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Debt Financing Does NOT Create Circularity Within Pro Forma Analysis

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Using debt to finance a firm's external financing need within a pro forma analysis can lead to "circularity" when finding the appropriate value for debt. The circularity incorrectly implies that there is no direct solution for finding the value of debt. In this paper, a direct solution for the value of debt is found; thereby showing that circularity need not exist. Further, the technique is demonstrated to be more accurate than the "additional funds needed" (AFN) approach featured in many texts.

INTRODUCTION

A common approach to preparing pro forma statements is to assume that debt is the means of financing an external funding need. Debt becomes the "plug" or "slack term" and is set to a value that makes total assets equal to total liabilities plus total equity. Because of the manner in which the debt plug is programmed into a spreadsheet, pro forma users often believe that solving for the debt amount requires an iterative solution.[2] The reason for the iterative solution is due to a "circularity" that emerges because of the interrelated nature of debt levels and interest expense.

Figure 1 uses simplified financial statements to illustrate how the circularity emerges. The beginning point is the sales forecast. Operating expenses, current assets, gross fixed assets, depreciation expense (indirectly via a link to gross fixed assets), and current liabilities are all forecasted to grow at the same rate as sales. Common stock does not change and retained earnings grow by the amount of retention. Both the dividend payout ratio and tax rate are constants. All other items use the standard relationships between the financial statements (see Benninga and Sarig (1997)). Debt is the equilibrating item on the balance sheet. The problem with solving for debt is that the amount of debt depends on retained earnings, which requires net income, but net income depends on interest expense which, in turn, depends on debt. The variables are interdependent.

If the pro forma analysis in Figure 1 is programmed into a spreadsheet software package, the software package immediately recognizes the circularity and suggests a solution, such as an iterative algorithm (e.g. using "Tools/Calculation/Iteration" in Excel). Another possible solution is to assume that interest expense is based on the previous

Figure 1. The Circularity Caused by Debt Financing

Income Statement:	Year 0:	Year 1:	Year 1 Calculation:
Sales:	$1,500.00	$1,737.00	← based on sales growth forecast of 15.80%
Operating Expenses:	$1,023.00	$1,184.63	← 68.20% of revenue
Depreciation Expense:	$ 78.75	$ 91.19	← gross fixed assets ÷ 20
EBIT:	$ 398.25	$ 461.17	← earnings before interest and taxes
Interest Expense:	$ 22.40		← 8%' debt in year 1
EBT:	$ 375.85		← EBIT - interest expense
Taxes:	$ 169.13		← 45.00% 'EBT
Net Income:	$ 206.72		← EBT – tax
Dividend:	$ 89.30		← 43.20% x net income
To Retained Earnings:	$ 117.42		← net income – dividends
Balance Sheet:	**Year 0:**	**Year 1:**	**Year 1 Calculation:**
Assets:			
Current Assets:	$ 162.00	$ 187.60	← 10.80% of revenue
Gross Fixed Assets:	$1,575.00	$1,823.85	← 105.00% of revenue
Accumulated Depreciation:	$ 300.00	$ 391.19	← increased by depreciation expense
Net Fixed Assets:	$1,275.00	$1,438.88	← fixed assets - accumulated depreciation
Total Assets:	$1,437.00	$1,620.25	← current assets + net fixed assets
Liabilities and Equity:			
Current Liabilities:	$ 132.00	$ 152.86	← 8.80% of revenue
Debt:	$ 280.00		← total assets – current liabilities – total equity
Total Liabilities:	$ 412.00		← current liabilities + debt
Stock:	$ 825.00	$ 825.00	← held constant
Retained Earnings:	$ 200.00		← increased by earnings retained in year 1
Total Equity:	$1,025.00		←stock + retained earnings
Total L and E:	$1,437.00	$1,620.25	← total liabilities + total equity (which must equal total assets)

Circularity in Year 1: Calculating *interest expense* requires the **debt** amount, which requires *retained earnings*, but *retained earnings* requires *net income* and that is affected by *interest expense*.

Note: $0.01 rounding error is common with pro forma analysis performed on a spreadsheet

period's debt level. Because last period's debt level is known and is unaffected by the current year's financing requirements, this approach eliminates the circularity. Unfortunately, this approach implicitly assumes that all new debt is issued at the end of the period with the result that any new debt has no effect on interest expense for the current year. This is generally not a realistic assumption. An assumption that better approximates reality is that debt can be issued throughout the year. If so, interest expense can be reasonably computed using the average level of debt between the current and previous periods. However, because the current debt amount is again part of the interest expense calculation there is still circularity.

For exposition purposes, we will continue with the assumption in Figure 1 that interest expense is based only on the current period's debt level, but we will address the assumption of averaging the debt level between two periods to calculate the interest expense later in the paper. The next section will demonstrate that regardless of the choice of these two assumptions, circularity does not need to exist when debt is used as the slack term in a pro forma analysis. Circularity is actually the result of programming and not a problem inherent in pro forma financial statements.

