Spring 2013

Power distribution in the European Union

Dayton Steele
University of Richmond

Power Distribution in the European Union

By

Dayton Steele

Honors Thesis

In

Department of Mathematics and Computer Science
University of Richmond
Richmond, VA

May 1, 2013

Advisor: Dr. Kathy Hoke

The signatures below, by the lead thesis advisor, the consulting thesis advisor, and the honors coordinator for mathematical economics, certify that this thesis, prepared by Dayton T. Steele, has been approved, as to style and content.

Kathy Hoke (lead advisor)

Olena Mykhaylova (consulting advisor)

Lester Caudill (honors coordinator)

Abstract

The Treaty of Lisbon, the latest treaty governing law-making in the European Union (EU), was ratified in 2009 and goes fully into effect in 2014. This treaty, with its change to voting procedures in the Council of Ministers, claims to make decision-making in the EU more democratic and more efficient. Since the EU serves as an economic and political entity, we will assess these claims by comparing each member state's GDP and population to its power as modeled using the concept of a power index from the game theory literature. We will utitlize the normalized Banzhaf index, the Shapley-Shubik index, the position value with no assumptions about which member states are likely to cooperate, and the position value under the assumption that Germany and France have a key central role in decision-making. We will show that in general these power indices support the claims of a more democratic and a more efficient voting process when looking at the entire EU, but they do not support the claims when restricting the analysis to the EU members who have adopted the euro. Finally, we add the European Parliament to our model and show that under the Treaty of Lisbon, power is not as evenly distributed between the Council of Miniters and the European Parliament as one may expect at first glance.

Contents

1 Introduction

According to the official website of the European Union (EU) (Europa 2013), the EU is a "unique *economic* and *political* partnership between twenty-seven countries." It has grown from 12 member states in 1993 to 27 as of January 2013 (with Croatia expected to enter as early as July 2013).

Everything the EU does is governed by a series of treaties, and amendment to the treaties requires the agreement and ratification (according to their national procedures) of every single member (Europa 2013). The two principal treaties on which the EU is based are the *Treaty on European Union* (also known as the Maastricht Treaty, effective since 1993) and the *Treaty on the Functioning of the European Union* (also known as the Treaty of Rome, effective since 1958). These two documents (plus their attached protocols and declarations) have been amended by a long series of treaties. The latest treaty is the Treaty of Lisbon, ratified by all EU countries before entering into force in December 2009. It will be fully in effect by 2014 replacing several parts of the Treaty of Nice, signed into effect in February 2003.

Three main institutions are involved in EU legislation: the European Commission proposes new laws, and the European Parliament (EP) and Council of the European Union, sometimes referred to as the Council of Ministers, adopt them. The European Commision has no formal voting power, so it will be less important for analysis in this paper. The Council of Ministers represents the governments of the individual member states, with a voting member per member state. Gunlicks (2011) points out that the Council of Ministers is "not just one body of ministers; rather, it consists of various combinations of cabinet ministers, who form subject matter councils," such as the Economics and Financial Affairs Council (ECOFIN) and the Foreign Affairs Council. The EP represents the EUs 500 million citizens. Each member state of the EU is given a set number of representatives, roughly in proportion to population, which are then elected in a national election. Members of the European Parliament, called MEPs, are elected once every five years, with a current total of 754.

Note that the European Council, another institution of the EU, sets the overall political direction but has no law-making power. It should not be confused with the Council of Ministers, which along with the EP acts as the legislative branch of the EU. To avoid confusion, we will use the name Council of Ministers rather than Council of the European Union.

Europa (2013) summarizes the changes brought forth by the Treaty of Lisbon as well as the need for these changes. It notes that the EU had expanded to twenty-seven member states using rules designed for fifteen members. It cites that the Treaty of Lisbon addresses the increased need to make the decision-making process more *democratic* as well as more *efficient*. Towards this end, voting rules in the Council of Ministers were changed. From 2014 on, the calculation of qualified majority will be based on the double majority of member states and people, thus representing the *dual legitimacy* of the Union. A double majority will be achieved when a decision is taken by 55% of the member states representing at least 65% of the Union's population. The Treaty of Nice required 50% of the members representing at least 62% of the population and 74% of specially assigned voting weights to agree on a proposal for it to become law. Furthermore, the Treaty of Lisbon strengthens the role for the EP by providing it with important new powers regarding EU legislation, the EU budget, and international agreements. In particular, for the vast bulk of EU legislation, it gives the same weight to the EP as to the Council of Ministers; each of the two bodies must approve proposed legislation before it becomes law, which is known as the *co-decision process*. Also, under the Lisbon Treaty, the maximum number of members of Parliament is set at 751; this limit, however, will not go into effect until the next election. Since Germany will be capped at 96 MEPs, we account for this change by reducing the number of MEP's that represent Germany from 99 to 96.

We will analyze the power of the member states under the formal voting rules in the Council of Ministers with the assumption that the formal rules influence compromise and concessions in the decision-making process, even though normally decision making in the Council takes place behind closed doors. Often by the time the formal vote that is recorded happens, the reported result is unanimous. VoteWatch Europe (2013) is an independent organization that started collected data in July '09 on how the 27 member states vote in the Council of Ministers. They point out that, in practice, a formal vote only takes place in cases where adoption of a proposal is guaranteed. In their 2012 annual report, they report that of all decisions taken under the qualified majority voting rule since 2009, 65% were adopted unanimously. Further, instead of voting against a proposed legislation, member states often make formal statements to signal their reservations about Council decisions.

We further note that the decision-making in the European Union cannot be fully interpreted simply from the objective voting systems in place. Any look into the decision-making process of the EU with an eye towards quantifying where the power lies must take into account the following three dimensions in addition to simply looking at the rules of how a law is passed:

- <u>Use of the euro</u>. On January 1, 2002, the Euro replaced national currencies in 12 of the 15 countries that at that time made up the EU. Today, 17 of the 27 EU member states comprise the Eurozone, the collection of EU members that have adopted the euro as their official common currency. According to Europa (2013), all members of the EU, except Denmark and the UK, are required to adopt the euro as soon as they meet certain conditions known as the "convergence criteria." Denmark and the United Kingdom obtained special opt-outs in the original Maastricht Treaty and are legally exempt from joining the Eurozone unless their governments decide otherwise. Sweden has thus far chosen not to make all the necessary changes to meet the convergence criteria. The remaining members of the EU who are not in the Eurozone joined the EU after the euro was launched and are required to work toward meeting the necessary criteria. Furthermore, the Treaty of Lisbon formalized the following policy: when the full ECOFIN council votes on matters only affecting the Eurozone, only those states using the euro are permitted to vote on it. In this case, according to Article 205 (3) (a) of the Treaty, a qualified majority shall be defined as at least 55% of the members of the Council representing the participating Member States, comprising at least 65%

of the population of these States. Note: The monetary policy of all countries in the Eurozone is managed by the Eurosystem which comprises the the European Central Bank (ECB) and the central banks of the EU states who have joined the euro zone. Countries outside the Eurozone are not represented in these institutions.

• Views on full integration of the EU. As expressed by Gunlicks (2011,45), "[T]he wish of some leaders and citizens in Europe to see the EU evolve into a more democratic federal state, which they see as the logical conclusion of ever closer union, is vehemently opposed by other leaders and Euroskeptics in general." The three largest members of the EU each have different views on integration. The Federalist view, held by Germany, envisions deeper integration of the member states, with a central decision-making authority in some well-defined areas separate from the member nations and decisions in these areas made by central institutions. The Nationalist view, held by the UK, states that the EU should function as an informal co-operative union and that national government representatives should make important European decisions. The Functionalist view, led by France, is somewhere between the other two opposing views.

• Traditional political parties. Though the European Parliament assigns votes to each Member State by population, the MEPs in Parliament are grouped by political affiliation. Once elected, MEPs organize along political lines, and not by nationality. Currently Europa (2013) lists seven political parties represented in the EP.

Taking each of these into account, we first analyze the power of each member state in the Council of Ministers, using three different power indices, to see how (if at all) the power shifts in the move to the new rules in the Treaty of Lisbon. Our analysis will argue that following the shift in power, decision-making is more efficient. In order to assess the claim of fairer distribution of power, we need to clarify how we interpret fairness. Fairness is two-fold: whether an individual member state is justly represented given its contribution to the EU in terms of population and Gross Domestic Product (GDP), and whether there is a balance of under-representation and over-representation across the entire EU. We will analyze fairness by calculating relevant power indices and then computing ratios comparing GDP to power and population to power. From this power analysis we will address how well the EU represents its member states economically and politically, since it is formally described as an "economic and political partnership." Since on matters pertaining to the euro, only the ministers from the member states whose currency is the euro can vote, we redo this analysis of power specifically for the Eurozone countries. We then add in the European Parliament, given its more equal role in the Treaty of Lisbon, and consider the distribution of power between the EP and the Council of Ministers. To perform this analysis, we will need to introduce mathematical tools, particularly from game theory, and apply these mathematical tools to the voting processes in each respective law-making body.

2 Mathematics Background

A **game**, (N, v), in characteristic form, is a finite set $N = \{1, 2, ..., n\}$ of players along with a real-valued set function $v : 2^N \to \Re$, defined for all subsets $S \subseteq N$ with $v(\emptyset) = 0$. A subset of players is called a **coalition**. A *simple game* is a game in which $v(S)$ is either 0 or 1; in this case a coalition is thought of as either winning or losing. Simple games are used to model voting situations; the game describes which groups of voters can assure the passage of a proposition by all voting yes regardless of how the other voters vote. A **weighted voting game** $[q; w_1, w_2, ...w_n]$ with quota q and player weights w_i ($0 \leq w_i < q$ for $1 \leq i \leq n$) is a simple game (N, v) defined by $v(S) = 1$ if $\sum_{i \in S} w_i \geq q$ and $v(S) = 0$ otherwise.

The **intersection game** $v = v_1 \wedge v_2 \wedge ... \wedge v_k$, is defined by $v(S) = min\{v_1(S), v_2(S), ...v_k(S)\}$. A *weighted m-majority voting game* is the intersection game of m weighted voting games. It is the simple game $(N, v_1 \wedge v_2 \wedge ... \wedge v_m)$ where the games (N, v_t) are the weighted voting games represented by $[q^t; w_1^t, w_2^t, ...w_n^t]$ for $1 \leq t \leq m$. The characterstic function is given by $v_1 \wedge v_2 \wedge ... \wedge v_m(S) = 1$ if $w^t(S) \geq q^t$ for all t; and 0 otherwise. In other words, a coalition S must reach the quota in each individual game in order to be a winning coalition.

For example, for the Council of Ministers in the European Union, each criteria for a proposition to pass can be thought of as a weighted voting game. In the Treaty of Nice, for a propositon to pass, all three of the following criteria must be met: 1) a 62 percent majority of the population votes yes, 2) a majority of all 27 countries must vote yes, and 3) a 74 percent majority of voting weights vote yes. Each of these criteria can be thought of as a weighted voting game where $v_1 = [620; 163, 129, ...1]$, $v_2 = [14; 1, 1, ...1]$, and $v_3 = [255; 29, 29, ...3]$ as described in Section 5. Thus, the Treaty of Nice can be described as the triple-majority weighted voting game $v_N = v_1 \wedge v_2 \wedge v_3$. Similarly, in the Treaty of Lisbon, for a proposition to pass, both of the following criteria must be met: 1) a 65 percent majority of the population votes yes and 2) a 55 percent majority of countries (or 15 countries) vote yes. Each of these criteria can be described as $v_4 = [650; 163, 129, ...1]$ and $v_5 = [15; 1, 1, ...1]$ as described in Section 5. Thus, the Treaty of Lisbon can be described as the double-majority weighted voting game $v_L = v_4 \wedge v_5$.

A **power index** is a vector-valued function $\Psi(v) = (\psi_1(v), \psi_2(v), \ldots, \psi_n(v))$ that assigns the real value $\psi_i(v)$ to each player in the game (N, v), representing a player's individual value or power in the game. Some well-known examples of such an index are the Shapley-Shubik value (Shapley and Shubik 1954) and the (normalized) Banzhaf value (for simple games) (Banzhaf 1965). Both measure a player's power by analyzing (in different ways) the number of coalitions in which a player is a swing voter. A player *swings* in a coalition if the addition of the player to the coalition changes its status from losing to winning. Borm *et.al.* (1992) defined a different index of power that analyzes the power of voters under assumptions of how they can (or cannot) communicate. The position value measures a voter's power by taking into account the voter's ability to facilitate communication between other voters as well as taking into account the coalitions in which the voter swings.

For $S \subseteq N$, on the voting game (N, v), a player i *swings* if $v(S \cup \{i\}) = 1$ and $v(S) = 0$.

The total number of swings for player i is given by:

$$\sum_{S \subseteq N - \{i\}} v(S \cup \{i\}) - v(S) \tag{1}$$

Note that in a weighted voting game, any set $S \subseteq N$ with $i \notin S$ is a *swing set* for player i if $q - w_i \leq w(S) < q - 1$.

For a simple game (N, v), the **Shapley-Shubik value** $\phi_i(v)$ for player i assigns a measure of a player's power that is proportional to the number of orderings of all the players in which that player is *pivotal*: if all of the players are assigned a number from 1 to n (*i.e.*, the players are placed in an order), then the pivotal player is the player i for which the set of players $\{1, 2, \ldots, i-1\}$ is a losing coalition and the set of players $\{1, 2, \ldots, i-1, i\}$ is a winning coalition. The Shapley value is given by

$$\phi_i(v) = \sum_{S \subseteq N - \{i\}} \frac{s!(n-s-1)!}{n!}[v(S \cup \{i\}) - v(S)] \text{ for } s = |S| \tag{2}$$

The **Banzhaf value** $\beta_i(v)$ for a simple game (N, v) is defined to be the number of swings for player i. The normalized version $\beta'_i(v)$ is given by $\frac{\beta_i}{\sum_{i=1}^{n} \beta_i}$. The Banzhaf index assigns a measure of a player's power that is proportional to the number of coalitions in which that player is a swing voter.

Owen (1995) points out that both $\phi_i(v)$ and $\beta_i(v)$ give averages of a player i's marginal contributions $v(S \cup \{i\}) - v(S)$: $\beta_i(v)$ weights them all equally and $\phi_i(v)$ weights according to the size of S.

In order to compute the position value, we will need to introduce a communication graph. An undirected **communication graph** is a pair (N, A), where N is a set of vertices and A is a set of arcs joining pairs of vertices. There are many types of communication graphs, but particularly interesting for this paper are the star graph and the complete graph. A *star graph* with center n is the graph with vertices N and arc set $A = \{\{i, n\}\}$ for $i = \{1, 2, ...n-1\}$. A *complete graph* is the graph with vertices N and the arc set $A = \{\{i, j\} : \forall i, j \in N, i < j\}$. Examples of the star graph and the complete graph on $N = \{1, 2, ...15\}$ vertices are shown in Figure 1.

