally lead to $K$ mixing in all bands with the loss of axial symmetry for $γ > 0$. The PSM in its current state of development gives good agreement with some isomer level energies, but it has not yet been used to calculate EM transition matrix elements. The predicted $γ$ softness is not in conflict with the Coriolis mixing model, and future calculations may show that $γ$ softness makes an additional contribution to the measured strength of $K$-forbidden transitions to the isomer bands.

Neither of these two models has made absolute predictions of a complete set of $K$-forbidden matrix elements. Nor does $γ$-barrier tunneling $^{[1,76]}$ make quantitative predictions above the isomer band head. The present experimental probe of the loss of $K$ conservation and the measured decrease in the hindrance of $K$-forbidden transitions with increasing spin in the low-$K$ bands demonstrates the need for further theoretical work.

C. Coulomb depopulation

Based on the present measurements of the $⟨K^\pi = 16^+|E2|\text{GSB}⟩$ matrix elements, the $16^+ E2 \rightarrow \text{GSB}$ $K$-forbidden paths would allow Coulomb depopulation of the $K^\pi = 16^+$, 31-yr isomer using heavy ions, resulting in a cascade of 93- to 718-keV $γ$ rays and a net energy gain of 2.4 MeV. Calculations for a 230 MeV $^{58}\text{Ni}$ beam on a thin $\sim 1\text{-mg/cm}^2$ isomeric target predict a depopulation probability of $\leq 1\%$ $^{[77]}$ compared to the in-band excitations. The known strength of the $K$-allowed $M2$ decay of the $14^−$ isomer to the $16^+$ isomer (Fig. $28$) is irrelevant in Coulomb excitation, but the allowed $E3$ transitions would have a $\leq 1\%$ probability $^{[77]}$ assuming typical $E3$ matrix elements in $^{178}\text{Hf}$, whereas $\approx 50\%$ of the $14^−$ excitation would decay back to the $16^+$ isomer. Neither of these paths would be effective for photodeexcitation of the isomer, because photon absorption is dominated by $E1$ transitions, but this does not rule out depopulation via a $K$-forbidden ($ν = 1$) 463-keV $E1$ photoexcitation to the $15^−_1$ state, for example. The $16^+ E3 \rightarrow 8^−$ transitions are highly forbidden because $K$ is well-defined in the high-$K$ bands, as it has been shown above, making this path ineffective. Although low-yield Coulomb deexcitation paths have been discovered, intermediate states that might mediate the reported stimulated emission of the $K^\pi = 16^+$ isomer $^{[78]}$ were not found.

VI. CONCLUSION

A $^{178}\text{Hf}(^{136}\text{Xe},^{136}\text{Xe})^{178}\text{Hf}$ experiment using CHICO $^{[23]}$ and Gammasphere $^{[24]}$ demonstrated substantial $K$-forbidden Coulomb excitation of the $K^\pi = 4^+$ rotational band, as well as the rotational bands built on the $K^\pi = 6^+$ and $8^-$ isomers. Several rotational bands in $^{178}\text{Hf}$ were extended from the previously known states to spin $15h−18h$. Remarkably, the $K^\pi = 16^+$ isomer band in $^{178}\text{Hf}$ was populated by Coulomb excitation with nearly the same strength as the $K^\pi = 4^+$ rotational band, as well as the rotational bands built on the $K^\pi = 6^+$ and $8^-$ isomers. Several rotational bands in $^{178}\text{Hf}$ were extended from the previously known states to spin $15h−18h$. Remarkably, the $K^\pi = 16^+$ isomer band in $^{178}\text{Hf}$ was populated by Coulomb excitation with nearly the same strength as the $K^\pi = 6^+$, $8^-$ isomer bands, despite the $ν = 14$ forbbiddness. Data from Coulomb excitation of the $K^\pi = 16^+$ isomer via $^{96}\text{Ta}(^{178}\text{Hf},^{178}\text{Hf})^{96}\text{Ta}$ beam excitation provided an unambiguous indication that the isomer was populated by safe Coulomb excitation with collision energies as low as 73% of the Coulomb barrier and confirmed the remarkably high population of the $16^+$ isomer band in the former experiment.

A set of model-dependent intrinsic matrix elements was measured, which couple the $K^\pi = 0^+, 2^+, 4^+, 6^+$, and $8^−$ bands in $^{178}\text{Hf}$. The $K^\pi = 16^+$ isomer band was found to be populated directly from the GSB, and upper and lower bounds on a consistent set of $∥E3∥ (K^\pi = 6) |E2⟩$ matrix elements were determined. The deduced matrix elements provided the first qualitative measurement of the $K$ distribution with respect to nuclear spin in low-$K$ bands and revealed the rapid breakdown of the goodness of the $K$ quantum number as the low-$K$ bands are excited to higher rotational frequencies. The rapid increase in the interband $Eλ$ matrix elements coincides with...
the rotational alignment of low-$K$ bands that has a noticeable effect on the moment of inertia above the $I \approx 10$ levels of the $\gamma$ band and the GSB. Higher-$K$ components are admixed in the nominally low-$K$ bands with increasing spin, until the reduced transition probabilities saturate near \(\sim 1\) W.u. for \(I \gtrsim 12\) signifying the total breakdown of the $K$ quantum number. The high-$K$ bands remain quite pure, even at the same spin ($I$) levels where mixing in the low-$K$ bands has saturated. The present measurements are consistent with Coriolis alignment.

Coulomb excitation probabilities were calculated for depopulation of nuclei in the \(16^+\) isomer state, based on the present set of \(|I_{K=16^+}\rangle \langle E2|I_{GSB}\rangle\) matrix elements. Although a \(\lesssim 1\%\) Coulomb depopulation of the \(K^\pi = 16^+\) isomer may be possible using heavy ions, no useful intermediate state was found that might mediate photodepopulation via \(E1\) excitations.

The present work, initially focused on understanding the $K$-forbidden population of the \(K^\pi = 8^-\) isomer in \(^{178}\)Hf, has provided a variety of nuclear structure information. New levels and $\gamma$-ray information were used to probe physical properties of the \(K^\pi = 0^+, 2^+, 4^+, 6^+, 8^-, \) and \(16^+\) rotational bands, including $K$ mixing. The observed breakdown of the $K$ selection rule appears to be a result of collective rotational effects, so that the present conclusions regarding $K$ mixing as a function of spin may offer an explanation of $K$-forbidden excitation and decay in other axially symmetric quadrupole-deformed nuclei.

**ACKNOWLEDGMENTS**

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