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Duration Measures for Corporate Project Valuation

Tom Arnold
University of Richmond, tarnold@richmond.edu

David S. North
University of Richmond, dnorth@richmond.edu

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Tom Arnold
The Robins School of Business
Department of Finance
1 Gateway Road
University of Richmond
Richmond, VA 23173
tarnold@richmond.edu
O: (804) 287-6399
F: (804) 289-8878

and

David S. North
The Robins School of Business
Department of Finance
1 Gateway Road
University of Richmond
Richmond, VA 23173
O: 804-287-1938
F: 804-289-8878
dnorth@richmond.edu

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Abstract:

Sensitivity analysis is a very common exercise performed with the forecasting of project cash flows. In this paper, a duration-type measure is generated that provides a single number for the assessment of project cash flows relative to changes in the discount rate (or adjusted for changes in a particular cash flow model parameter). The calculation is no more difficult than the duration measures that already exist for bonds. Yet, the calculation provides valuable insight that many times is lost when performing sensitivity analysis. Further, at a minimum, the measure provides a gauge for the consequences of mis-specifiying the discount rate for a project.

INTRODUCTION:

When evaluating a project, forecasting parameters such as cash flows, timing of cash flows, and a discount rate is a necessary process. Unfortunately, forecasted parameters are not always reliable even under the best of circumstances. Consequently, even the best forecasting methods are subject to sensitivity analysis to determine the effects of changes in forecasting parameters. Generally, this is performed by assessing various scenarios for the parameters and may even lead to a probability distribution for the project's profitability. The process is tedious and has the potential to provide analysis that is equally complex to interpret.

Ultimately, the goal of sensitivity analysis is to provide a measure of how susceptible the forecasted cash flows are to a change in the environment in which the forecasted cash flows have been generated. When analyzing a bond portfolio, duration measures provide the equivalent of sensitivity analysis relative to interest rate changes. The purpose of this paper is to generate an equivalent duration measure for project cash flows. Such a measure can be simple to calculate (depending on the cash flow structure or the modeling of the cash flows) and captures the intuition of project sensitivity analysis in a single number.

The current literature takes duration analysis beyond bonds and analyzes duration of equities (e.g. Havert, McLaughlin, and Taggart (1998)). This investigation takes duration analysis to the project level and can in theory provide a duration measure for the cash flows of the entire firm or for the cash flows to equity holders. The primary goal is to provide a duration measure relative to the discount rate (keeping cash flows static) because the discount rate is very difficult to assess for a project even when cash flows are relatively certain. However, duration measures can be calculated based on a cash flow model parameter (e.g. the sales growth rate) or based on the discount rate which can in turn also affect a cash flow model parameter.

In the first section of the paper, a duration measure for project valuation is derived and discussed. An associated duration measure specific to a project input parameter is derived in the second section. The third section concludes the paper. In addition, there is an appendix containing VBA code for Excel-type functions to measure project duration and project convexity.

SECTION 1: The Duration Measure for Project Cash Flows

Macaulay's (1938) bond duration, "D", is the negative of the first derivative of the bond price multiplied by (1 + discount rate) and divided by the bond price. More specifically, let the discount rate be symbolized by "k" and let the bond price be symbolized by "P".

$$D = -\left[\frac{\Delta P}{\Delta k}\right] * \left[\frac{(1+k)}{P}\right] \tag{1}$$

The duration is then used to find an approximate percentage change in the bond price given a change in the discount rate (see Bodie, Kane, and Marcus (2004)).

$$\frac{\Delta P}{P} = -D * \left[\frac{\Delta k}{(1+k)}\right] \tag{2}$$

One can perform a similar calculation based on modified duration, "D_M", which is Macaulay's duration divided by (1 + k).