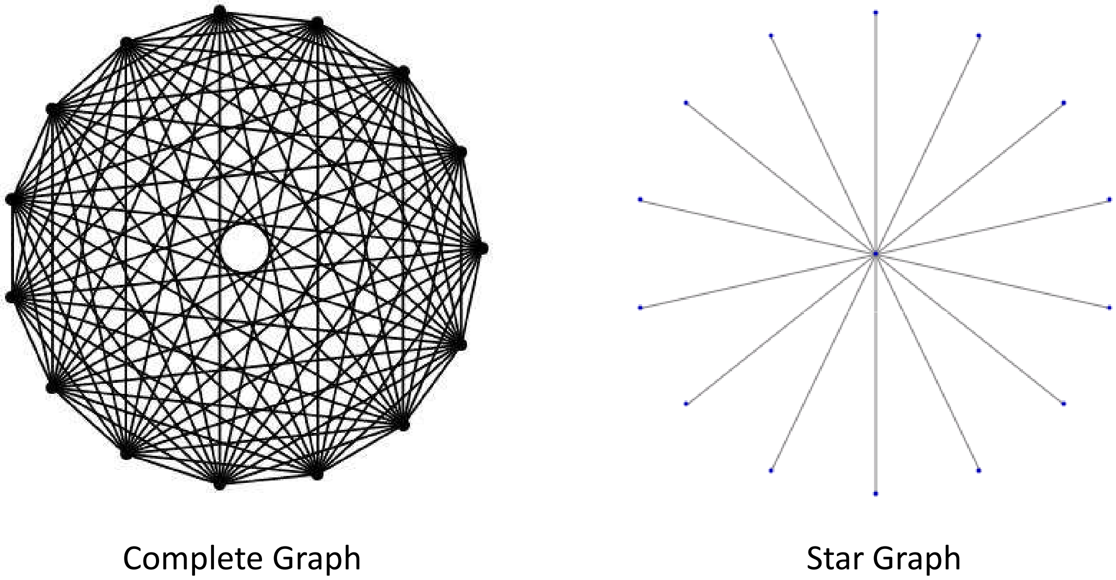

Figure 1: Complete graph and star graph on $|N| = 15$ vertices

A **communication situation** is a triple (N, v, A), where (N, v) is a game in characteristic form and (N, A) is an undirected communication graph. For convenience, we will also assume that the underlying games (N, v) in the communication situations are zero-normalized; i.e. $v(\{i\}) = 0$ for all $i \in N$. In a communication situation (N, v, A), a coalition S can obtain its economic capability $v(S)$ only if its members cooperate; this is made difficult by restrictions on communication among members (as represented by the graph (N, A)). For example, if A includes the complete graph on the induced subgraph of vertices S, then the coalition S attains its full economic capability because all players in S cooperate. In the presence of a graph A, if the coalition S is totally disconnected, however, then it cannot achieve its full economic capability. In our paper this will be relevant to communication situations on the star graph. Since we are concerned with weighted voting games in the European Union, we can utilize weighted communication situations. A *weighted communication situation* is a communication situation (N, v, A) on a weighted voting game with $[q; w_1, w_2, ...w_n]$ where w_i is the weight of player $i \in N$.

Communication situations were first studied by Myerson (1977) who focused on the role of the vertex or player. Owen (1986) investigated further along these lines, relating power indices of the original game to the restricted game. Borm, Owen, and Tijs (1992) first focused on the the role of the arc in this context, which is the approach we will take.

Even if $(i, j) \notin A$, players i and j may still be able to communicate via a path. In a communication graph two vertices v_1 and v_k are said to be joined by a *path* if there is a sequence of vertices $v_1, v_2, \ldots, v_k$ such that vertices v_i and v_{i+1} are joined by an arc for $i = 1, 2, \ldots, k-1$. A *connected component* of the graph is a maximal set of vertices such that any two vertices in the set are joined by a path. A component is a maximal set of vertices that can communicate with each other. In order to study the role of the arc in a communication situation, Borm, *et.al.* (1992) defined the *arc game*, a new game defined

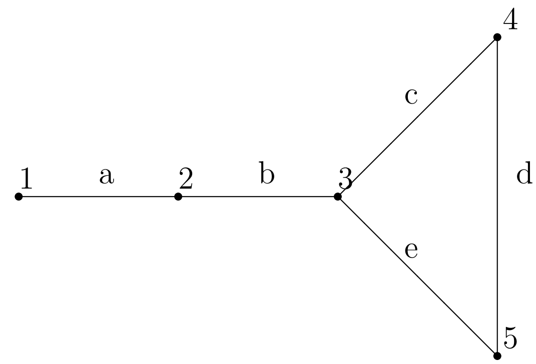

Figure 2: Example of a Communication Graph

from the communication situation in which the "players" become the arcs. The **arc game** (A, v^A) is defined in coalition form as follows:

$$v^A(T) = \sum_{C_i \in \mathcal{C}(T)} v(C_i)$$

where $T \subseteq A$ and $\mathcal{C}(T)$ is the set of connected components of the induced subgraph (N, T).

Communication situations, in the case of a simple game, model voting situations in which communication among the voters is restricted. For example, let G be the graph shown in Figure 2 (arcs are labeled by letters and players are labeled with numbers) and let v be the majority-rules game on five players (any group of three or more players wins). Then $v^A(\{a,b\}) = v(\{1,2,3\}) + v(\{4\}) + v(\{5\}) = 1 + 0 + 0 = 1$ and $v^A(\{a,c\}) = v(\{1,2\}) + v(\{3,4\}) + v(\{5\}) = 0 + 0 + 0 = 0$. Notice that without arcs c or e, players 4 and 5 cannot communicate with player 3. Also, for example, player 1 can only communicate with player 2, and in effect any of the other players, with the inclusion of arc a.

With an arc game, the arcs are the "players," so that a power index like the Shapley value for the arc game measures the value of each arc. Using this interpretation, for the arc game (A, v^A), the *Shapley value* $\phi_i(v^A)$ for arc a is computed by summing over the permutations of the swings sets for arc a:

$$\sum_{T \subseteq A - \{a\}} \frac{|T|!(|A| - |T| - 1)!}{|A|!} [v(A \cup \{a\}) - v(A)] \tag{3}$$

The **position value**, defined by Borm, *et.al.* (1992), translates these arc values into values for the original players N. Let A_i be the set of all arcs incident with vertex i. Then the position value $(\pi_1(v), \pi_2(v), \ldots, \pi_n(v))$ for the communication situation (N, v, A) is defined

$$\pi_i(v) = \frac{1}{2} \sum_{a \in A_i} \phi_a(v^A) \tag{4}$$

where $\phi_a(v^A)$ is the Shapley value of arc a in the arc game (A, v^A). The position value takes into account both a player's power in the original game (N, v) and a player's location in the graph (N, A).

Borm, *et.al.* (1992) characterize the position value in the case that the underlying graph (N, A) has no cycles. Hoke and Noonan (1997) compute the position values of the Canadian provinces in the Constitutional amendment game using a graph representing the geographic location of the provinces (and containing one simple cycle). Borm, *et.al.* (1992) point out that if the underlying graph is the complete graph (*i.e.*, A contains all possible pairs of vertices in N), then technically no restrictions on communication exist, and the position value becomes a measure of power on the original game. It can be compared to other power indices such as the Shapley and the Banzhaf.

According to Straffin (1993), the Banzhaf index is most appropriate when the voters act independently and the Shapley-Shubik is most appropriate when the voters act according to some common criteria. Both measure the power of a player using the number of sets in which the player swings. The position value allows us to also look at communication between the players. It is a power measure that credits a player's ability to facilitate communication between "valuable" players as well as the player's ability to add value to a coalition. For our paper, we utilize the Shapley-Shubik index, the normalized Banzhaf index, the position value on the star graph, and the position value on the complete graph. To more easily calculate these values for the European Union, we take advantage of useful techniques that are outlined in the next two sections.

3 The Multilinear Extension, Unanimity Games, and Generating Functions for an Intersection of Games

For this paper, we will utilize the multilinear extension (MLE) to compute the Banzhaf and Shapley values, as well as the position value. The multilinear extension of v on an n-person game is the function f defined as follows:

$$\begin{aligned} f(x_1, x_2, ...x_n) &= \sum_{S \subseteq N} \{\prod_{i \in S} x_i \prod_{i \notin S} (1 - x_i)\} v(S) \\ &= \sum_{S \subseteq N} \Delta_S (\prod_{i \in S} x_i) \end{aligned} \tag{5}$$

where $\Delta_S = \sum_{T \subseteq S} (-1)^{|S|-|T|} v(T)$.

Theorem 1 *(Owen 1995) Let (N, v) be a game where $N = \{1, 2, ..., n\}$. Let $f_i = \frac{\partial}{\partial x_i} f$ be the ith partial derivative of the MLE $f(x_1, x_2, ..., x_n)$. Setting $(x_1, x_2, ..., x_n) = (t, t, ...t)$ and integrating from 0 to 1, we obtain the Shapley value for player i:*

$$\int_0^1 f_i(t, t, ...t) dt = \phi_i(v) \tag{6}$$

We introduce the function g which will turn out to be a convenient function written in terms of all of the winning coalitions. Let g be the function

$$g(x_1, x_2, ...x_n) = \sum_{S \subseteq N} (\prod_{j \in S} x_j) v(S) = \sum_{\substack{winning \\ S}} \prod_{j \in S} x_j \tag{7}$$

Note that g(1,1,...1) is the total number of winning coalitions.

We also introduce g_i as a convenient function written in terms of all of the swing coalitions for player i:

$$\begin{aligned} g_i(x_1, x_2, ...x_n) &= \sum_{S \subseteq N - \{i\}} (\prod_{j \in S} x_j)[v(S \cup i) - v(S)] \qquad (8) \\ &= \sum_{\substack{swings \\ S}} \prod_{j \in S} x_j \end{aligned}$$

We can directly calculate the non-normalized Banzhaf value for a player i with $g_i(x_1, x_2, ...x_n)$:

$$\beta_i(v) = g_i(1, 1, ...1) \tag{9}$$

More importantly, $(1-t)^{n-1} g_i(\frac{t}{1-t}, \frac{t}{1-t}, ..., \frac{t}{1-t}) = f_i(t, t, ...t)$. Using this result, we can use g_i to calculate the Shapley value:

$$\phi_i(v) = \int_0^1 f_i(t, t, ...t) dt = \int_0^1 (1-t)^{n-1} g_i(\frac{t}{1-t}, \frac{t}{1-t}, ..., \frac{t}{1-t}) dt. \tag{10}$$

We use the functions g and g_i to calculate the Banzhaf value and Shapley-Shubik value when the game is a weighted m-majority voting game because we can use generating functions (as introduced by Mann and Shapley (1962)) to easily find g and g_i.

For a single weighted voting game and player i, we use the *generating function*

$$\prod_{j \neq i} (1 + x y^{w_j}) \tag{11}$$

where x is a counter for the size of the set and w_j is the weight of player $j \in N - \{i\}$. Our generating function can also be written as:

$$\prod_{j \neq i} (1 + x y^{w_j}) = \sum_j \sum_k c_{jk} x^j y^k \tag{12}$$

where c_{jk} is the number of subsets $S \subseteq N$ of size j and weight k. Summing only the swings of our generating function, we find that our generating function relates back to g_i:

$$g_i(t,t,...t) = \sum_{\substack{swings\\S}} t^s = \sum_j \sum_{k=q-w_i}^{q-1} c_{jk} t^j \tag{13}$$

For a double majority weighted game $v_1 \wedge v_2$ where $v_1 = [q_1; w_1^{(1)}, w_2^{(1)}, ..., w_n^{(1)}]$ and $v_2 = [q_2; w_1^{(2)}, w_2^{(2)}, ..., w_n^{(2)}]$, for player i we use our generating function and find

$$g_i(t,t,...t) = \sum_{\substack{swings\\S}} t^s = \sum_j \sum_{k=q_1-w_i^{(1)}}^{q_1-1} \sum_{l=q_2-w_i^{(2)}}^{q_2-1} c_{jkl} t^j \tag{14}$$

where c_{jkl} is the number of subsets $S \subseteq N$ of size j and weights k, l from v_1, v_2 respectively.

For an m-majority weighted voting game $v_1 \wedge v_2 \wedge ... \wedge v_m$ where $[q^t; w_1^t, w_2^t, ...w_n^t]$ for $1 \leq t \leq m$, for player i we use our generating function to find

$$g_i(t,t,...t) = \sum_{\substack{swings\\S}} t^s = \sum_j \sum_{k=q_1-w_i^{(1)}}^{q_1-1} \sum_{l=q_2-w_i^{(2)}}^{q_2-1} ... \sum_{p=q_m-w_i^{(m)}}^{q_m-1} c_{jkl...p} t^j \tag{15}$$

Using the above equation, we can calculate the Banzhaf value and Shapley value for an m-majority weighted voting game. This methodology will be especially useful for programming since a computer can easily compute a generating function. Specifically, this methodology will help to calculate power indices for the Council of Ministers under the Treaty of Nice and the Treaty of Lisbon where $v_N = v_1 \wedge v_2 \wedge v_3$ and $v_L = v_4 \wedge v_5$.

We also use the multilinear extension to compute the position value. For this, we will need unanimity games μ_S defined for all $S \subseteq N$ with $S \neq \emptyset$ as follows:

$$\mu_S(T) = \begin{cases} 1 & \text{if } S \subseteq T \\ 0 & \text{otherwise} \end{cases}$$

The set of unanimity games μ_S for $S \subseteq N$, $S \neq \emptyset$, forms a basis for the set of games (*cf.* Owen (1995)); any game (N, v) can be written (uniquely) in terms of these games: $v = \sum_{\substack{S\subseteq N\\S\neq\emptyset}} \Delta_S \mu_S$, where $\Delta_S = \sum_{T\subseteq S} (-1)^{|S|-|T|} v(T)$, the coefficients from the multilinear extension.

Theorem 2 *(Hoke 2012) If* $f(x_1, x_2, ...x_n) = \sum_{S\subseteq N} \Delta_S \prod_{j\in S} x_j$ *is the MLE for the game (N,v) and the arc game* $v^A = \sum_{\substack{S\subseteq N\\S\neq\emptyset}} \Delta_S \mu_S^A$*, then the position value* $\pi_i(v) = \sum_{S\subseteq N} \Delta_S \pi_i(\mu_S)$*.*

Thus, it is possible to compute the position values for any game from the position values on the unanimity games using the multilinear extension.

4 Computing the Position Value with the Underlying Graph as Star or Complete

The first graph on which we compute the position value is the star graph, which may be the easiest graph since we can compute the Shapley value using only $v = [q; w_1, w_2, ...w_n]$ with player n being the center player.

Theorem 3 *In a weighted communication situation (N, v, A) on a star graph with center n, the position value for player $j \in N - \{n\}$ is $\frac{1}{2}\phi_j(v')$ where $\phi_j(v')$ is the Shapley value for the weighted majority game $v' = [q - w_n; w_1, w_2, ..w_{n-1}]$ on the player set $N - \{n\}$. The position value for player n is $\frac{1}{2}$.*

Proof: Let (N, v, A) be a weighted communication situation on a star graph with center player n. By definition, for a player $i \in N$ in a communication situation, $\pi_i = \frac{1}{2}\sum_{a \in A_i} \phi_a(v^A)$. For all $T \subseteq A$, on the star graph, there is a unique connected component C_T with more than one vertex in the induced subgraph (N, T) because every arc includes n. Thus, $v^A(T) = \sum_{C_i \in \mathcal{C}(T)} v(C_i) = v(C_T) = v(n, j_1, j_2, ...j_k)$ where $j_k \in N - \{n\}$ and $j_k \in C_T$. For player j_k, $v(n, j_1, j_2, ...j_k) = v'(j_1, j_2, ...j_k)$ where $v' = [q - w_n; w_1, w_2, ..w_{n-1}]$ on the player set $N - \{n\}$. Therefore, since each player $j \in N - \{n\}$ is in a distinct arc $a_j = \{j, n\}$ on the star graph, $\pi_j = \frac{1}{2}\sum_{j \in a} \phi_a(v^A) = \frac{1}{2}\phi_{a_j}(v^A) = \frac{1}{2}\phi_j(v')$ where $\phi_j(v')$ is the Shapley value on the game v'. Since player n is in every arc, the position value for player n will be $\frac{1}{2}\sum_{n \in A} \phi_a = \frac{1}{2}$.

Unfortunately, computing the position value for most graphs requires the use of the MLE, which makes the problem more computationally difficult. Computing the MLE reqiures on the order of 2^n calculations because the MLE is computed by looking at every subset of the player set. Thus, for large games 2^n calculations becomes difficult; particularly, we find computing the MLE infeasible for player sets that are larger than approximately 22 players. However, with the complete graph, we can take advantage of symmetries and use $f(t, t, ...t)$ and $f_i(t, t, ...t)$ rather than the full MLE $f(x_1, x_2, ...x_n)$. Further, these functions can be easily computed for weighted majority games by utilizing our functions $g(t, t, ...t)$ and $g_i(t, t, ...t)$.