FINDING DEBT WITHOUT CREATING CIRCULARITY

Based on Figure 1, debt (D_t) equals total assets (TA_t) less the sum of: current liabilities (CL_t), stock (S_t), and retained earnings (RE_t). Retained earnings equal the previous period's retained earnings (RE_{t-1}) plus the current addition to retained earnings (ΔRE_t). The key to finding the solution for debt is in expanding addition to retained earnings:

$$\Delta RE_t = EBIT_t(1 - T)(1 - d) - kD_t(1 - T)(1 - d) \tag{1}$$

where "T" is the tax rate, "d" is the dividend payout ratio, "k" is the interest rate on debt, and "$EBIT$" equals: revenues less operating expenses less depreciation.
Debt equals:

$$D_t = TA_t - CL_t - S_t - RE_t = TA_t - CL_t - S_t - [RE_{t-1} + \Delta RE_t] \tag{2}$$

Substituting equation (1) into equation (2) produces a solution for debt:

$$D_t = \frac{TA_t - CL_t - S_t - RE_{t-1} - EBIT_t(1 - T)(1 - d)}{1 - k(1 - T)(1 - d)} \tag{3}$$

Applying equation (3) to the Figure 1 example gives the following result.

$$D_1 = \frac{\$1,620.25 - \$152.86 - \$825.00 - \$200.00 - \$461.17(1 - 0.45)(1 - 0.432)}{1 - 0.08(1 - 0.45)(1 - 0.432)}$$

$$D_1 = \frac{\$298.32}{0.9750} = \$305.97$$

With the debt amount determined, the interest expense and the remainder of the pro forma financial statements can be completed as in Figure 2.

Although in many cases the iteration solution is adequate, there are situations where equation (3) is necessary. One instance is when the analyst is doing extensive computer simulations with Monte Carlo software such as with *Crystal Ball* or *@Risk*.

Figure 2. Pro Forma Financial Statements Completed Without Circularity

Income Statement:	Year 0:	Year 1:	Year 1 Calculation:
Sales:	$1,500.00	$1,737.00	← based on sales growth forecast of 15.80%
Operating Expenses:	$1,023.00	$1,184.63	← 68.20% of revenue
Depreciation Expense:	$ 78.75	$ 91.19	← gross Fixed Assets ÷ 20
EBIT:	$ 398.25	$ 461.17	← earnings before interest and taxes
Interest Expense:	$ 22.40	$ 24.48	← 8% $310.36
EBT:	$ 375.85	$ 436.70	← EBIT - interest expense
Taxes:	$ 169.13	$ 196.51	← 45.00% EBT
Net Income:	$ 206.72	$ 240.18	← EBT - tax
Dividend:	$ 89.30	$ 103.76	← 43.20% net income
To Retained Earnings:	$ 117.42	$ 136.42	← net income - dividends

Balance Sheet:	Year 0:	Year 1:	Year 1 Calculation:
Assets:			
Current Assets:	$ 162.00	$ 187.60	← 10.80% of revenue
Gross Fixed Assets:	$1,575.00	$1,823.85	← 105.00% of revenue
Accumulated Depreciation:	$ 300.00	$ 391.19	← increased by depreciation expense
Net Fixed Assets:	$1,275.00	$1,432.66	← fixed assets - accumulated depreciation
Total Assets	$1,437.00	$1,620.25	← current assets + net fixed assets
Liabilities and Equity:			
Current Liabilities:	$ 132.00	$ 152.86	← 8.80% of revenue
Debt:	$ 280.00	$ 305.97	← based on equations (3)
Total Liabilities	$ 412.00	$ 458.83	← current liabilities + debt
Stock:	$ 825.00	$ 825.00	← held constant
Retained Earnings:	$ 200.00	$ 336.42	← increased by earnings retained in year 1
Total Equity:	$1,025.00	$1,161.42	← stock + retained earnings
Total L and E:	$1,437.00	$1,620.25	← total liabilities + total equity (which must equal total assets)

Note: $0.01 rounding error is common with pro forma analysis performed on a spreadsheet

When compared to the "additional funds needed" (AFN) technique suggested in many textbooks, the solution in this paper is more accurate. Using the presentation in chapter seventeen of Brigham and Houston (2007) with the variables defined in this paper along with the additional definitions that "R_t" is revenue, "g" is expected sales growth rate, and "NPM_t" is profit margin, equation (4) defines AFN.

$$AFN_t = TA_{t-1} \times g - CL_{t-1} \times g - NPM_{t-1} \times Rt \times (1 - d) \qquad (4)$$

Applying equation (4) with the data from Figure 1, AFN equals:

$$AFNt = \$1{,}437.00 \times 17.80\% - \$132.00 \times 15.80\% \tag{5}$$

$$- 13.78\% \times \$1{,}737.00 \times (1 - 43.20\%) = \$70.23$$

This makes the new debt level $350.23 ($280.00 + $70.23), which is roughly $44.00 higher than the correct value computed using equation (3) as shown in Figure 2.