The magnitude of the duration measure indicates the susceptibility of the bond price to an interest rate change. A larger duration indicates a greater percentage change in bond price should interest rates move. To mitigate the risk of interest rate fluctuations, securities of varying duration can be combined to produce an optimal or target portfolio duration. Consequently, duration is a simple metric that provides valuable insight in regard to bond portfolios. This is the same intuition desired for evaluating project cash flows in regard to sensitivity analysis.

To create a "project duration" measure, "D_P", it is simply a matter of following the formula provided in equation (1). Project cash flows (CF_i), positive or negative, are evaluated on a periodic basis with a terminal value ($TERM_N$) used to value all cash flows beyond period "N". Notice the project valuation, "V", resembles a bond valuation without fixed coupons.

$$V = \sum_{i=0}^{N} \left[\frac{CF_i}{(1+k)^i} \right] + \frac{TERM_N}{(1+k)^N} \tag{3}$$

In practice, the terminal value is usually based on the Nth cash flow, "CF_N". CF_N is assumed to grow at a rate, "g", into perpetuity and the terminal value is calculated as a growing perpetuity (assuming k > g).

$$TERM_N = \frac{CF_N * (1+g)}{(k-g)} \tag{4}$$

Equation (5) restates equation (3):

$$V = \sum_{i=0}^{N-1} \left[\frac{CF_i}{(1+k)^i} \right] + \left[\frac{CF_N}{(1+k)^N} \right] * \left[1 + \frac{(1+g)}{(k-g)} \right] \tag{5}$$

Equation (6) is the negative of the partial derivative of the project cash flows relative to the discount rate:

$$-\frac{\Delta V}{\Delta k} = \left\{ \sum_{i=1}^{N-1} \left[\frac{i * CF_i}{(1+k)^{i+1}} \right] + \left[\frac{N * CF_N}{(1+k)^{N+1}} \right] * \left[1 + \frac{(1+g)}{(k-g)} + \frac{(1+k)*(1+g)}{N*(k-g)^2} \right] \right\} \tag{6}$$

Multiplying equation (6) by (1 + k) and dividing it by equation (5) produces the project duration:

$$D_P = \frac{\left\{ \sum_{i=1}^{N-1} \left[\frac{i * CF_i}{(1+k)^i} \right] + \left[\frac{N * CF_N}{(1+k)^N} \right] * \left[1 + \frac{(1+g)}{(k-g)} + \frac{(1+k)*(1+g)}{N*(k-g)^2} \right] \right\}}{\left\{ \sum_{i=0}^{N-1} \left[\frac{CF_i}{(1+k)^i} \right] + \left[\frac{CF_N}{(1+k)^N} \right] * \left[1 + \frac{(1+g)}{(k-g)} \right] \right\}} \tag{7}$$

Although equation (7) appears daunting, the components of the equation are all based on what would be existing project cash flows. Further, Excel subroutines are supplied in the appendix to perform the calculation. To demonstrate the calculation of the project duration and to allow for project comparison where the project duration measure is useful, Table 1 contains three projects that have the same net present value ($1,733.33), but with slightly different cash flows (Note: although the same discount rate and maturity are assumed for all three projects for presentation purposes, one should not infer that the discount rates or the maturities of the projects need to be the same for this type of analysis).

Table 1: An Evaluation of Three Projects with Equivalent Net Present Values and Perpetual Cash Flows

Time:	***Project 1 Cash Flows:***	***Project 2 Cash Flows:***	***Project 3 Cash Flows:***
Year 0	$ (2,000.00)	$ (2,000.00)	$ (2,000.00)
Year 1	$ 110.00	$ 0.00	$ 110.00
Year 2	$ 121.00	$ 121.00	$ 121.00
Year 3	$ 133.10	$ 133.10	$ 133.10
Year 4	$ 146.41	$ 146.41	$ 146.41
Year 5	$ 161.05	$ 161.05	$ 161.05
Year 6	$ 177.16	$ 177.16	$ 177.16
Year 7	$ 194.87	$ 194.87	$ 194.87
Year 8	$ 214.36	$ 214.36	$ 214.36
Year 9	$ 235.79	$ 235.79	$ 235.79
Year 10	$ 259.37	$ 259.37	$ 259.37
Year 11	$ 285.31	$ 285.31	$ 285.31
Year 12	$ 313.84	$ 313.84	$ 0.00