By symmetry of the position value on the complete graph, there are only two different position values for the game (N, μ_S): $\pi_i(\mu_S)$ when $i \in S$ which is denoted $\pi_{in}(\mu_S)$ and $\pi_i(\mu_S)$ when $i \notin S$ which is denoted $\pi_{out}(\mu_S)$. Further, we only need to find $\pi_i(\mu_k)$ for $k = |S|$. Now we show how $f_i(t, t, ...t)$ can be used rather than the full MLE in the case of the complete graph. Using Theorem 2,

$$\begin{aligned}
\pi_i(v) &= \pi_i(\sum_{S\subseteq N} \Delta_S \mu_S) && (16)\\
&= \sum_{S\subseteq N} \Delta_S \pi_i(\mu_S) && (17)\\
&= \sum_{\substack{S\subseteq N\\ i\in S}} \Delta_S \pi_i(\mu_S) + \sum_{\substack{S\subseteq N\\ i\notin S}} \Delta_S \pi_i(\mu_S) && (18)\\
&= \sum_{k=1}^{n} \sum_{\substack{S\subseteq N\\ i\in S\\ k=|S|}} \Delta_S \pi_i(\mu_S) + \sum_{k=1}^{n-1} \sum_{\substack{S\subseteq N\\ i\notin S\\ k=|S|}} \Delta_S \pi_i(\mu_S) && (19)\\
&= \sum_{k=1}^{n} c_k \pi_{in,k}(\mu_k) + \sum_{k=1}^{n-1} d_k \pi_{out,k}(\mu_k) && (20)
\end{aligned}$$

We only need to compute the coefficients c_k, d_k for $1 \leq k \leq n$, which we can do without computing all 2^n Δ_S with our earlier work. From the MLE, the coefficient of t^k in $f(t, t, t...t)$ will give us the coefficient for μ_k for $k = |S|$. However, by simply substituting t into the MLE, we lose being able to distinguish whether $i \in S$, which is important for computing the position value on the complete graph. For a given player i, think of our MLE as the sum of two pieces. From (6),

$$\begin{aligned}
f(x_1, x_2, ...x_n) &= \sum_{S\subseteq N} \Delta_S(\prod_{j\in S} x_j) = \sum_{\substack{S\subseteq N\\ i\in S}} \Delta_S(\prod_{j\in S} x_j) + \sum_{\substack{S\subseteq N\\ i\notin S}} \Delta_S(\prod_{j\in S} x_j) && (21)\\
&= f(x_1, x_2, ...x_n)_{in} + f(x_1, x_2, ...x_n)_{out} && (22)
\end{aligned}$$

Note that $f(x_1, x_2, ...x_n) = x_i \cdot f_i(x_1, x_2, ...x_n)$, and therefore $f(t, t, ...t) = t \cdot f_i(t, t, ...t)$. Notice that we can use the work from earlier in the paper to see that $t \cdot f_i(t, t, ...t) = t \cdot (1-t)^{n-1} g_i(\frac{t}{1-t}, \frac{t}{1-t}, ..., \frac{t}{1-t})$ to make using the MLE to compute the position value much easier on weighted voting games. Now that we have one piece, we need to find f_{out}, which can be found from above where $f(t, t, ...t)_{out} = f(t, t, ...t) - t \cdot f(t, t, ...t)_{in}$. Notice that again we can use the work from earlier to find $f(t, t, ...t)$ from g. Substituting $\frac{t}{1-t}$ into $g(x)$, $(1-t)^n g(\frac{t}{1-t}, \frac{t}{1-t}, ... \frac{t}{1-t}) = f(t, t, ...t)$. Therefore, we see that for a given player i:

$$f(t, t, ...t)_{in} = t \cdot f_i(t, t, ...t) \qquad (23)$$

and

$$f(t, t, ...t)_{out} = f(t, t, ...t) - t \cdot f_i(t, t, ...t) \qquad (24)$$

We want to find the coefficients on the unaniminity games $\mu_{in,k}$ and $\mu_{out,k}$ since $v = \sum_{k=1}^{n} c_k \mu_{in,k} + \sum_{k=1}^{n-1} d_k \mu_{out,k}$. In order to find c_k and d_k we use our two pieces of the MLE as generating functions:

$$f(t,t,...t)_{in} = \sum_{k=1}^{n} c_k t^k \quad (25)$$

and

$$f(t,t,...t)_{out} = \sum_{k=1}^{n-1} d_k t^k \quad (26)$$

where $k = |S|$ so that the exponent t gives us k in the unanimity game μ_k. Notice that our MLE gives us the coefficients for the vertex unanimity games μ_k, which allows us to find $\pi_i(\mu_k)$.

For a given weighted voting game, recall that we compute $f_i(t,t,...t)$ from $g_i(t,t,...t)$ using generating functions in order to compute the Banzhaf and Shapley values. Our methodology only changes slightly now because we need to keep track of whether a player $i \in S$ or $i \notin S$. We wrote a computer program to compute the position value after inputting the values of $\pi_i(\mu_k)$ as described in Hoke (2012). This program is presented in the appendix.

5 Calculating Power Indices for the Council of Ministers and the Eurogroup

To calculate each power index will require applying the mathematical tools established earlier in the paper. Importantly, each power index requires a set of weights as well as a quota (since these are all weighted voting games). Each law-making body has its own rules for a measure to pass, which must be highlighted in order to calculate each power index. The position value on the complete graph power requires an additional table for unanimity games, which is difficult to generate for games with a large player set; thus for games with more than twenty players, the position value on the complete graph is excluded.

To calculate each of these power indices we will use the function g_i as defined in (8), which will change for each different voting body, to find all of the swing sets in a given game. This will directly give us the non-normalized Banzhaf value for a given player i. As noted earlier, we can manipulate g_i to calculate the Shapley value for a given player i since $(1-t)^{n-1} g_i(\frac{t}{1-t}, \frac{t}{1-t}, ..., \frac{t}{1-t}) = \frac{\partial}{\partial i} f(t,t,...t)$. As shown in Theorem 3, to calculate the position value on the star graph, we simply calculate the Shapley value on a new game where we subtract the weights of the players in the center from the quota and remove their weights from the game. We need to divide these Shapley values by 2 since the players in the center split 50% of the power. Finally, we utilize $f_i(t,t,...t)$ that we used in computing the Shapley value. We utilize the coefficients that correspond to the unanimity games of size k that fully exclude the coalition of players and those that do not fully inlude the subset of players, as generated in our table previously.

Pseudocode has also been provided in Appendix A to help readers follow the methodology more easily.

5.1 Council of Ministers

As noted in the preliminaries, for the Council of Ministers in the European Union, each criteria for a proposition to pass can be thought of as a weighted voting game. Thus, the Treaty of Nice can be described as the triple-majority weighted voting game $v_N = v_1 \wedge v_2 \wedge v_3$ where

$$\begin{aligned} v_1 &= \{620; 163, 129, 124, 121, 92, 77, 43, 33, 23, 22, 21, 21, 20, 19, 17, 15, \\ &\qquad 11, 11, 11, 9, 6, 4, 4, 3, 2, 1, 1\} \\ v_2 &= \{14; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\} \\ v_3 &= \{255; 29, 29, 29, 29, 27, 27, 14, 13, 12, 12, 12, 12, 12, 10, 10, 10, 7, 7, \\ &\qquad 7, 7, 7, 4, 4, 4, 4, 4, 3\} \end{aligned} \tag{27}$$

Similarly, the Treaty of Lisbon can be described as the double-majority weighted voting game $v_L = v_4 \wedge v_5$ where

$$\begin{aligned} v_4 &= \{650; 163, 129, 124, 121, 92, 77, 43, 33, 23, 22, 21, 21, 20, 19, 17, 15, \\ &\qquad 11, 11, 11, 9, 6, 4, 4, 3, 2, 1, 1\} \\ v_5 &= \{15; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\} \end{aligned} \tag{28}$$

The games v_1 and v_4 model the population criteria: 65% of the EU population must approve a measure under the Treaty of Nice and 65% under the Treaty of Lisbon. The games v_2 and v_5 model the "number of member states" criteria: 50% of member states must approve under the Treaty of Nice and 55% under the Treaty of Lisbon. Note that the population weights we are using do not sum exactly to 1000 because the percentages were rounded to the nearest .1%.

Recall that $g_i(t, t, ...t)$ is a polynomial that represents all of the swing sets for a given player i. Let $g_i^{(Nice)}(t, t, ...t)$ represent the swing sets for the Treaty of Nice for a player i, which is derived from the triple intersection of v_1, v_2, and v_3. Let $g_i^{(Lis)}(t, t, ...t)$ represent the swing sets for the Treaty of Lisbon for a player i, which is derived from the double intersection fo v_4 and v_5. From here we can calculate the non-normalized Banzhaf values and the Shapley values for the Treaty of Nice where

$$\beta_i^{(Nice)} = g_i^{(Nice)}(1, 1, ...1) \tag{29}$$

and (30)

$$\phi_i^{(Nice)} = \int_0^1 (1-t)^{N-1} g_i^{(Nice)}\left(\frac{t}{1-t}, \frac{t}{1-t}, ...\frac{t}{1-t}\right) dt \tag{31}$$

We can also calculate the Banzhaf and Shapley Values for the Treaty of Lisbon where

$$\beta_i^{(Lis)} = g_i^{(Lis)}(1,1,...1) \tag{32}$$
$$\text{and} \tag{33}$$
$$\phi_i^{(Lis)} = \int_0^1 (1-t)^{N-1} g_i^{(Lis)}(\frac{t}{1-t}, \frac{t}{1-t}, ...\frac{t}{1-t})dt \tag{34}$$

To compute the position value for the star, we must slightly adjust our voting games since we are omitting the weights of the players from the center and then subtract their weights from the quota. Thus, we will compute the Shapley value for each player i that is not in the center from v_1^*, v_2^*, and v_3^* under the Treaty of Nice and v_4^* and v_5^* under the Treaty of Lisbon such that

$v_1^* = \{328; 124, 121, 92, 77, 43, 33, 23, 22, 21, 21, 20, 19, 17, 15, 11, 11, 11, 9, 6, 4, 4, 3, 2, 1, 1\}$
$v_2^* = \{12; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$
$v_3^* = \{197; 29, 29, 27, 27, 14, 13, 12, 12, 12, 12, 12, 10, 10, 10, 7, 7, 7, 7, 7, 4, 4, 4, 4, 4, 3\}$
$v_4^* = \{358; 124, 121, 92, 77, 43, 33, 23, 22, 21, 21, 20, 19, 17, 15, 11, 11, 11, 9, 6, 4, 4, 3, 2, 1, 1\}$
$v_5^* = \{13; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$

From here, the position value for each player not in the center is half. Germany and France as central players will receive 25% each.

The results for the power indices in the Council of Ministers are presented in Appendix C.

5.2 Eurogroup

Computing each power index for the Eurogroup will be the same as in the Council of Ministers except that we use a specific subset of 17 players for our player set. Note, that we calculate population percentages as a percentage of the Eurogroup, not as a percentage of the EU, so the weights in v_1 and v_4 will change. Also, the quotas for the other games will change. To find g_i to compute the Shapley and Banzhaf values, we use the following games similarly to our methodology for the Council of Ministers in Section 5.1

$$\begin{aligned}
v_1 &= \{620; 246, 196, 183, 139, 50, 34, 33, 32, 25, 16, 16, 14, 6, 4, 3, 2, 1\} \\
v_2 &= \{9; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\} \\
v_3 &= \{158; 29, 29, 29, 27, 13, 12, 12, 12, 10, 7, 7, 7, 4, 4, 4, 4, 3\} \\
v_4 &= \{650; 246, 196, 183, 139, 50, 34, 33, 32, 25, 16, 16, 14, 6, 4, 3, 2, 1\} \\
v_5 &= \{10; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}
\end{aligned} \tag{35}$$

For the star graph, we repeat what we did in the Council of Ministers, only using the games listed directly above so that

$$
\begin{aligned}
v_1^* &= \{620; 246, 196, 183, 139, 50, 34, 33, 32, 25, 16, 16, 14, 6, 4, 3, 2, 1\} \\
v_2^* &= \{9; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\} \\
v_3^* &= \{158; 29, 29, 29, 27, 13, 12, 12, 12, 10, 7, 7, 7, 4, 4, 4, 4, 3\} \\
v_4^* &= \{650; 246, 196, 183, 139, 50, 34, 33, 32, 25, 16, 16, 14, 6, 4, 3, 2, 1\} \\
v_5^* &= \{10; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}
\end{aligned} \tag{36}
$$

For the complete graph, we will need to use $g_i(x)$ as computed for the Shapley and Banzhaf values using v_1, v_2, v_3, v_4, and v_5. We will create $f_{in}(t, t, ...t)$ by generating $f_i(t, t, ...t)$ and multiplying by t. We will create $f_{out}(t, t, ...t)$ by generating $f(t, t, ...t)$ and subtracting $f_{in}(t, t, ...t)$. For each player i, we will use the arc unanimity game position values from the table below:

size s	In	Out
1	0	0
2	0.208902	0.038813
3	0.145221	0.04031
4	0.120672	0.039793
5	0.106236	0.039068
6	0.096304	0.03838
7	0.088904	0.037767
8	0.083117	0.037229
9	0.078438	0.036757
10	0.074561	0.036341
11	0.071288	0.035973
12	0.068482	0.035644
13	0.066047	0.035349
14	0.063911	0.035083
15	0.062021	0.034842
16	0.060336	0.034622
17	1/17	1/17

Figure 3: Unanimity Table

The results for the power indices in the Eurogroup are presented in Appendix C.

6 Analysis of Fairness and Efficiency in the Council of Ministers

Since the EU was created in part to promote economic growth, GDP should be an important determinant of power for its member states. Further, in order to maintain stability the EU also serves as a political institution that maintains the interests of all those it oversees – making population an important factor in determining power in decision-making. To analyze power distribution, we will use four power indices: the normalized Banzhaf index, the Shapley-Shubik index, the position value on the star graph, and the position value on the complete graph. Every power index shows that voting power is not distributed evenly across member states, which seems reasonable given the diversity in population and GDP. In both the Treaty of Nice and the Treaty of Lisbon, the voting procedures are designed so that population directly determines a member state's power through weights, whereas GDP is not a direct determinant. Since a member state's swing sets are determined by the member state's weights, when the member states are ranked by population they are also ranked by power. The same does not necessarily hold for GDP, although the ranking is approximately correct as GDP is strongly correlated to population. Within the European Union, GDP and population across member states have a correlation coefficient of .959, which is very close to 1.0. This coefficient was calculated using data in Appendix B.

A reasonable topic for analysis for the legislative bodies in the European Union is *fairness*. As noted earlier, fairness is two-fold: whether an individual member state is justly represented given its contribution to the EU in terms of percentage of GDP and percentage of population, and whether there is a balance of under-representation and over-representation across the entire EU. We will analyze fairness by calculating relevant power indices and then computing ratios comparing GDP to power and population to power. The ratio for GDP to power will be calculated by dividing each member state's percentage of EU GDP by the member state's percentage of power from the power index. The ratio for population will be calculated by dividing each member state's percentage of EU population by the member state's percentage of power from the power index. A ratio of 1.0 will be the desirable benchmark because this means that a member state's percentage of total population or GDP is the same as its percentage of power, which is an indication of a fair representation. Note, that a ratio above 1.0 means a member state is under-represented, whereas a ratio below 1.0 means a member state is over-represented, which may seem counter-intuitive to some readers. Specifically, we will use the following metrics to assess fairness:

- **Mean.** Since a ratio of 1.0 represents the fairness benchmark, power is distributed more fairly (in the sense of balance of under-representation and over-representation across all member states) if the mean of all ratios is close to 1.0. In both the Council of Ministers and the Eurogroup, the mean will generally be less than 1.0, meaning that an increase in the mean from Nice to Lisbon is desirable.

- **Standard Deviation.** More dispersion means that representation is less consistent across member states. In other words, a lower standard deviation supports a fairer distribution of power. Note that we need to be careful when using the standard deviation because we are most interested in dispersion about the 1.0 standard, whereas the standard deviation measures dispersion about the mean; in many cases the mean is not 1.0.

