2012

Extending the Arnold-Eisemann Algorithm for Pro Forma Circularity with a Specific Mix of New Debt and New Equity

Tom Arnold
University of Richmond, tarnold@richmond.edu

Recommended Citation
Arnold, Tom. "Extending the Arnold-Eisemann Algorithm for Pro Forma Circularity with a Specific Mix of New Debt and New Equity." *Advances in Financial Education* 10, no. 1&2 (2012): 95-99.

Extending the Arnold-Eisemann Algorithm for Pro Forma Circularity with a Specific Mix of New Debt and New Equity

Tom Arnold
University of Richmond

Arnold and Eisemann (2008) developed an algorithm that calculates the value of long-term debt when long-term debt is considered the "plug" or "slack" term within a pro forma analysis. In this paper, the algorithm is presented in a slightly different form and adjusted for the use of a target mix of new debt and new stock.

INTRODUCTION:

Pro forma analysis is a method for forecasting financial statements that can be used for budgeting purposes or for valuation purposes. Generally, revenues or sales grow at a particular rate and other portions of the income statement and balance sheet either grow at the same rate as sales (or computed as a proportion of sales) or at individual rates. The level of long-term debt or the level of stock on the balance sheet is determined in a manner to make the balance sheet "balance." This is achieved by having all other items determined and then have the long-term debt, stock, or a combination of both set as a "plug" figure (or slack term) to make the assets equal the liabilities and the equity. Any account associated with the plug figure is interpreted as the source of funding to support the sales growth.

When using long-term debt as the plug figure, generally a circularity issue occurs when considering only accounts in the current period:

- long-term debt requires information about total equity and total assets
- total equity requires information about retained earnings
- retained earnings requires information about paid to retained earnings
- paid to retained earnings requires information about taxable income
- taxable income requires information about the interest expense on long-term debt
- interest expense on long-term debt requires knowledge of long-term debt which returns one to the first bullet point.

Iteration routines will solve for the long-term debt value, however, Arnold and Eisemann (2008) developed an algorithm that uses current and past information to solve for the "debt-plug" directly.

$$A = \text{Total assets}_t - \text{Current liabilities}_t - \text{Total equity}_{(t-1)} \quad (1)$$
$$B = -\text{EBIT}_t \times (1 - \text{Tax rate}_t) \times (1 - \text{Dividend payout ratio}_t) \quad (2)$$
$$C = [1 - \text{Interest rate}_t \times (1 - \text{Tax rate}_t) \times (1 - \text{Dividend payout ratio}_t)] \quad (3)$$
$$\text{Debt plug}_t = (A + B) \div C \quad (4)$$

where EBIT = revenues - operating expenses - depreciation

In this paper, the Arnold-Eisemann algorithm is extended to allow for a target mix of debt and stock as the means for raising funds to support the sales growth. The target mix will be measured using the proportion of the funds that is debt (%D). The algorithm extension is provided in the next section and is followed by the conclusion of the paper.

EXTENDING THE ARNOLD-EISEMANN ALGORITM

Because the desire is to raise new capital with a specific mix of debt and equity using %D, the Arnold-Eisemann algorithm has to be adjusted to only be applied to new debt (ΔDebt plug_t). Consequently, the debt plug becomes the previous long-term debt (Long-term $\text{debt}_{(t-1)}$) plus ΔDebt plug_t.

$$\text{Debt plug}_t = \text{Long-term debt}_{(t-1)} + \Delta\text{Debt plug}_t \quad (5)$$

Based on equation (4):

$$\Delta\text{Debt plug}_t = [(A + B) \div C] - \text{Long-term debt}_{(t-1)} \quad (6)$$

Keeping C as it is currently defined, define X and Y as follows:

$$X = \text{Total assets}_t - \text{Current liabilities}_t - \text{Long-term debt}_{(t-1)} \quad (7)$$
$$Y = -[\text{Total equity}_{(t-1)} + (\text{EBIT}_t - \text{Interest}_{(t-1)}) \times (1 - \text{Tax rate}_t) \times (1 - \text{Dividend payout ratio}_t)] \quad (8)$$
$$\Delta\text{Debt plug}_t = [(X + Y) \div C] \quad (9)$$