Year 13	$ 345.23	$ 345.23	$ 345.23
Year 14	$ 379.75	$ 379.75	$ 379.75
Year 15	$ 417.72	$ 417.72	$ 417.72
Year 16	$ 459.50	$ 459.50	$ 459.50
Year 17	$ 505.45	$ 505.45	$ 505.45
Year 18	$ 555.99	$ 555.99	$ 555.99
Year 19	$ 611.59	$ 1,223.18	$ 1,223.18
Year 20	$ 672.75	$ 672.75	$ 672.75
Terminal Value*:	$11,661.00	$ 11,661.00	$11,661.00
Net Present Value:	$ 1,733.33	$ 1,733.33	$ 1,733.33
Payback Period:	10.87 Years	11.23 Years	10.87 Years
Discounted Payback Period:	20 Years	20 Years	20 Years
Project Duration:	50.45	51.49	50.85
Effect of 10 Basis Point Increase in Discount Rate:	-4.59%	-4.68%	-4.62%

Note: The discount rate is 10% APR.
*The terminal value is based on the cash flow in Year 20 assuming a 4% perpetual growth rate. This terminal value assesses all of the cash flows from Year 21 and onward.

When examining Project 1, the numerator for equation (7) is $87,444.44 and the denominator for equation (7) is the net present value (NPV) of the project, $1,733.33. As stated in Table 1, the project duration is 50.45 and indicates that a 10 basis point increase in the discount rate will reduce the NPV by 4.59% (-50.45*{0.1% ÷ (1 + 10%)} = -4.59%). To provide a comparison with Project 1, Project 2 has slightly different cash flows with an equivalent NPV (zero in Year 1, but offset with a higher cash flow in Year 19). However, due to having the larger cash flow in Year 19, Project 2 is more susceptible to a change in the discount rate. Consequently, the project duration for Project 2 is higher than the project duration of Project 1 (51.49 compared to 50.45). For Project 2, a ten basis point increase in the discount rate leads to a value reduction of 4.68% (-51.49*{0.1% ÷ (1 + 10%)} = -4.68%).

Given the higher project duration in Project 2, Project 1 becomes the better choice if the projects are mutually exclusive or if there is a capital constraint preventing the acceptance of both projects should the projects be independent. Should the two projects be contingent, the duration of the combined project is simply the weighted average of the project durations (i.e. 50.97 = 50.45*{$1,733.33 ÷ $3,466.66} + 51.49*{$1,733.33 ÷ $3,466.66}). Notice that the duration measure aids in assessing projects that are perfectly (or nearly) equivalent by the traditional NPV method. In this case the NPV of both projects are equal, but one can also envision a case in which a project with a lower NPV may be deemed more acceptable because its duration is significantly lower than another project with higher NPV (assuming non-contingent projects). However, an assessment of the trade-off between project duration and NPV is not pursued in this paper.

NPV is not the only criteria in which projects can be assessed. Using the Payback Period method, Project 1 is deemed better because the payback period of 10.87 years is shorter than the payback period of Project 2, 11.23 years. The payback period method does capture some of what project duration measures because cash flows that accumulate faster lower the project duration. The problem with the payback period method is that not all cash flows are considered as part of the measure. The project duration measure does incorporate all cash flows and can quickly be converted to measure the effect of a change in the discount rate on the value of the project. To illustrate the advantage of the project duration measure relative to the payback period method, Project 3 is introduced from Table 1.