- **Skewness.** Symmetry shows a more fair distribution of power in balancing between under-represented (ratio greater than 1.0) and over-represented (ratio lower than 1.0) member states. Using histograms, we analyze skewness by looking at the number of member states above and below the 1.0 standard. If relatively many member states are either over- or under-represented, then we conclude that power is distributed less fairly.

• **Bias in Power Distribution.** If a member state has a characteristic that makes it more likely to be either over- or under- represented then power is not distributed fairly. For example, if member states with a large population are always under-represented, this would be an unfair distribution of power.

• **Further Under-representation.** Member states that are under-represented (ratio greater than 1.0) may notice an increase in their ratio. Since an already under-represented member state becomes even more under-represented, we deem this an unfair transition of power. More intuitively, if a member state's representation was unfavorable under Nice, it would be unfair for it to have an even more unfavorable representation under Lisbon. Note that we could also interpret a further over-representation as an unfair transition of power. However, as we will see in the analysis, the results are often skewed towards much fewer under-represented countries, which makes focusing on further under-representation more interesting.

In addition to these standard metrics, we will use the **Gini Index** as a summary statistic to provide a more aggregrate portrayal of fairness. A good description of the Gini Index, including its origins, can be found in Farris (2010). The Gini Index is most often used to measure how equitably (or inequitably) income is distributed in a population by using a *Lorenz curve*. The Lorenz curve orders the population in increasing order of income such that the first cumulative $P\%$ of the population represent the poorest $P\%$, and the last cumulative $P\%$ represent the richest $P\%$. For the poorest $P\%$ of the population, we define the Lorenze curve to be $L(P)$ such that L represents the percentage of income that the $P\%$ holds. In "perfect equitability," $L(P) = P$ so that every member of the population has the same income. An example of a Lorenze Curve is offered in Figure 4. The Gini Index measures the area between the Lorenz curve and "perfect equitability" to measure the equitability of income distribution in a population, which can be shown as area A in Figure 4. In practice, this area is doubled so that the index ranges between 0 and 1. Since our analysis is concerned with equity in power distribution relative to popoulation and GDP, we analogously treat income as power under each index and we treat population as population percentage, or GDP percentage. Note that the closer the Gini Index is to 1, the less fairly power is distributed; the closer the Gini Index is to 0, the more fairly power is distributed.

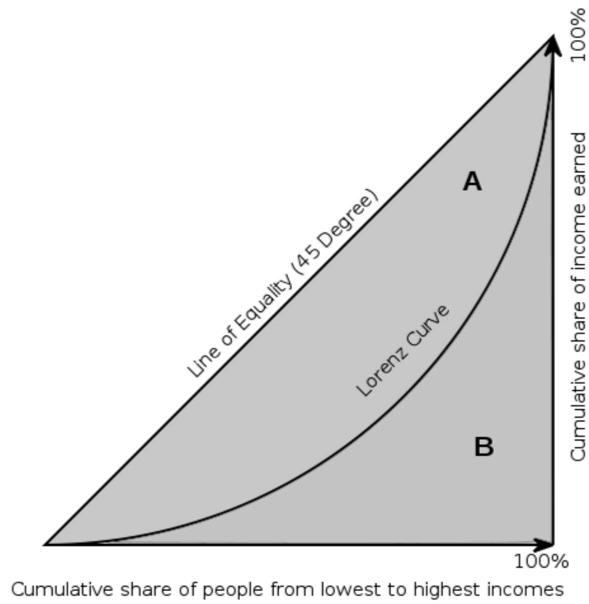

Figure 4: Example of Lorenz Curve
source by: `http://en.wikipedia.org/wiki/File:Economics_Gini_coefficient.svg`

To compute the Gini Index, we first order the member states by the power index and then by population percentage (to break ties). We then compute the cumulative population $P_i = \sum_{k=1}^{i} p_i$ for each member state (where p_i is the percent of the EU population for member state i), and the cumulative power index $L_i = \sum_{k=1}^{i} l_i$, where l_i is the value of the power index for member state i. Notice that in the special case of the European Union, since each power index is directly determined by population percentage within the weighted voting games, the orderings of power in each index will always follow the same ordering as population percentage (assuming population percentage breaks ties). Thus, for each power index, when ordering to calculate the Gini Index for the EU, we can simply order by population percentage. The formula for estimating the area B under the Lorenz curve (using trapezoids) as described in Figure 4 is given by

$$\frac{1}{2}\sum_{i=1}^{N}(P_i - P_{i-1}) \cdot (L_i + L_{i-1}) \tag{37}$$

where $P_0 = L_0 = 0$. Since we are interested in the area between the Lorenz curve and the curve $L(P) = P$, we use this area to calculate the Gini Index. In the customary problem of income distribution, the Lorenz curve always lies below $L(P) = P$; however, this may not necessarily be the case in our problem. In the case of income distribution, the Gini Index is $2(\frac{1}{2} - B)$ since the area between the curve $L(P) = P$ for $0 \leq P \leq 1$ is $\frac{1}{2}$. In the case where the Lorenz curve lies above $L(P) = P$, the Gini Index is $2(B - \frac{1}{2})$. Unfortunately, there may also be the case where the Lorenz curve lies both above and below $L(P) = P$. For our analysis, we will use the net difference in area above and below $L(P) = P$, so that the Gini Index is $|2(\frac{1}{2} - B)|$. We compute the Gini Index for each power index relative to population percentage and GDP percentage. When using GDP, P_i is the cumulative GDP, rather than the cumulative population.

Another reasonable analysis will be *efficiency*. The Treaty of Lisbon states that one of its goals is to increase efficiency of lawmaking. This may be simply referring to ease of counting votes by moving from a triple qualified majority voting system to a double qualified majority voting system. We hope to provide further insight into other areas of efficiency that result from the Treaty of Lisbon. Specifically, we will show that there are now more winning coalitions, making it easier for a vote to pass, and the minimum power required for a vote to pass decreases from the earlier to the later treaty. We will analyze law-making bodies of the Council of Ministers and the Eurogroup. For simplicity and brevity, below we will refer to the Treaties of Nice and Lisbon simply as "Nice" and "Lisbon."

6.1 Fairness in the Council of Ministers

Looking at the GDP/Shapley ratios in the Treaty of Nice, we notice immediately that Germany and France have the highest ratios, meaning that they are the most under-represented countries in the EU. This is especially interesting because they are the two counties with the highest GDP. In contrast, most of the countries with low GDP also have low ratios. Furthermore, we notice that only a handful of countries meet the 1.0 standard with the mean ratio of .68. In addition to few member states meeting the 1.0 standard, the distribution of favorable and unfavorable ratios is sporadic with a standard deviation of .60. In general, it seems that the Treaty of Nice distributes power unfavorably to countries with high GDP, but favorably to countries with low GDP. One country to note is Luxembourg, however, where relative to other countries with low GDP, its ratio is high.

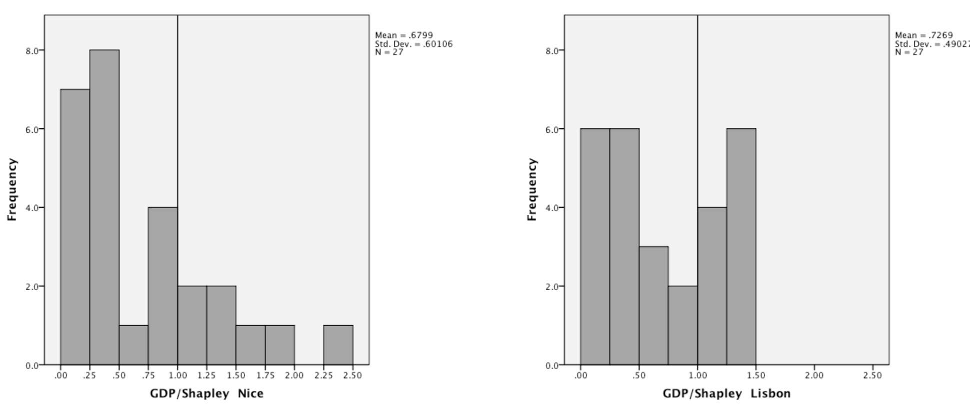

Figure 5: Council of Ministers, Shapley, GDP

Moving from Nice to Lisbon, the GDP/Shapley ratio tells a different story. The mean of the ratios increases to .73, meaning that on average a country's power more closely matches its GDP contribution. The standard deviation of the ratios decreases as well, meaning that the countries as a whole are more concentrated near the center. Similarly, the ratios are less skewed, as there are several more countries above the 1.0 standard and significantly less with ratios less than .5. Furthermore, the ratios for high GDP countries decrease and the ratios for low GDP countries increase, which is seen by the range of ratios decreasing from 2.5 to 1.75. In general, the Treaty of Lisbon seems fairer to countries with high GDP and low GDP and better balances the distribution of power throughout the entire EU. Note that countries with high GDP relative to their population are treated unfavorably (and arguably unfairly) in the transition from Nice to Lisbon – notably Denmark, Netherlands, and Sweden who are all above 1.0 but still notice their ratios increasing. In other words, they were under-represented under Nice and become more under-represented under Lisbon.

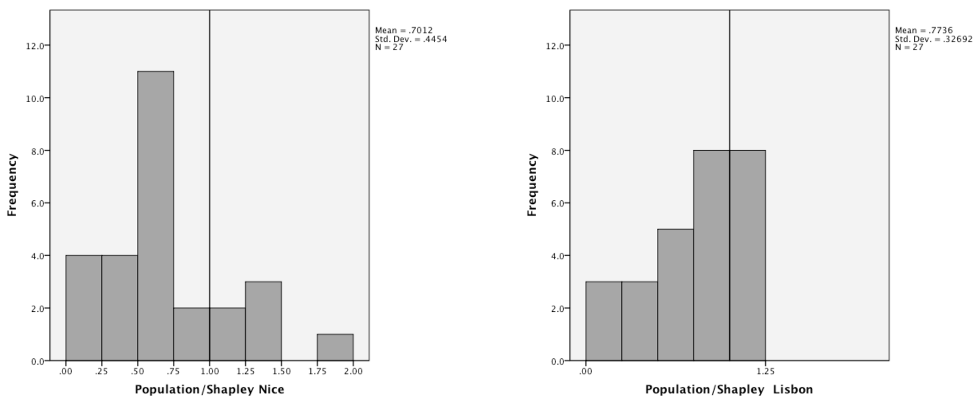

Figure 6: Council of Ministers, Shapley, Population

As noted earlier, GDP and population are closely related, so not surpringly the graphs for the ratios of population/GDP look similar to those with GDP. However, as population is more directly related to the voting games in Nice and Lisbon, the distribution of the ratios more closely reflects fairness in power distribution. Under the Treaty of Nice, the mean of the ratios is .70 with a standard deviation of .441. Similar to with GDP, however, only a handful of countries meet 1.0. According to the Treaty of Lisbon, the graph looks fairer than in Nice, with mean of .77 and standard deviation of .327. Also, many of the countries have ratios close to 1.0. As with GDP, the transition from Nice to Lisbon seems much fairer in its distribution of power relative to population. Although Denmark, Netherlands, and Sweden notice their ratios rising, they did not have ratios above 1.0 in Nice so the increase is more justified. In fact, none of the countries that had a ratio above 1.0 under Nice notice an increase in their ratio under Lisbon.

The Banzhaf ratios, in general, have a lower mean and a lower standard deviation. Thus, the Banzhaf ratios show a slightly less optimistic view toward the fairness of the distribution of power in the European Union. Further, it shows a less dramatic shift in mean and standard deviation compared to Shapley.

Also, when looking at the histograms for GDP/Banzhaf, the shifts in power to do not seem quite as dramatic as with the Shapley value. Nine countries still remain below the .25 ratio and the general trend shows much fewer above the 1.0 line than below it. In fact, the mean actually decreases. However, similar to the Shapley value, the range of ratios decreases, and the standard deviation decreases significantly. The high GDP countries are better represented and notice a decrease in their GDP/Banzhaf ratios. Now, Netherlands and Spain seem to be treated unfairly as their ratios increase even though their ratios were above 1.0 under Nice.

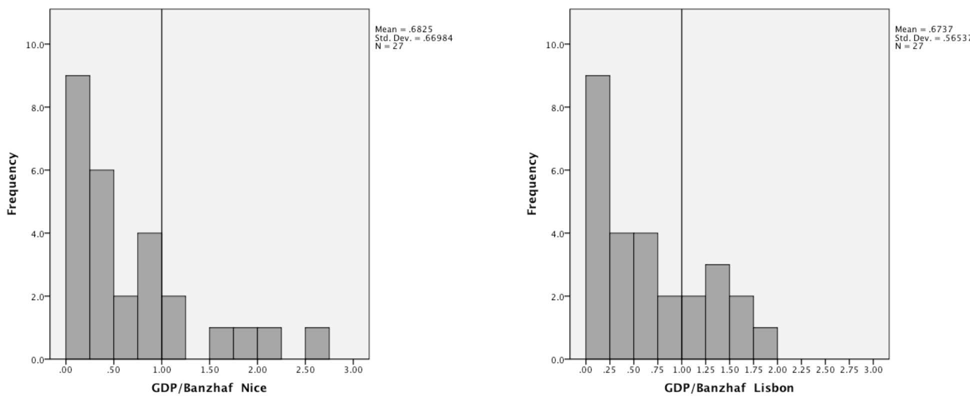

Figure 7: Council of Ministers, Banzhaf, GDP

For population, we notice that the range of ratios decreases and their spread decreases with the standard deviation decreasing. The large countries all become better represented. However, the histograms show that the number of countries above 1.0 remains at 7 and two more countries have ratios below .25. Poland and Spain notably notice an increase in their ratio even with ratios above 1.0 under Nice.

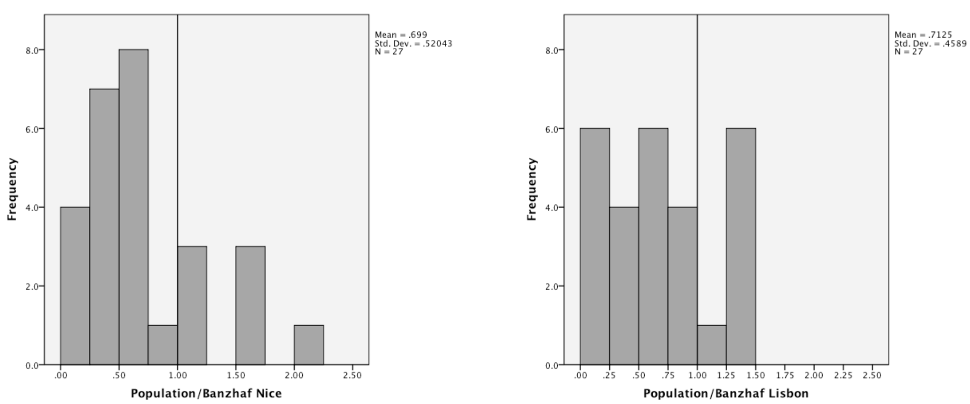

Figure 8: Council of Ministers, Banzhaf, Population

By using the position value with the star graph as a measure of power, we assume that the center controls communication. In other words, a country outside the center cannot communicate with another country without going through the center. Another way to look at this is that a coalition cannot win without the players in the center. It is not unrealistic to assume that approval cannot be obtained without the support of both Germany and France. Further, as proven earlier in the paper, the position value on the star graph is simply a special case of Shapley value analysis after removing players in the center. Taking the power of the central players for granted and redoing the analysis for the other countries without their influence should give different insight into the dynamics of the voting system.

The mean of the GDP ratio for the star graph is much higher than that of the Banzhaf and Shapley indices—in fact it is right at 1.0. However, the standard deviation is also much higher at over .7. Further, in the transition from Nice to Lisbon, the standard deviation increases. Compared to Banzhaf and Shapley, the histograms do not show dramatic changes in the shape of the distribution. Also looking at the histograms, many of the countries with ratios above 1.0 notice their ratios increasing. With Germany and France "removed" from the game, the Netherlands and the United Kingdom are the most under-represented countries with ratios above 2.0 in both Nice and Lisbon.