Why is the AFN result different? AFN requires some implicit assumptions about assets and liabilities, more specifically, assets and some liabilities (usually current liabilities) grow at the same rate as the sales level. For fixed assets, this can be problematic in regard to how depreciation expense relates to gross and net fixed assets. Second, the AFN model assumes a constant profit margin, but the actual profit margin varies because it depends on interest expense and interest is related to the amount of debt. The same circularity that plagues the spreadsheet programming of pro forma statements creates similar problems for the AFN model (debt level affects interest expense which affects net income which determines AFN through the net profit margin). Consequently, the AFN technique will generally produce an incorrect answer when debt is the source of new financing.

Ultimately, the AFN approach is too simplistic a view of balance sheet dynamics. It provides an approximate answer of the firm's debt needs, but the accuracy is dependent upon the pro forma model being restricted to a "sales driven" model for assets and current liabilities. This may be appropriate for an introduction to pro forma analysis, however, AFN will become more inaccurate as the pro forma analysis increases in complexity (e.g., when financial statement inputs grow at a rate different from the sales growth rate as will be the case when efficiencies are expected to vary or the inputs change over time). Consequently, it may be beneficial to use equation (3) (or equation (6) below) even for a simplistic pro forma analysis because the solution provided by these equations will always be correct no matter how parameters change individually through time. In other words, there is a trade off between a somewhat more complex equation that will always be correct and a simpler approximation that will always at best be close to the correct answer.

As mentioned earlier, the assumption of interest expense based on the average of the current and previous level of debt often better matches actual financial activity. The following equation modifies equation (3) for the average debt assumption:

$$D_t = \frac{TA_t - CL_t - S_t - RE_t - (R_t - OE_t - DP_t)(1 - T)(1 - d) + 1/2kD_{t-1}(1 - t)(1 - d)}{1 - \left[\dfrac{k(1 - T)(1 - d)}{2}\right]} \tag{6}$$

USE IN THE CLASSROOM

When introducing this technique into the classroom, the instructor should distinguish between student experience levels rather than undergraduate versus graduate

levels. Assuming little to no experience, we suggest that the instructor follow the standard textbook presentation of pro forma analysis similar to Figure 1 in which "circularity" is programmed and the iteration technique solves for the debt financing need. This allows the student to think about the relationship between the different financial statement accounts and teaches the student the functionality of Excel.

For students with more experience or as a follow-up for students who are first exposed to the iteration solution, the equations presented here can be useful. To make the solution relevant to the student, use a numerical example to demonstrate that equation (3) or (6) provide a similar solution to the iterated solution. Students, at a minimum, will observe that Excel will no longer provide a circular reference error and will be exposed to how the financial statements are connected through time in reality and within a pro forma analysis.

There are additional activities that can further deepen the student's understanding. One idea that has impact is to find a platform other than Excel to produce the pro forma analysis (*i.e.*, get the student away from the capability of using the iteration routine in Excel). For example, the student may be asked to perform the pro forma analysis with "pen and paper" (possibly a quiz or a test with the relevant formula provided). Another example is to perform the pro forma analysis using a different software platform, such as, MATLAB. The benefit of this lesson plan is that it forces the student to understand how a pro forma analysis truly works and how financial statements are inter-connected through time. Further, providing a platform to perform pro forma analysis under any conditions allows the student to adapt to any software environment they may face in the future. At a minimum, the student will also understand the shortcomings of the AFN technique when there is debt financing.

CONCLUSION

A common presumption is that solving pro forma financial statements for a company's external financing need creates a situation in which an iterative routine is necessary to produce the forecasted financial statements. Although spreadsheet programming of pro forma statements does create "circularity" when debt is used for financing growth, circularity does not need to exist. In this paper, a direct solution for the financing need is found that is no more complicated to calculate than many other cash flow calculations. The benefits of the direct solution are: increased efficiency when performing extensive analysis, a better understanding of how debt financing structurally "feeds back" into the system, an ability to perform pro forma analysis using software other than Excel, and an understanding of the limitations of the AFN technique.

NOTE: Bradford Jordan notified the authors in an e-mail that earlier editions (not the current edition) of Ross, Westerfield, and Jordan (2008) acknowledged a solution to the "circularity" problem. Bradford Jordan attributes his knowledge of the solution to Jimmy Hilliard.

ENDNOTES

1 The authors wish to thank Bradford Jordan and two anonymous referees for helpful comments.

2 See, for example, Ehrhardt and Brigham [2006], p. 394.

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