Project 3 is similar to Project 1 except in regard to the cash flows in Year 12 and Year 19. Both projects have the same NPV and payback period, but have different project duration measures, 50.45 for Project 1 and 50.85 for Project 3. Consequently, the payback period method views both projects as equivalent. The project duration measure reveals that a ten basis point increase in the discount rate will affect Project 3's value (decreases by 4.62% [i.e. $-50.85*\{0.1\% \div (1 + 10\%)\} = -4.62\%$]) more so than Project 1's value. Thus, Project 1 is viewed as superior to Project 3 using the project duration measure even though NPV and the payback period method view both projects as essentially the same.

When using the discounted payback period method, the same criticism of "not all cash flows being evaluated" still applies. Casting this criticism aside, one should notice that all three projects have the same discounted payback period of 20 years (recall the projects are perpetual; Table 1 contains explicit cash flows until Year 20 and a terminal value is assessed for Years 21 and onward). Again, the project duration measure allows one to distinguish between the three projects even though the discounted payback period is the same for all three projects. Essentially, the project duration measure provides critical information that current project evaluation methods cannot provide in a simplified manner.

Yet, based on the example above, one can argue that the impact of a slight discount rate change is so small that it is a detail not worth pursuing. The reason for the small impact on the project value is because of the terminal value substituting for the infinite horizon of the project. To demonstrate this fact, eliminate the terminal value for all three projects and add a cash flow of $740.02 in the 21^{st} year. All three projects have an NPV of $100.00 and a Discounted Payback Period of 20 years. The Payback Period is again, 10.87 years, 11.25 years, and 10.87 years respectively for the three projects. Consequently, the ability to compare the three projects using NPV, Payback Period, and Discounted Payback Period is still equally difficult, yet the project duration measures are still distinct for each project and change dramatically: 231 for Project 1, 249 for Project

2, and 238 for Project 3. Respectively, a ten basis point increase in the discount rate reduces the NPV by 21.00%, 22.64%, and 21.64%. Although the duration measures appear high, the measures are reasonably accurate when the three NPVs are compared using a 10.10% discount rate: $79.15 (a 20.85% decrease in value) for Project 1, $77.52 (a 22.48% decrease in project value) for Project 2, and $78.52 (a 21.48% decrease in project value) for Project 3.

Further, depending on the magnitude of the cash flows, even a slight percentage decrease in value could be in the millions of dollars. Consequently, one should not dismiss the project duration measure based on the potentially small impact it has on the project value (measured as a percentage) and recognize that the impact of the project duration measure is commensurate with the scale of the project. The larger the cash flows, the greater the impact of small differences between project duration measure comparisons.

As mentioned earlier, an appendix for the paper contains VBA code for Excel-type functions that measure project duration for finite maturity projects and for projects with an infinite horizon using a terminal value similar to equation (4). Also, like bond duration, project duration will vary through time and if parameters, such as the discount rate, change within the project. To capture how the duration measure can change based on a discount rate change, a second measure called "convexity" is employed. Convexity or "project convexity" (C_P) in this context, is based on the second derivative of the project's value relative to the discount rate.

$$C_P = \frac{\Delta^2 V}{\Delta k^2} * \frac{1}{V} \tag{10}$$

It is difficult to say how useful convexity is as a measure in the context of a project, but for completeness, the project convexity measure for an infinite horizon project similar to equation (5) is:

$$C_P = \frac{\left\{\sum_{i=1}^{N-1}\left[\frac{i*(i+1)CF_i}{(1+k)^{i+2}}\right]+\left[\frac{N*(N+1)*CF_N}{(1+k)^{N+2}}\right]*\left[1+\frac{(1+g)}{(k-g)}+\frac{2*(1+k)*(1+g)}{(N+1)*(k-g)^2}+\frac{2*(1+k)^2*(1+g)}{N*(N+1)*(k-g)^3}\right]\right\}}{\left\{\sum_{i=0}^{N-1}\left[\frac{CF_i}{(1+k)^i}\right]+\left[\frac{CF_N}{(1+k)^N}\right]*\left[1+\frac{(1+g)}{(k-g)}\right]\right\}} \tag{11}$$