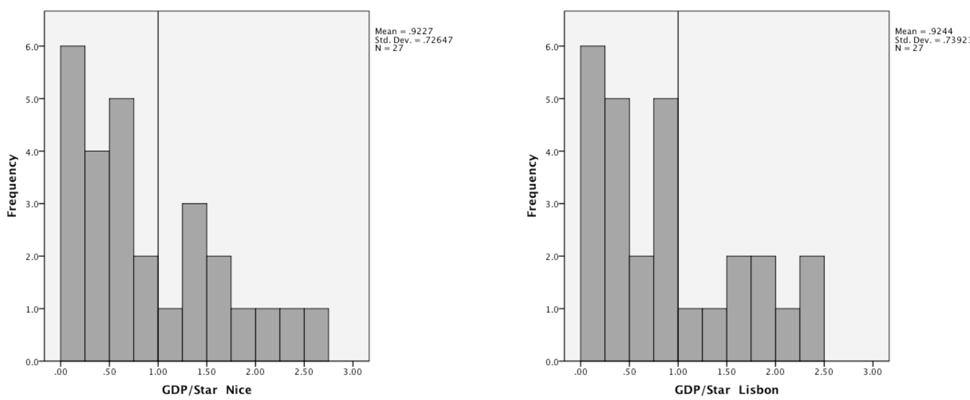

Figure 9: Council of Ministers, Star, GDP

The population ratios, on the other hand, seem to depict a completely different story. In Nice, the distribution appears almost normal, with most of the countries near the center and much less away form the center. Under Lisbon it appears almost uniform with an equal distribution at the center and away from the center. Even though the mean is again very close to 1.0, the standard deviation is much less and closer to that of Shapley and Banzhaf. Countries that notice an "unfair" increase in power include: Belgium, the Czech Republic, Greece, the Netherlands, Poland, and Spain. Analyzing population ratios in the star graph may support the hypothesis that the transition from Nice to Lisbon does not redistribute power more fairly.

To summarize our work with histograms, we utilize the Gini Index. Gini Index results for the Council of Ministers are listed in Figure 11. We notice that relative to both population

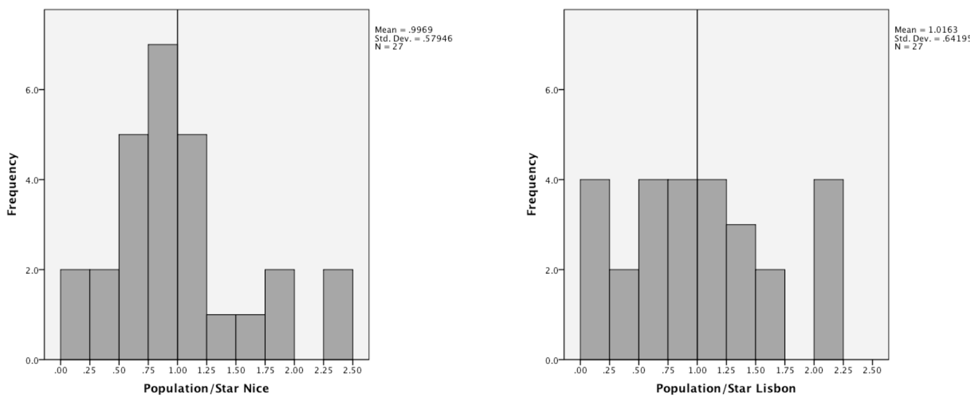

Figure 10: Council of Ministers, Star, Population

and GDP, Banzhaf and Shapley show a significant decrease in the Gini Index meaning that power is distributed more fairly in the transition from Nice to Lisbon. The star shows a less dramatic shift, meaning that the effects of the transition are small. Interestingly, utilizing the Gini Index, each power index depicts similar effects on fairness of power distribution across population and GDP.

	POPULATION			GDP		
	Nice	Lisbon	Change	Nice	Lisbon	Change
Council of Ministers						
Banzhaf	0.32	0.25	-0.07	0.38	0.32	-0.07
Shapley	0.26	0.09	-0.18	0.34	0.17	-0.17
Star Graph	0.11	0.10	-0.01	0.04	0.03	-0.01

Figure 11: Gini Indices in the Council of Ministers

6.2 Conclusions of fairness (in terms of population and GDP)

In general, the large member states in the Council of Ministers are the most under-represented, and there are more countries below the 1.0 benchmark (over-represented) than there are above the benchmark (under-represented). The shift from Nice to Lisbon alleviates the burden on the larger member states by giving them more representation, which all power indices support. Further, aside from the GDP/Banzhaf ratios, the other ratios show an increase in their mean toward the 1.0 mark as a result of the new treaty meaning that on average a member state's representation is closer to what it should be. Also, the ratios with Shapley and Banzhaf see a decrease in dispersion, meaning that as a body there is less deviation in fair representation. Banzhaf and Shapley show that the shift is more fair to individual member states and the Council of Ministers as a whole. For the star graph, the conclusions are less clear. But note that putting the two largest countries in the center gives them 25% each regardless of the game so that they show no change in representation across the two treaties. Since our analysis on the star graph in practice “ignores” France and Germany, the new treaty may be unfair to the rest of the member states. Yet, these two member states were significantly under-represented according to the other two power indices, and their increase in representation may validate the fairness of the move from Nice to Lisbon.

We noted that the voting procedures are directly determined by population, and the ratios using population measures seem to best support the hypothesis as described above. However, as GDP measures economic strength, and the EU is an economic institution, it seems that we should analyze how fairly these measures distribute economic power. Why does the Council of Ministers not have a criterion based on GDP? Perhaps, since population and GDP in the EU are correlated, the population criterion is expected to encompass GDP. Yet, although our results are not too dramatically different, the representation of power when looking at GDP is notably different from the representation when looking at population. Particularly, some member states such as the Netherlands find an unfair increase in under-representation under the Treaty of Nice, as they were under-represented even prior to the new treaty.

Assuming a central role for France and Germany, the position value with the star graph shows a much less dramatic difference in power distribution when compared to GDP in the change to the Treaty of Lisbon. Likewise, the position value on the star graph gives some indication that when compared to population, power distribution may be less fair under the Treaty of Lisbon.

The Gini Index seems to support this analysis. Both Shapley and Banzhaf portray the shift from Nice to Lisbon optimistically, whereas the results for the position value on the star graph show little change. Furthermore, according to the Gini Index, the change in fairness from Nice to Lisbon seems to be the same across population and GDP.

Since the European Union votes on a wide array of social issues, we are cautious in adding too much emphasis on using GDP as it relates to power distribution. Although economic issues are an important component of law-making within the EU, they are only a sector of the larger pool of issues. As such, population may be the more appropriate metric within the Council of Ministers. Still, we include analysis relative to GDP since the EU is described as both a political and economic institution.

6.3 Efficiency in the Council of Ministers

The Treaty of Lisbon removes one of the criteria for a vote to pass in the Council of Ministers: the qualified majority of designated weighted votes. This is what we have called v_3. By removing this criterion, the obvious effect is that counting votes will be easier. By changing the quotas, however, the effect on power distribution is unclear.

Under the Treaty of Lisbon, in the Banzhaf and Shapley power indices, the four largest member states gain power. Notably, Germany has the largest power shift and power gain, especially in the Shapley index. Under Banzhaf, the four largest countries have 38% of voting power versus 31% previously. Under Shapley the four largest countries have 48% of voting power versus 35% previously. It could be argued that centralizing power in fewer member states could make decision-making more efficient.

Additionally, the number of winning coalitions increases significantly in the Treaty of Lisbon. Under the Treaty of Nice, there are approximately 2.7 million winning coalitions, which represents 2.03% of all possible 2^{27} coalitions. On the other hand, under the Treaty of Lisbon there are approximately 17.5 million winnning coalitions, which represents 23.06% of all possible 2^{27} coalitions. Therefore, the number of winning coalitions in the Treaty of Lisbon is over six times the number of winning coalitions in the Treaty of Nice. More winning coalitions should mean that it is easier for a measure to pass, making decision-making more efficient in the Council of Ministers. The number of winning coalitions for a given game can be found by substituting 1 for each variable in g, the function that represents all winning coalitions. In other words, $g(1, 1, ...1)$ gives the number of winning coalitions.

7 Analysis of Fairness and Efficiency in the Eurogroup

7.1 Fairness in the Eurogroup

A different story emerges when analyzing fairness in the Eurogroup. In fact, contrary to the Council of Ministers, most of the power indices when compared to population and GDP show an unfavorable shift in power under the Treaty of Lisbon. First, we compare GDP to Shapley, Banzhaf, and the position value with the star graph:

1. The Shapley value shows that the fairness of shifts in power under the new treaty are unclear. In the Eurogroup under the Treaty of Nice, four member states have GDP/Shapley ratios above 1.0: France (1.449), Germany (1.881), Italy (2.018), and the Netherlands (1.095), and interestingly these member states make up over 75% of the Eurogroup's GDP. Thus, 13 of the 17 member states in the Eurogroup make up 25% of total GDP and are over-represented. The Treaty of Nice distributes power favorably to the smaller member states, and unfavorably to the large member states. Moving from Nice to Lisbon, the mean for GDP/Shapley ratio increases from .64 to .71, meaning that an average member state is closer to being represented appropriately. Also, Belgium and Austria now have GDP/Shapley ratios above 1.0, meaning that 6 of the 17 member states are under-represented. However, the number of member states in the 0-.25 range increases. Thus, the standard deviation shows little change. Under Lisbon, surprisingly, the Netherlands (1.65) becomes the most under-represented country!

2. The Banzhaf value, in general, shows that the transition from Nice to Lisbon is unfavorable. Similar to the Shapley value, the GDP/Banzhaf ratios have the same four large countries above 1.0 in the Treaty of Nice. Notably, Germany has a ratio above 2.0. The Banzhaf ratios have about the same mean, but a smaller standard deviation. Interestingly, aside from France, Germany, Italy, and Spain, the Banzhaf ratios are lower than the Shapley ratios.

   Moving from Nice to Lisbon, Spain now has a ratio above 1.0 so that there are five countries with GDP/Banzhaf ratios above 1.0. Of the previous four, only Germany's

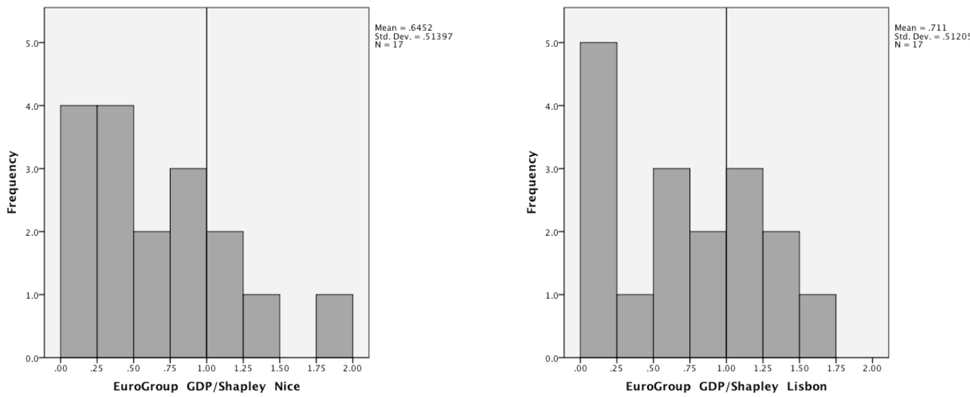

Figure 12: Eurogroup, Shapley, GDP

ratio goes down, meaning that France, Italy, and the Netherlands are even more under-represented. Still, however, Germany has the highest ratio. As with the Shapley ratios, the GDP/Banzhaf ratios notice a mean increase in moving from Nice to Lisbon. Unlike the Shapley ratios, however, the standard deviation increases as many of the already over-represented member states become even more over-represented. From the perspective of the larger countries, with respect to GDP, the shift from Nice to Lisbon was unfavorable.

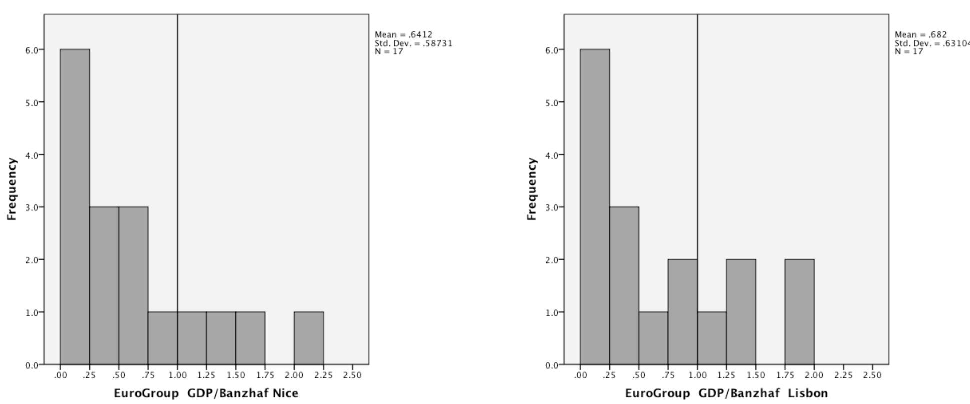

Figure 13: Eurogroup, Banzhaf, GDP

3. The position value value using the star graph also shows an unfavorable redistribution of power under the Treaty of Lisbon. As with the Council of Ministers, we use the position value on the star graph to give insight into the transition from the Treaty of Nice to Lisbon if the two largest players, France and Germany, do not notice a shift in power. We expect the game to especially change in the Eurogroup since France and Germany consist of almost 50% of the population of the Eurogroup.

In the Treaty of Nice, the GDP/star ratios have a much higher mean than the Banzhaf and Shapley ratios relative to GDP, and also notice a lower standard deviation. The range is smaller and there are also six member states with a ratio greater than 1.0. With the ratios for France and Germany much lower, Italy and the Netherlands now have the highest ratios.

The negative impacts in moving from Nice to Lisbon are much more dramatic in the case of the star graph with respect to GDP. Every country besides Germany that had a ratio above 1.0 becomes dramatically more under-represented. The range of the ratios increases, and the number of member states at either extreme increases. The standard deviation also dramatically increases. The mean does increase toward the 1.0 mark, however.

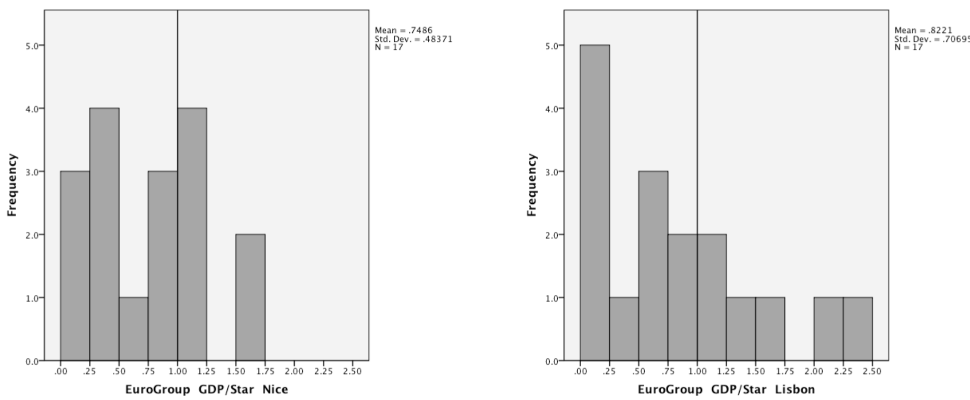

Figure 14: Eurogroup, Star, GDP

Next, we compare population to Shapley, Banzhaf, and the position value on the star graph:

1. The Shapley value shows that, relative to population, the Treaty of Lisbon more fairly distributes power in the Eurogroup. When looking at the population/Shapley ratios in the Eurogroup under the Treaty of Nice, there are also four countries with ratios above 1.0: France (1.339), Germany (1.683), Italy (1.248), and Spain (1.011). Moving from Nice to Lisbon, the mean for the population/Shapley ratios increase and the standard deviations are almost identical. As shown in the histograms, the range of ratios decreases and there is more central tendency toward 1.0. Under Lisbon, there are now six under-represented member states, an increase from four under Nice. The larger member states are now represented more fairly as they notice a decrease in their ratios, and the number of member states below .25 remains at only four. Also, the number of countries in the .25-.5 and .5-.75 ranges decrease significantly.