The mix of new debt and new equity is calculated based on %D (note: the Equity plug is the additional amount of stock that is raised to fund the growth):

$$\text{Equity plug}_t = \text{Stock}_{(t-1)} + \Delta\text{Stock plug}_t \quad (10)$$
$$\Delta\text{Stock plug}_t = (X + Y) \div [1 + \{\%D \div (1 - \%D)\} \times C] \quad (11)$$
$$\text{Debt plug}_t = \text{Long-term debt}_{(t-1)} + \Delta\text{Debt plug}_t \quad (12)$$
$$\Delta\text{Debt plug}_t = (X + Y) \div [\{(1 - \%D) \div \%D\} + C] \quad (13)$$

A demonstration of the adjusted algorithm with %D = 80% is displayed in Table 1.

Table 1. Pro Forma Financial Statements with %D = 80%

Panel A: Inputs	
Revenue growth rate [g(rev)]:	10%
Cost of Goods Sold growth rate [g(COGS)]:	8%
Salary, General, Administrative growth rate [g(SGA)]:	2%
Depreciation periods*:	20
Interest rate on debt:	8%
Tax rate:	40%
Dividend Payout Ratio:	20%
Current Assets growth rate [g(C/A)]:	10%
Fixed Assets growth rate [g(F/A)]:	10%
Current Liabilities growth rate [g(C/L)]:	10%

Panel B: Pro Forma Financial Statements

Income Statement:	**Year 0:**	**Year 1:**
Revenues:	$ 10,000.00	$ 11,000.00[a]
Cost of Goods Sold:	$ 6,000.00	$ 6,480.00[b]
Salary, General, Administrative:	$ 1,000.00	$ 1,020.00[c]
Depreciation:	$ 1,000.00	$ 1,100.00[d]
Earnings Before Interest and Taxes (EBIT):	$ 2,000.00	$ 2,400.00
Interest (8%):	$ 720.00	$ 762.50[e]
Earnings before Taxes (EBT):	$ 1,280.00	$ 1,637.50
Tax (40%):	$ 512.00	$ 655.00[f]
Earnings After taxes (EAT):	$ 768.00	$ 982.50
Dividends (20%):	$ 153.60	$ 196.50[g]
Paid to Retained Earnings:	$ 614.40	$ 786.00

Balance Sheet:	**Year 0:**	**Year 1:**
Current Assets:	$ 6,000.00	$ 6,600.00[h]
Fixed Assets:	$ 20,000.00	$ 22,000.00[i]
Accumulated Depreciation:	$ 5,000.00	$ 6,100.00
Net Fixed Assets:	$ 15,000.00	$ 15,900.00
Total Assets:	$ 21,000.00	$ 22,500.00
Current Liabilities:	$ 500.00	$ 550.00[j]
Long-term Debt:	$ 9,000.00	$ 9,531.20
Common Stock:	$ 10,500.00	$ 10,632.80
Retained Earnings:	$ 1,000.00	$ 1,786.00
Total Liabilities and Equity:	$ 21,000.00	$ 22,500.00

Table 1. Pro Forma Financial Statements with %D = 80% (continued)

%D = 80%
X = $22,500.00 - $550.00 -$9,000.00 = $12,950.00
Y = -[$11,500.00 + ($2,400.00 - $720.00) × (1 - 40%) × (1 - 20%) = - $12,306.40
C = [1 – 8% × (1 - 40%) × (1 - 20%)] = 0.9616
(X + Y) = $643.60
ΔStock plug = $643.60 ÷ [1 + {80% ÷ (1 - 80%)} × 0.9616] = $132.80
Common Stock (Equity plug) = $10,500.00 + $132.80 = $10,632.80
ΔDebt plug = $643.60 ÷ [{(1 - 80%) ÷ 80%} + 0.9616] = $531.20
Long-term debt (Debt plug) = $9,000.00 + $531.20 = $9,531.20