2. The Banzhaf value shows an unfavorable power shift. Looking at population/Banzhaf, as with the Shapley ratios with population, the same four countries have ratios above

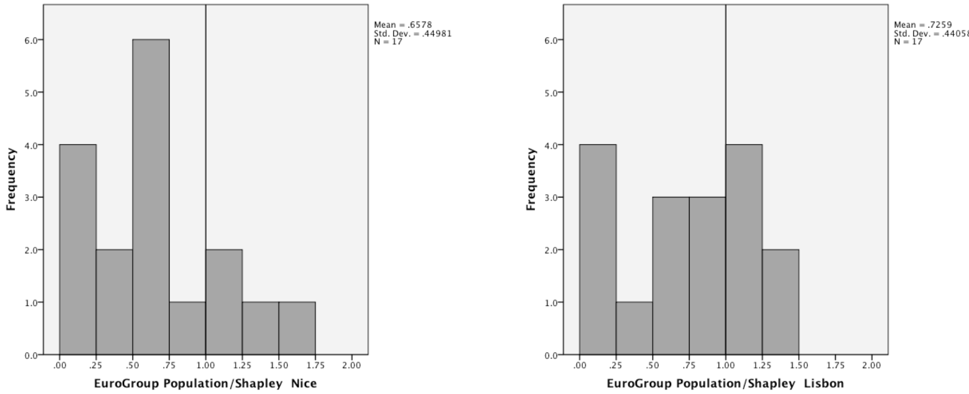

Figure 15: Eurogroup, Shapley, Population

1.0 under Nice. Aside from those four countries, every country has a lower population/Banzhaf ratio than population/Shapley. The population/Banzhaf mean is about the same, but the ratios are more sporadic with a higher standard deviation. As seen in the histograms, there are 4 countries in each of the ranges below .75, and there is one country in each of the ranges above .75.

Moving from Nice to Lisbon, the range of ratios decreases. Also, the mean increases from .65 to .69. But, the number of ratios in the range of 0-.25 and the range of 1.5-1.75 increases, which in turn increases the standard deviation. As with the Banzhaf/GDP ratios, all of the large countries aside from Germany notice an increase in their ratio and several smaller countries notice a decrease in their ratio.

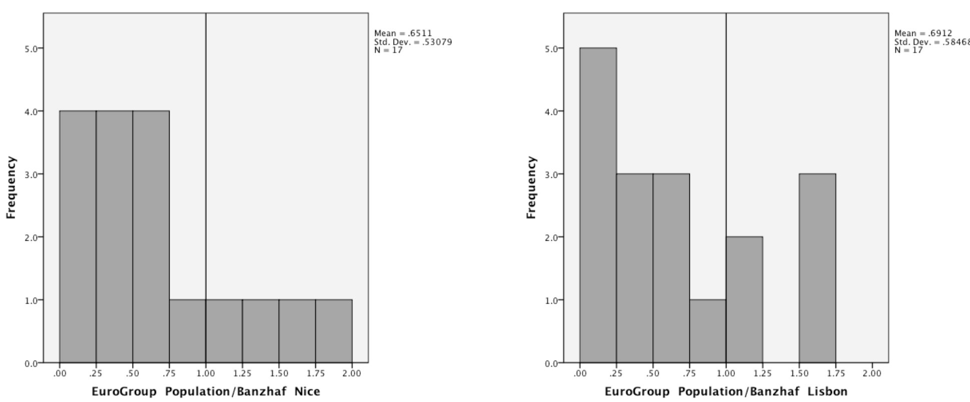

Figure 16: Eurogroup, Banzhaf, Population

3. The position value on the star graph shows similar results to looking at the position values on the star graph with respect to GDP, with an unfavorable shift in power. The mean increases, but the standard deviation increases, the range of the ratios increases,

and the under-represented member states become even more under-represented. An interesting difference with population, however, is that there are only three countries with ratios above 1.0 under Nice, but there are six countries above 1.0 under Lisbon.

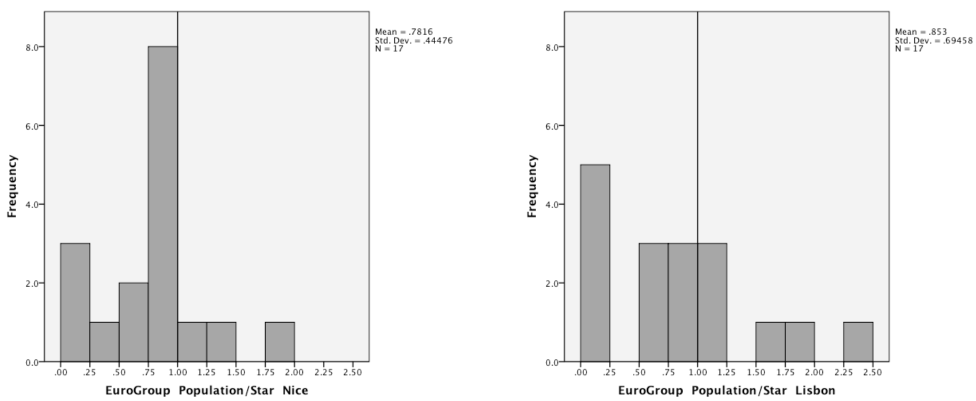

Figure 17: Eurogroup, Star, Population

7.2 Fairness in the Eurogroup as Measured by the Position Value on the Complete Graph

With the Eurogroup, we add a fourth power index: the position value on the complete graph. As noted earlier, this index credits a player's ability to facilitate communication between "valuable" players in addition to a player's ability to influence the outcome. The complete graph is interesting in that in distributing power it takes away influence from the weights and distributes power more evenly through the graph. We expect that we will notice less dramatic power shifts and the range of the power index will be much smaller than that of the Shapley, Banzhaf, and star graph. Since there is significant variation in the distribution of population and GDP in the European Union, the ratios of population and GDP to the position value on the complete graph will be much more pessimistic of the fairness of the EU, and specifically the Eurogroup. Although the mean will be closer to 1.0, the range of the ratios will be much greater and the standard deviation will be much greater compared to the other power indices. Nonetheless, we can still utilize the position value on the complete graph to provide insight into the *changes* in fairness as a result of the shift from Nice to Lisbon.

For the GDP/complete ratios, in the shift from Nice to Lisbon, the mean decreases from .79 to .78, which could imply that power is distributed unfairly as an average member states has a ratio even farther from the 1.0 mark. However, the standard deviation decreases from .98 to .90 and the range of ratios decreases. Also, more member states notice ratios closer to the 1.0 line, even though there are still the same number of member states in the 0-.50 and .50-1.0 range. Aside from the Netherlands, all of the under-represented member states are represented more fairly under Lisbon with respect to GDP.

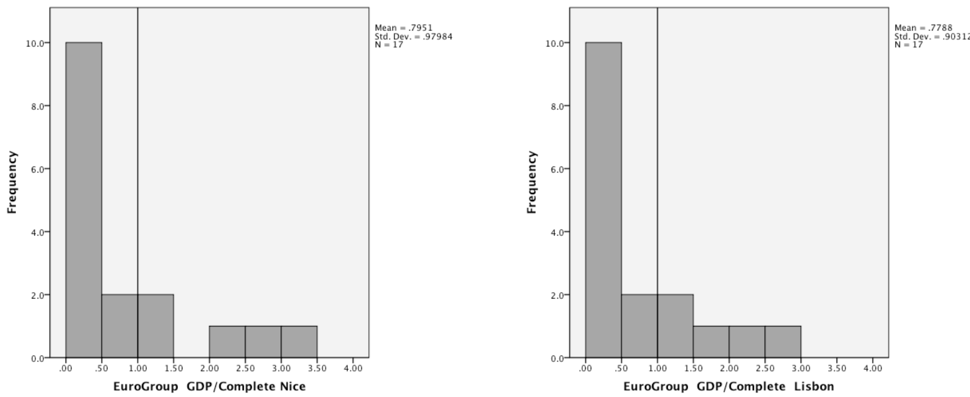

Figure 18: Eurogroup, Complete, GDP

With population, the results are less exciting. Looking at the histograms, aside from Germany, every member state remains in the same .25 range as under the Treaty of Nice. The mean decreases, which is mostly due to Germany's dramatic decrease from 2.9 to 2.5. In general, the other ratios notice little change, although every member state with a ratio above 1.0 notices a decrease in their ratio.

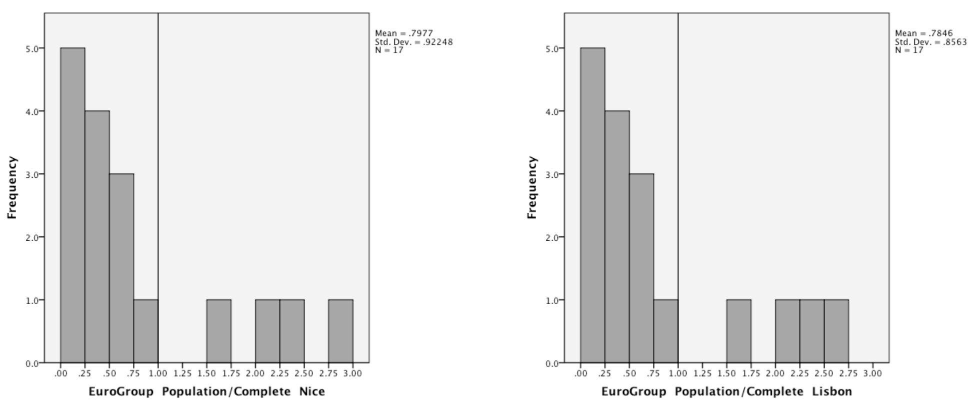

Figure 19: Eurogroup, Complete, Population

7.3 Fairness in the Eurogroup as Measured by the Gini Index

The Gini Indices for the Eurogroup are displayed in Figure 20. In the Eurogroup, the results using the Gini Index are unclear. Using the Banzhaf value, the Gini Index portrays the shift from Nice to Lisbon as unfair as each notices an increase under both population and GDP.

Using the Shapley Value, the Gini Index portrays the shift from Nice to Lisbon as fairer as it notices a decrease with both population and GDP. Since the original Gini Indices for the star graph are so close to 0 relative to the others, the increase may not be significant. Further, the change for the complete graph seems relatively undramatic, showing that the effect of the transition is small.

Gini Index for the CM and the EG under Nice and Lisbon

	POPULATION			GDP		
	Nice	Lisbon	Change	Nice	Lisbon	Change
Council of Ministers						
Banzhaf	0.32	0.25	-0.07	0.38	0.32	-0.07
Shapley	0.26	0.09	-0.18	0.34	0.17	-0.17
Star Graph	0.11	0.10	-0.01	0.04	0.03	-0.01
Eurogroup						
Banzhaf	0.31	0.35	0.04	0.35	0.38	0.03
Shapley	0.25	0.16	-0.09	0.28	0.20	-0.09
Star Graph	0.02	0.07	0.05	0.06	0.11	0.05
Complete Graph	0.53	0.51	-0.02	0.55	0.53	-0.02

Figure 20: Gini Indices

7.4 Conclusions of Fairness (in terms of population and GDP)

Contrary to the Council of Ministers, the transition of power in the Eurogroup seems unfair relative to GDP, according to the Banzhaf value and position value on the star graph. The few over-represented countries generally become more over-represented and the most under-represented countries notice little change. Although the mean typically increases, the standard deviation increases as well, often substantially. The Shapley value and the position value on the complete graph show an aggregrate unclear effect, with both favorable and unfavorable effects under Lisbon.

Relative to population, again the Banzhaf and position value on the star graph show unfavorable shifts in power. It is important to note, however, that the Shapley value shows that the power shifts due to the Treaty of Lisbon are favorable and generally more fair. The treaty grants much more power to the large players than the other indices, which is reasonable given that they now constitute a much larger percentage of population and GDP. We notice the same general effects as in the Council of Ministers with the mean increasing and the range decreasing relative to both population and GDP. Finally, the position value on the complete graph shows little aggregrate redistribution of power. Germany is the only member state with a significant change in its ratio, and it is a favorable change.

As with the Council of Ministers, the Gini Index seems to support our analysis. Under the Gini Index, Shapley portrays the shift optimistically, Banzhaf portrays the shift pessimistically, and the position value on both the star graph and the complete graph shows little change. Furthermore, according to the Gini Index, the change in fairness from Nice to Lisbon seems to be the same across population and GDP. We also note an interesting observation with the position value on the star graph. When the underlying graph is the star graph, our central player(s) always have the same power, which in effect "removes" their effect on power distribution. It is interesting that in both the Council of Ministers and the Eurogroup, the effects for the position value on the star graph are minimal compared to the effects for Banzhaf and Shapley. This may show that the aggregate effects of the shift in treaties are mostly from effects toward France and Germany.

The big losers in the Eurogroup as a result of the shift seem to be the four large countries which are France, Germany, Italy, and Spain. With respect to GDP, as with the Council of Ministers, again Netherlands notices unfavorable representation.

In contrast to the Council of Ministers, the Eurogroup *only* votes on economic issues. As such, GDP may be the more appropriate metric for comparing to power distribution. To remain consistent, however, we include analysis for both population and GDP.

7.5 Efficiency in the Eurogroup

Unlike in the Council of Ministers, the Eurogroup does not notice the same centralization of power when looking at the four largest member states of Germany, France, Italy, and Spain. Banzhaf shows a decrease in voting power from 51% to 48% and the position value on the star graph notices a decrease from 70% to 65%. The complete graph shows a subtle increase in voting power from 33% to 35%. Shapley, in contrast, notices an increase in voting power from 58% to 65%. Of course, there may be less incentive to centralize power with a much smaller voting body.

Since the Eurogroup has a smaller player set, there are now 2^{17} possible coalitions rather than 2^{27}. Under the Treaty of Nice there are 7,628 winning coalitions, which represents 5.8% of the 131,072 possible coalitions. Under the Treaty of Lisbon there are 20,768 winning coalitions, which represents 15.8% of the possible 131,072 possible coalitions. Thus, there are more winning coalitions under the Treaty of Lisbon, which could make decision-making more efficient.

8 Composition of Games to Include the European Parliament

Thus far, our paper has ignored the EU's co-decision process where both the European Parliament (EP) and the Council of Ministers must approve a measure in order for it to pass. Because these two law-making bodies have disjoint player sets, we will use the idea of **composition of games**, as described by Owen (1995), to analyze power distribution when we include the EP. As earlier in the paper, in order to compute the Shapley and Banzhaf values when we include the EP, we use the multilinear extension.

Let $M_1, M_2, ..., M_n$ be n dispoint nonempty player sets and $(w_1, M_1), (w_2, M_2), ..., (w_n, M_n)$ be simple games, and let (N, v) be a non-negative game ($v(S) \geq 0, \forall S \subseteq N$). Then the *v-composition* of $w_1, w_2, ..., w_n$. denoted $u = v[w_1, w_2, ..., w_n]$ is a game with player set

$M = \cup_{j=1}^{n} M_j$ defined by $u(S) = v(\{j|w_j(S \cup M_j) = 1\})$ for $S \subseteq M$. The co-decision process with the EP and Council of Ministers can be modelled as a v-composition of games where v is the simple game on two players in which the only winning coalition is the one containing both players, w_1 is the game modelling voting in the Council of Ministers under the Treaty of Lisbon, and w_2 is the game modelling voting in the EP. M_1 is the set containing the 27 member states (the players in Council of Ministers), and M_2 is the set containing the 751 MEP's in the EP. To compute the multilinear extension, we need the following theorem from Owen(1995):

Theorem 4 *Let v be a nonnegative n-person game, let w_1, w_2 be simple games on disjoint player sets, and let $u = v[w_1, w_2]$. Let $h, f^{(1)}, f^{(2)}$ be the MLE's of v, w_1, w_2, respectively, and let $f = (f_1, f_2)$. Then $\alpha = h \circ f$ is the multilinear extension of u.*

Let $v =$ the simple game on two players in which the only winning coalition is the one containing both players. Since its MLE is given by $h(z_1, z_2) = z_1 z_2$, the theorem tells us that the MLE α for the composite game including both the EP and the Council of Ministers is found by simply multiplying the respective multilinear extensions ($f^{(1)}$ and $f^{(2)}$) for the Council of Ministers and the EP. We note that α is a function of $27 + 751$ players represented by variables $x = (x_1, x_2, ..., x_{27})$ and $y = (y_1, y_2, ..., y_{751})$: $\alpha = f^{(1)}(x) f^{(2)}(y)$. To compute the Shapley value and the Banzhaf value using the MLE, we need partial deriviatives of α: $\alpha_i(t, t, ..., t)$. For player i in the Council of Ministers, $\alpha_i = f_i^{(1)} f^{(2)}$ since we take the partial derivative of α with respect to x and $f^{(2)}$ is only in terms of y. Similarly, for player i in the EP, $\alpha_i = f^{(1)} f_i^{(2)}$ since we take the partial derivative of α with respect to y and $f^{(1)}$ is only in terms of x. In the preceding sections, we showed how to use the special function g_i to compute the function $f_i^{(1)}(t, t, ..., t)$ for the Council of Ministers under the Treaty of Lisbon. In the EP, $f_i^{(2)}(t, t, ..., t)$ for the majority rules game on 751 players can be computed in the same way. However, note that we also need to compute the functions $f^{(1)}(t, t, ..., t)$ and $f^{(2)}(t, t, ..., t)$, which we can generate from the function g. Recall that g is the function given by

$$g(x_1, x_2, ...x_n) = \sum_{S \subseteq N} (\prod_{j \in S} x_j) v(S) \tag{38}$$

For the Council of Ministers, then $f^{(1)}(t, t, ..., t) = (1-t)^n g^{(1)}(\frac{t}{1-t}, \frac{t}{1-t}, \cdots \frac{t}{1-t})$, an exercise that will be left for the reader. Similarly, for the EP, $f^{(2)}(t, t, ..., t)$ can be computed by using $g^{(2)}$.

Example. Let (N, w_1) be the simple majority rules game on $2a + 1$ players with quota $a + 1$ and let (M, w_2) be the simple majoirty rules game on $2b + 1$ players with quota $b + 1$ where the player sets of each are disjoint. Let $f^{(1)}(x)$,$f^{(2)}(y)$ be the multilinear extensions for w_1 and w_2 respectively. Let v be the composition game of w_1 and w_2 with multilinear extension $\alpha(x, y) = f^{(1)} \cdot f^{(2)}$.

Since a player i only swings in w_1 if the size of the coalition $S \subseteq N - \{i\}$ is equal to a, $g_i^{(1)}(t,t,...t) = \binom{2a}{a} t^a$. Relating $g_i^{(1)}$ back to $f_i^{(1)}(x)$, we find that $f_i^{(1)}(t,t,...t) = \binom{2a}{a} t^a (1-t)^a$. In addition, a coalition S is only winning in w_2 if the size of the coalition $S \subseteq M$ is greater than or equal to b, or $s \geq b+1$. Thus, $g^{(2)}(t,t,...t) = \sum_{j=b+1}^{2b+1} \binom{2b+1}{j} t^j$. Relating $g^{(2)}$ back to $f^{(2)}$, we see that $f^{(2)}(t,t,...t) = \sum_{j=b+1}^{2b+1} \binom{2b+1}{j} t^j (1-t)^{2b+1-j}$. Because $\alpha(x,y) = f^{(1)} \cdot f^{(2)}$, for a player $i \in N$, we have $\alpha_i = f_i^{(1)} \cdot f^{(2)}$. From the work in this example, for a player $i \in N$

$$\alpha_i = \binom{2a}{a} t^a (1-t)^a \cdot \sum_{j=b+1}^{2b+1} \binom{2b+1}{j} t^j (1-t)^{2b+1-j} \tag{39}$$

Previously in the paper we used $g_i(1,1,...1)$ to calculate the Banzhaf index, but from Owen, we can also compute the un-normalized Banzhaf index for player i by using $\beta_i = f_i(\frac{1}{2}, \frac{1}{2}, ...\frac{1}{2})$. After some algebra, through this new method

$$\beta_i = \binom{2a}{a} \cdot \frac{1}{2}^{2a+1} \tag{40}$$

To normalize the index for player $i \in N$, we need to divide by the total number of winning coalitions in v. Summing $g^{(1)}(1,1,...1)$ and $g^{(2)}(1,1,...1)$, we compute the total number of winning coalitions in v to be $(2a+1)\binom{2a}{a}(\frac{1}{2})^{2a+1} + (2b+1)\binom{2b}{b}(\frac{1}{2})^{2b+1}$. Similarly, the un-normalized Banzhaf index for a player $i \in M$ is $\binom{2b}{b} \cdot \frac{1}{2}^{2b+1}$. We note, importantly, that a is the only variable in the numerator for a player in N and that b is the only variable in the numerator for a player in M, whereas both a and b are in the denominator. As a result, as a becomes large relative b, the Banzhaf value for a player in M decreases and approaches 0. Conversely, the Banzhaf value for each player in N approaches $\frac{1}{n}$.

The result in this example gives us insight into the implications of using the Banzhaf index for our analysis of the European Union when including the European Parliament. Since the EP has significantly more players, we expect the players in the Council of Ministers to have low Banzhaf values and in consequence the Council of Ministers will have significantly less power relative to the EP. In fact, using the Banzhaf value, under the Treaty of Lisbon, the Council of Ministers as a body has a Banzhaf value of 15% and the EP as a body has a Banzhaf value of 85%. Since we have evidence to believe that this dispropportionate distribution of power is directly resulting from the dispropportionate size of player sets for the Council of Ministers and the EP, we will only focus on the Shapley value in our analysis of the EU when including the EP.

Hoke (2012) repeats the above example using the Shapley value and proves that player $i \in N$ takes $\frac{1}{2n}$ power and player $i \in M$ takes $\frac{1}{2m}$ power so that N receives 50% power and M receives 50% power *regardless* of the disparity in size of player sets N and M. We prefer the Shapley value in our analysis for this reason.

Applying our methodology of the composition of games to the EU, for the Shapley value we find that the Council of Ministers takes 87% power and the EP takes 13%. Results for each individual country are listed in Appendix C. Similar to the Banzhaf index, we again notice a disparity in power distribution across the two law-making bodies. To understand why this disparity occurs, we inspect two scenarios using the composition of games that are simple modifications to the quotas that define the formal voting procedures of the Council of Ministers under the Treaty of Lisbon.

First, consider the Council of Ministers to be defined as the double intersection game $v_1' \wedge v_2'$ where $v_1' = \{620; 163, 129, ..., 1\}$ and $v_2' = \{14; 1, 1, ..., 1\}$, which are simply the same games as those in the Council of Ministers under Lisbon only now $q_1' = 620$ and $q_2' = 14$. This game could also be thought of as the intersection game that defined the Council of Ministers under Nice, without the majority game with designated weights. Repeating our analysis using the composition of games, we find that the Council of Ministers now takes 80% power and the EP takes 20% power. From this exercise, it seems that lowering the quotas toward 50% redistributes power back to the EP, which uses a quota of 50%.

Next, consider the Council of Ministers to be the double intersection game $v_1'' \wedge v_2''$ where $v_1'' = \{500; 163, 129, ..., 1\}$ and $v_2'' = \{14; 1, 1, ..., 1\}$, in which both games are simple majority rules with quotas of 50%. In other words, these are $v_1'' = v_1'$ with $q_1'' = 14$ and $v_2'' = v_2'$ with $q_2'' = 500$. Under these new rules, the Council of Ministers receives 64% power and the EP receives 36% power. Notice two implications from these results. One, again lowering the quota of a game closer to 50% redistributes power back to the Parliament. Two, the intersection of games in the Council of Ministers advantageously distributes power to the Council even though both games have quotas of 50%. From these two realizations, the structure of the relation in decision-making between the Council of Ministers and the EP clearly favors the Council, despite the explicit goal of Lisbon to balance power between the two.

9 Concluding Remarks on the Transition from Nice to Lisbon

In this paper, we introduced the Shapley value, the Banzhaf value, the position value using the star graph, and the position value using the complete graph. The position value requires a graph, making power distribution dependent on more than just the weights of the game. The star graph assumes restricted communication through central players, and the complete graph assumes unrestricted communication. We utilized unanimity games and several functions that relate back to the multilinear extension (MLE) in order to more easily compute each of these power indices. Further, we described how to use generating functions in the case of the intersection of weighted voting games.

Each of these power indices makes different underlying assumptions, so we would expect them to return different results—especially in games as diverse as the Council of Ministers and the Eurogroup. Still, although each power index is inherently different, their results must have certain similarities that give us insight into the aggregate dynamics of the actual law-making body, regardless of the assumptions.

Particularly, we wanted to answer questions regarding the effects of the transition in the European Union from the Treaty of Nice to the Treaty of Lisbon. As formal voting procedures change, what happens to the distribution of power? More specifically, the EU claims that the Treaty of Lisbon will be more efficient and fairer, so we asked if our analysis supported these claims.

On the surface, dropping a criterion for a measure to pass makes counting votes easier and thus the voting process more efficient. We probed deeper into game theory to obtain more insight into this question. We counted the number of winning coalitions under both Nice and Lisbon to see the change in probability that a given measure would pass. Moving from Nice to Lisbon, in both the Council of Ministers and the Eurogroup, there are more winning coalitions, meaning that the probability a measure will pass increases, and thus decision-making becomes more efficient.

To assess the fairness of the shifts in power under Lisbon, we computed power indices that we introduced in Sections 2-4 and compared them to two relevant metrics—population and GDP—because the EU serves as an economic and political institution. For the Council of Ministers, we computed the Shapley value, the Banzhaf value, and the position value using the star graph, but omitted the position value on the complete graph due to computational difficulties. Each of these indices portrayed the shift from Nice to Lisbon optimistically with member states as a whole more fairly represented when compared to both population and GDP. If we were to order the indices from most optimistic to least optimistic about the transition, a reasonable ranking would be: Shapley, Banzhaf, position value on the star graph. Our analysis was supported by the Gini Index, which interestingly showed similar effects relative to population versus GDP in the transition from Nice to Lisbon, per power index. Furthermore, it is interesting that the results for the star graph under the Gini Index show little change when we remove the central players from our analysis of power distribution. Perhaps, the aggregate effects of the shift in treaties are mostly from effects toward France and Germany.

For the Eurogroup, we redid this analysis but supplemented it with the position value using the complete graph since our player set was smaller. Contrastingly, in general the indices when compared with population and GDP showed that for the Eurogroup the transition from Nice to Lisbon was unfavorable. The Shapley value and the position value on the complete graph were more optimistic, but in general seemed to show that the aggregate effect of the new treaty was unclear. The Gini Index supported our analysis for reasons similar to those in our analysis of the Council of Ministers.

Within this analysis, we realized that although population and GDP are correlated, they yield different results when looking at our ratios. Since the voting procedures in the Council of Ministers and the Eurogroup are directly determined by population, we expected our power indices to match population percentages best—and they did. In fact, in every power index, the rankings of population percentages and power are the same. On the other hand, this is not the case for GDP. As a result, countries with disproportionate population and GDP percentages may notice different results when GDP and population are compared to the power indices. Particularly, we found that the Netherlands noticed an unfavorable loss in power when compared to GDP. Although population may be more relevant to decision-making in the Council of Ministers, whereas GDP may be more relevant to decision-making in the Eurogroup, we included both metrics in analysis of each law-making body to remain consistent.

Finally, we inspected the balance of power between the Council of Ministers and the European Parliament under the Treaty of Lisbon by introducing the composition of games since the two bodies have independent decision-making processes. We ignored the Parliament's role under the Treaty of Nice because we were more interested in Lisbon's claim to strengthen Parliament through formalizing the co-decision process. Further, we argued that since Parliament has significantly more voters than the Council of Ministers, the Banzhaf value should not be used because it is not robust when one disjoint player set is significantly larger than another. The Shapley index tells us that the Council of Ministers retains the majority of power under Lisbon, although at first glance one may conclude that the two should split power equally because each body must approve a measure in order for it to pass. Exploring why the power is unevenly distributed, we discover that both the higher quotas and the intersection of games in the Council of Ministers grants power more favorably to the Council of Ministers using the Shapley value.

10 Limitations and Future Work

We note that since our analysis only uses formal voting procedures, our results for power distributions likely have limitations. Voting members in an institution as large and sophisticated as the European Union are surely influenced by more than just formal voting procedures. For example, in addition to formalizing the co-decision process for the European Parliament, Lisbon grants the EP voting power on many more issues, which may be a significant increase in power not directly reflected from the formal voting procedures. At worst, our results only apply to a narrowly focused perspective toward the EU, and are only useful to this degree. Still, our methodology may be useful if it were adjusted to account for more factors such as political affiliation or geographical location. At best, our results give general insights into voting dynamics of the EU that may not be obvious at first glance. Our paper offers a thorough analysis of power transition when making different underlying assumptions about cooperation and communication among players.

Future work could include increasing the efficiency of computation in order to include the complete graph analysis for the Council of Ministers. Further, an interesting institution associated with the EU would be the European Central Bank. Although the ECB is not directly affected by the Treaty of Lisbon, since it serves the Eurogroup, it may be another area to utilize the complete graph with a smaller player set. Also, as we noted, a limitation of this paper is its narrow focus on formal voting procedures. It may be interesting to look at party influence in both the Council of Ministers and the EP. Furthermore, we alluded to the use of blocking in our use of France and Germany as one central player. There may be realistic blocking examples that better model the European Union such as blocking by past voting alliances or by geographical region. Finally, we could simply introduce other well-known graphs such as the chain graph or bipartite graph to compute the position value and compare the power distribution outcomes accordingly.

Acknowledgements

We would like to thank Dr. Olena Mykhaylova, Assistant Professor of Economics, and Dr. Art Gunlicks, Professor Emeritus in the Department of Policial Science, for their consultations.

11 References

Banzhaf, J. R. 1965. Weighted Voting Doesn't Work: a Mathematical Analysis. *Rutgers Law Review* 19: 317-343.

Borm, Peter, Guillermo Owen, and Stef Tijs. 1992. On the Position Value for Communication Situations. *SIAM J. Disc. Math* 5 (3): 305-320.

Europa. 2013. "How the EU works." Accessed March 13.
http://europa.eu/about-°©-eu/index_en.htm

Farris, Frank A. 2010. The Gini Index and Measures of Inequality. *American Mathematical Monthly* 117(10): 851-864.

Gunlicks, Arthur B. 2011. *Comparing Liberal Democracies: United States, United Kingdom, France, Germany, and the European Union*. Bloomington, In: IUniverse.

Hoke, Kathy and Kimberly Noonan. 1997. Communication Games and the Canadian Constitution. *The UMAP Journal* 18(2): 95-104.

Hoke, Kathy. 2012. The Position Value for Communication Games on the Complete Graph. Preprint.

Mann, I. and Shapley, L.S. 1962. Values of Large Games VI: Evaluating the Electoral College Exactly. RAND Corp. Memo RM-3158-PR. Santa Monica, CA: RAND.

Myerson, R.B. 1977. Graphs and Cooperation in Games. *Math.Oper. Res* 2: 225-229.

Owen, G. 1995. *Game Theory, 3rd Edition*. New York: Academic Press.

Owen, G. 1986. Values of Graph-Restricted Games. *SIAM J.Algebraic and Discrete Methods* 7: 210-220.

Shapley, L.S. and M. Shubik. 1954. A Method for Evaluating the Distribution of Power in a Committee System. *American Political Science Review* 48: 787-792.