[*] Depreciation is assumed to be straight-line based on 20 periods
[a] = Revenues (Year 0) × (1 + g(Rev)) = $10,000.00 × (1 +10%)
[b] = Cost of Goods Sold (Year 0) × (1 + g(COGS)) = $6,000.00 × (1 + 8%)
[c] = Salary, General, Administrative (Year 0) × (1 + g(SGA)) = $1,000.00 × (1 + 2%)
[d] = Fixed Assets (Year 1) ÷ Depreciation periods = $ 22,000.00 ÷ 20
[e] = Long-term Debt (Year 1) × Interest rate on debt= $ 9,531.20 × 8%
[f] = EBT (Year 1) × Tax rate = $ 1,637.50 × 40%
[g] = EAT (year 1) × Dividend Payout Ratio = $ 982.50 ×20%
[h] = Current Assets (Year 0) × (1 + g(C/A)) = $6,000.00 × (1 + 10%)
[i] = Fixed Assets (Year 0) × (1 + g(F/A))= $20,000.00 × (1 + 10%)
[j] = Current Liabilities (Year 0) × (1 + g(C/L)) = $500.0 × (1 + 10%)

If preferred, defining Q as [%D ÷ (1 - %D)] simplifies equations (11) and (13):

$$\Delta\text{Stock plug}_t = (X + Y) \div [1 + Q \times C] \quad (14)$$
$$\Delta\text{Debt plug}_t = Q \times (X + Y) \div [1 + Q \times C] \quad (15)$$
$$\Delta\text{Debt plug}_t = Q \times \Delta\text{Stock plug}_t$$

Q is similar to a debt-to-equity ratio, however, in this case it is new debt divided by new equity (or the proportion of new debt funding divided by the proportion of new stock funding). Q equals 4.0 when %D is 80% (i.e. 4.0 = 80% ÷ [1 - 80%]) and fits into Table 1 in the following manner:

$$\Delta\text{Stock plug}_t = \$643.60 \div [1 + 4.0 \times 0.9616] = \$132.80 \quad (16)$$
$$\Delta\text{Debt plug}_t = 4.0 \times \$132.80 = \$531.20 \quad (17)$$

CONCLUSION

The purpose of this paper is to extend the Arnold-Eisemann algorithm to allow for a specific mix of new debt and new equity to support sales growth in a pro forma analysis. The topic is advanced compared to traditional pro forma analysis and can be incorporated into a curriculum or the assessment of a curriculum in a number of ways for AACSB purposes:

- used as a means to assess aptitude with pro forma analysis in that one will not be able to grasp nor perform the Arnold-Eisemann algorithm nor the extension without a firm basic understanding of pro forma analysis
- used as a means of assessing the implications of the funding choice for new growth...if the firm decides to use only debt, the additional funding based on equation (13) will be $669.30...if the firm decides to use only equity, the additional funding based on equation (14) is $643.60... this could also be used as an assessment of testing "critical thinking" in regard to this topic
- used as a means to advance the understanding of the concept of "additional funds needed" (AFN, Brigham and Houston (2011)) when considering sales growth of "g"
- $AFN = [\text{Total assets}_{(t-1)} \times (1 + g) \div \text{Sales}_{(t-1)}] \times \Delta\text{Sales}$
  $- [\text{Total liabilities}_{(t-1)} \times (1 + g) \div \text{Sales}_{(t-1)}] \times \Delta\text{Sales}$
  $- \text{Net profit margin}_{(t-1)} \times \text{Sales}_{(t-1)} \times (1 + g) \times (1 - \text{Dividend payout ratio})$

  AFN (using Table 1 values) = $589.16

The intention here is to demonstrate how much of an approximation AFN actually is (Note: Winkler (1994) also provides content in this area)

Whether used as a means of developing pro forma statements or as a means of assessing pro forma statement/AFN acumen, the extension of the Arnold-Eisemann algorithm provides a rich context for the classroom at the undergraduate and MBA level.

REFERENCES

Arnold, T. and P. Eisemann, 2007, "Debt Financing Does NOT Create Circularity Within Pro Forma Analysis," *Advances in Financial Education,* V. 6 (Summer), 96–102.

Brigham, E. and J. Houston, 2011, *Fundamentals of Financial Management, Concise Edition* (Thomson South-Western, Mason, OH).

Winkler, D., 1994, "Financing Costs of Additional Funds Needed: A Modified Equation Approach.", *Financial Practice and Education,* Vol. 4, No. 1, 149-154.