Straffin, Phillip D. 1993. *Game Theory and Strategy*. Washington, D.C.: Mathematical Association of America.

VoteWatch Europe. 2013. Accessed March 13, 2013. http://www.votewatch.eu

Appendices

A Pseudocode to Compute Power Indices

Number of Winning Coalitions Calculator

[Input]: game(s) with quota(s)

Module[Generating Function]

*creates generating function $gf = \prod_{i=1}^{n}(1 + xy^{w_i})$ to represent all possible 2^n coalitions

Module[Check Winning]

*returns the polynomial where all exponents on y are greater than or equal to the quota, or the function $g = \prod_{i=1}^{n}(1 + xy^{w_i}) = \sum_{k=1}^{2^n}\sum_{j=1}^{n} c_{kj}x^k y^j$ so that $w_S \geq q$ for each coalition S

Module[Number Winning]

*since the coefficient on $x^k y^j$ in this polynomial represents the number of subsets of size k and weight $j \geq q$, setting $x = 1$ and $y = 1$ returns the number of winning coalitions

Banzhaf Calculator (for a player i)

[Input]: game(s) with quota(s)

Module[Generating Function]

*creates generating function $gf_i = \prod_{j \neq i}(1 + xy^{w_j})$ to represent all possible 2^{n-1} coalitions without player i

Module[Find Swings]

*returns the swings polynomial where all exponents on y are between $q - w_i$ and $q - 1$, or $g_i = \sum_{k=1}^{2^{n-1}}\sum_{j=q-w_i}^{q-1} c_{kj}x^k y^j$

Module[Find Banzhaf]

*since the coefficient on $x^k y^j$ in this polynomial represents the number of subsets of size k and weight $q - w_i \geq j \geq q - 1$, setting $x = 1$ and $y = 1$ returns a player's un-normalized Banzhaf value because the un-normalized Banzhaf value for a player is simply the number of its swing coalitions

Shapley Calculator (for a player i)

[Input]: game(s) with quota(s)

Module[Generating Function]

*creates generating function $gf_i = \prod_{j \neq i}(1 + xy^{w_j})$ to represent all possible 2^{n-1} coalitions without player i

Module[Find Swings]

*returns the swings polynomial where all exponents on y are between $q - w_i$ and $q - 1$, or $g_i = \sum_{k=1}^{2^{n-1}}\sum_{j=q-w_i}^{q-1} c_{kj}x^k y^j$

Module[Find Shapley]

*sets $x = \frac{t}{1-t}$ and multiplies the swing polynomial by $(1-t)^{N-1}$ to get the function $f_i(t, t, ...t)$ and integrates from 0 to 1 for each player i. This returns the Shapley value for player i

Complete Graph Calculator (for a player i)

[Input]: game(s) with quota(s)

[Input]: complete graph position values table for a game of size N

Module[Generating Function]

*creates generating function $gf_i = \prod_{j \neq i}(1 + xy^{w_j})$ to represent all possible 2^{n-1} coalitions without player i

Module[Find Swings]

*returns the swings polynomial where all exponents on y are between $q - w_i$ and $q - 1$, or $g_i = \sum_{k=1}^{2^{n-1}} \sum_{j=q-w_i}^{q-1} c_{kj} x^k y^j$

Module[Find player MLE(in)]

*sets $x = \frac{t}{1-t}$ and multiplies the swing polynomial by $(1-t)^{N-1}$ to get the function $f_i(t, t, ...t)$ for each player i. Multiplying by t gives us $f(t, t, ...t)_{in}$

Module[Find player MLE(out)]

*sets $x = \frac{t}{1-t}$ and multiplies the winning polynomial by $(1-t)^N$ to get the function $f(t, t, ...t)$ for each player i. Subtracting the player MLE(in) from above gives the player MLE(out)

Module[player PV(in)]

*multiplies each coefficient c_{jk} in MLE(in) by its respective value in the position value table

Module[player PV(out)]

*multiplies each coefficient c_{jk} in MLE(out) by its respective value in the position value table

Module[Find PV]

*sums player PV(in) and player PV(out) to compute the position value for player i

Composition Calculator

[Input]: game(s)$_1$ with quota

[Input]: game(s)$_2$ with quota

Module[Generating Function$_1$]

*creates generating function $gf_i^{(1)} = \prod_{j \neq i}(1 + xz^{w_j^{(1)}})$ to represent all possible 2^{n-1} coalitions without player i

Module[Generating Function$_2$]

*creates generating function $gf_i^{(2)} = \prod_{j \neq i}(1 + yz^{w_j^{(2)}})$ to represent all possible 2^{m-1} coalitions without player i

Module[Find Swings$_1$]

*returns the swings polynomial where all exponents on z are between $q^{(1)} - w_i^{(1)}$ and $q^{(1)} - 1$, or $g_i^{(1)} = \sum_{k=1}^{2^{n-1}} \sum_{j=q^{(1)}-w_i^{(1)}}^{q^{(1)}-1} c_{kj} x^k z^j$

Module[Find Swings$_2$]

*returns the swings polynomial where all exponents on z are between $q^{(2)} - w_i^{(2)}$ and $q^{(2)} - 1$, or $g_i^{(2)} = \sum_{k=1}^{2^{m-1}} \sum_{j=q^{(2)}-w_i^{(2)}}^{q^{(2)}-1} d_{kj} y^k z^j$

Module[Find Shapley$_1$]

*solve for $f_i^{(1)}(t, t, ...t)$ $f^{(2)}(t, t, ...t)$, multiply the two together, and integrate from 0 to 1 for

$i \in N$
Module[Find Shapley_2]
*solve for $f^{(1)}(t,t,...t)$ $f_i^{(2)}(t,t,...t)$, multiply the two together, and integrate from 0 to 1 for $i \in M$

B EU Information

Figure 21: EU map

MEMBER STATES OF THE EUROPEAN UNION RANKED BY GDP AND POPULATION

Country	GDP 2011 (billions of Euro)	Country	Population (in thousands)
Germany	$2,592.6	Germany	81,752
France	$1,996.6	France	65,048
UK	$1,747.1	UK	62,499
Italy	$1,579.7	Italy	60,626
Spain	$1,063.4	Spain	46,153
Netherlands	$602.0	Poland	38,530
Sweden	$387.6	Romania	21,414
Belgium	$369.8	Netherlands	16,656
Poland	$369.7	Greece	11,310
Austria	$300.7	Belgium	11,001
Denmark	$240.5	Portugal	10,572
Greece	$208.5	Czech Rep	10,487
Finland	$189.5	Hungary	9,986
Portugal	$171.0	Sweden	9,416
Ireland	$159.0	Austria	8,404
Czech Rep	$156.2	Bulgaria	7,369
Romania	$131.3	Denmark	5,561
Hungary	$99.8	Slovakia	5,392
Slovakia	$69.1	Finland	5,375
Luxem	$42.6	Ireland	4,570
Bulgaria	$38.5	Lithuania	3,053
Slovenia	$36.2	Latvia	2,075
Lithuania	$30.8	Slovenia	2,050
Latvia	$20.2	Estonia	1,340
Cyprus	$18.0	Cyprus	840
Estonia	$16.0	Luxem	512
Malta	$6.5	Malta	415
EU	$12,642.8	EU	502,405

Figure 22: EU Countries Ranked by GDP and Population

MEMBER STATES OF THE EUROGROUP RANKED BY GDP AND POPULATION

Country	GDP 2011 (billions of Euro)	Country	Population (in thousands)
Germany	$2,592.6	Germany	81,752
France	$1,996.6	France	65,048
Italy	$1,579.7	Italy	60,626
Spain	$1,063.4	Spain	46,153
Netherlands	$602.0	Netherlands	16,656
Belgium	$369.8	Greece	11,310
Austria	$300.7	Belgium	11,001
Greece	$208.5	Portugal	10,572
Finland	$189.5	Austria	8,404
Portugal	$171.0	Slovakia	5,392
Ireland	$159.0	Finland	5,375
Slovakia	$69.1	Ireland	4,570
Luxembourg	$42.6	Slovenia	2,050
Slovenia	$36.2	Estonia	1,340
Cyprus	$18.0	Cyprus	840
Estonia	$16.0	Luxembourg	512
Malta	$6.5	Malta	415
Eurogroup	$9,421.2	Eurogroup	332,017

Figure 23: Eurogroup Countries Ranked by GDP and Population

Country	No. of MEPS	MEP Percent of Total MEP's	Percent GDP	Percent Population
Germany	96	12.78%	20.51%	16.27%
France	74	9.85%	15.79%	12.95%
Italy	73	9.72%	12.49%	12.07%
United Kingdom	73	9.72%	13.82%	12.44%
Spain	54	7.19%	8.41%	9.19%
Poland	51	6.79%	2.92%	7.67%
Romania	33	4.39%	1.04%	4.26%
Netherlands	26	3.46%	4.76%	3.32%
Belgium	22	2.93%	2.93%	2.19%
Czech Rep	22	2.93%	1.24%	2.09%
Greece	22	2.93%	1.65%	2.25%
Hungary	22	2.93%	0.79%	1.99%
Portugal	22	2.93%	1.35%	2.10%
Sweden	20	2.66%	3.07%	1.87%
Austria	19	2.53%	2.38%	1.67%
Bulgaria	18	2.40%	0.30%	1.47%
Denmark	13	1.73%	1.90%	1.11%
Finland	13	1.73%	1.50%	1.07%
Slovakia	13	1.73%	0.55%	1.07%
Ireland	12	1.60%	1.26%	0.91%
Lithuania	12	1.60%	0.24%	0.61%
Latvia	9	1.20%	0.16%	0.41%
Slovenia	8	1.07%	0.29%	0.41%
Cyprus	6	0.80%	0.14%	0.17%
Estonia	6	0.80%	0.13%	0.27%
Luxembourg	6	0.80%	0.34%	0.10%
Malta	6	0.80%	0.05%	0.08%
Total	751	100.00%	100.00%	100.00%

Figure 24: Parliament Ranked by Number of MEP's

C Results

Council of Ministers Power Indices

Country	Banzhaf		Shapley		Star	
	Nice	Lisbon	Nice	Lisbon	Nice	Lisbon
Germany	7.78%	11.34%	8.73%	15.31%	25.00%	25.00%
France	7.78%	9.06%	8.72%	11.45%	25.00%	25.00%
UK	7.78%	8.74%	8.69%	10.94%	5.29%	5.80%
Italy	7.78%	8.55%	8.69%	10.64%	5.29%	5.65%
Spain	7.42%	6.68%	8.01%	7.87%	4.88%	4.34%
Poland	7.42%	5.57%	7.99%	6.70%	4.88%	3.70%
Romania	4.26%	4.11%	3.99%	4.07%	2.41%	2.47%
Netherlands	3.97%	3.50%	3.67%	3.23%	2.23%	2.10%
Greece	3.68%	2.91%	3.40%	2.44%	2.05%	1.75%
Belgium	3.68%	2.85%	3.40%	2.36%	2.05%	1.71%
Czech Rep	3.68%	2.79%	3.40%	2.28%	2.05%	1.68%
Portugal	3.68%	2.79%	3.40%	2.28%	2.05%	1.68%
Hungary	3.68%	2.73%	3.40%	2.20%	2.05%	1.65%
Sweden	3.09%	2.67%	2.81%	2.13%	1.70%	1.61%
Austria	3.09%	2.55%	2.81%	1.97%	1.70%	1.54%
Bulgaria	3.09%	2.43%	2.81%	1.82%	1.70%	1.48%
Denmark	2.18%	2.20%	1.95%	1.51%	1.18%	1.34%
Finland	2.18%	2.20%	1.95%	1.51%	1.18%	1.34%
Slovakia	2.18%	2.20%	1.95%	1.51%	1.18%	1.34%
Ireland	2.18%	2.08%	1.95%	1.37%	1.18%	1.27%
Lithuania	2.18%	1.90%	1.95%	1.14%	1.18%	1.18%
Latvia	1.25%	1.78%	1.10%	0.99%	0.66%	1.11%
Slovenia	1.25%	1.78%	1.10%	0.99%	0.66%	1.11%
Estonia	1.25%	1.72%	1.10%	0.91%	0.66%	1.08%
Cyprus	1.25%	1.66%	1.10%	0.84%	0.66%	1.05%
Luxembourg	1.25%	1.60%	1.10%	0.77%	0.66%	1.01%
Malta	0.94%	1.60%	0.82%	0.77%	0.50%	1.01%

Figure 25: CM Power Indices

Eurogroup Power Indices

Country	Banzhaf		Shapley		Star		Complete	
	Nice	Lisbon	Nice	Lisbon	Nice	Lisbon	Nice	Lisbon
Germany	13.01%	14.13%	14.63%	19.48%	25.00%	25.00%	8.31%	9.50%
France	13.01%	11.61%	14.63%	16.15%	25.00%	25.00%	8.31%	8.62%
Italy	13.01%	11.31%	14.63%	15.42%	10.40%	7.36%	8.31%	8.42%
Spain	12.36%	11.20%	13.75%	14.41%	9.48%	7.30%	8.08%	8.10%
Netherlands	6.25%	4.98%	5.84%	3.87%	4.02%	2.91%	5.83%	5.33%
Greece	5.82%	4.46%	5.22%	3.31%	3.68%	2.91%	5.66%	5.20%
Belgium	5.82%	4.43%	5.22%	3.28%	3.68%	2.91%	5.66%	5.19%
Portugal	5.82%	4.40%	5.22%	3.25%	3.68%	2.91%	5.66%	5.18%
Austria	4.96%	4.24%	4.21%	3.02%	3.09%	2.82%	5.40%	5.12%
Finland	3.44%	3.91%	2.88%	2.59%	2.09%	2.69%	5.05%	5.01%
Slovakia	3.44%	3.91%	2.88%	2.59%	2.09%	2.69%	5.05%	5.01%
Ireland	3.44%	3.85%	2.88%	2.46%	2.09%	2.68%	5.05%	4.98%
Slovenia	2.05%	3.60%	1.71%	2.13%	1.20%	2.58%	4.75%	4.89%
Estonia	2.05%	3.53%	1.71%	2.05%	1.20%	2.57%	4.75%	4.88%
Cyprus	2.05%	3.51%	1.71%	2.02%	1.20%	2.57%	4.75%	4.87%
Luxembourg	2.05%	3.48%	1.71%	1.99%	1.20%	2.55%	4.75%	4.86%
Malta	1.42%	3.45%	1.16%	1.95%	0.86%	2.54%	4.61%	4.85%

Figure 26: EG Power Indices

Country	CM Power	EP Power	Combined Power
Austria	1.61%	0.34%	1.95%
Belgium	1.96%	0.39%	2.35%
Bulgaria	1.47%	0.32%	1.79%
Cyprus	0.58%	0.11%	0.69%
Czech Rep	1.89%	0.39%	2.28%
Denmark	1.19%	0.23%	1.42%
Estonia	0.65%	0.11%	0.76%
Finland	1.19%	0.23%	1.42%
France	10.34%	1.31%	11.65%
Germany	13.95%	1.70%	15.65%
Greece	2.04%	0.39%	2.42%
Hungary	1.82%	0.39%	2.21%
Ireland	1.06%	0.21%	1.27%
Italy	9.60%	1.29%	10.89%
Latvia	0.72%	0.16%	0.88%
Lithuania	0.85%	0.21%	1.06%
Luxembourg	0.51%	0.11%	0.62%
Malta	0.51%	0.11%	0.62%
Netherlands	2.76%	0.46%	3.22%
Poland	5.99%	0.90%	6.90%
Portugal	1.89%	0.39%	2.28%
Romania	3.53%	0.58%	4.12%
Slovakia	1.19%	0.23%	1.42%
Slovenia	0.72%	0.14%	0.86%
Spain	7.04%	0.95%	7.99%
Sweden	1.75%	0.35%	2.11%
UK	9.87%	1.29%	11.16%
Total	86.72%	13.28%	100.00%

Figure 27: Council of Ministers and Parliament Results Under Lisbon