And, the project convexity measure for a project with a finite horizon is:

$$C_P = \frac{\sum_{i=1}^{N}\left[\frac{i*(i+1)CF_i}{(1+k)^{i+2}}\right]}{\sum_{i=0}^{N}\left[\frac{CF_i}{(1+k)^i}\right]} \tag{12}$$

For the projects illustrated in Table 1, the convexity measures are 1,655.58 for Project 1, 673.60 for Project 2, and 1,666.26 for Project 3. The greater the project convexity, the more change one will see in the project duration measure should the discount rate change. In one sense, the project convexity measures the "instability" or the "magnitude of change" of the project duration measure should the discount rate need to be adjusted because it actually changes or because it is measured with some error initially. Consequently, not only does Project 2 have the highest project duration, the project duration will change more (relative to Projects 1 and 3) should the discount rate change based on Project 2 having the highest project convexity.

SECTION 2: Project Parameter Duration

Even though the paper has been focused on the sensitivity of the project value to the discount rate, project duration measures can also be generated for project input parameters assuming a model for project cash flows (e.g. the project cash flow at time "i" could be modeled as a function of parameter "θ", $CF_i(\theta)$; or parameters if "θ" is a vector). Measuring duration relative to a model parameter (aside from the discount rate) is different from what was performed in the previous section. The key difference is that the duration measure is now taken in the context of parameters that generate the cash flows as opposed to a duration measure that simply takes the cash flows as given and assesses the effect of a discount rate change. Equation (6) now becomes equation (13).

$$-\frac{\Delta V}{\Delta \theta} = -\left\{ \sum_{i=0}^{N} \left[\frac{\Delta CF_i(\theta) / \Delta \theta}{(1+k)^i} \right] \right\} \tag{13}$$

The project "parameter" duration measure becomes:

$$D_{PRM} = -\left[\frac{\Delta V}{\Delta \theta} \right] * \left[\frac{1}{V} \right] \tag{14}$$

In equation (14), the project parameter duration appears to be similar to modified duration from bond pricing (rather than Macaulay duration) and relates to the percentage change in project value in a similar manner as modified duration.

$$\frac{\Delta V}{V} = -[D_{PRM}] * \Delta \theta \tag{15}$$

What one has to realize is that the "(1 + k)" term in Macaulay's duration formula exists to simplify the duration calculation and is actually not necessary for converting the duration to a percentage change in price. This becomes apparent when equation (1) is substituted into equation (2); the "(1 + k)" term cancels out. In theory, a "(1 + θ)" term can be introduced into equations (13) and (14), but the only reason to perform such a task is for simplifying the duration calculation. Consequently, the "(1 + θ)" term is left out of equations (13) and (14), but can be introduced if necessary.

What if the cash flow model parameter and the discount rate are both affected by a common factor, say "λ"? This is not an unreasonable question because "λ" could be a macroeconomic factor such as inflation. Equation (13) becomes:

$$-\frac{\Delta V}{\Delta \lambda} = -\left\{ \sum_{i=0}^{N} \left[\frac{\left(\Delta CF_i(\theta[\lambda]) \big/ \Delta\theta[\lambda] \right) * \left(\Delta\theta[\lambda] \big/ \Delta\lambda \right)}{(1+k[\lambda])^{i}} \right] - \left[\frac{i * CF(\theta[\lambda])_i * \left(\Delta k[\lambda] \big/ \Delta\lambda \right)}{(1+k[\lambda])^{i+1}} \right] \right\} \tag{16}$$

Equation (14) becomes:

$$D_{PRM} = -\left[\frac{\Delta V}{\Delta \lambda} \right] * \left[\frac{1}{V} \right] \tag{17}$$

Equation (15) becomes:

$$\frac{\Delta V}{V} = -[D_{PRM}] * \Delta\lambda \tag{18}$$

To visualize how project parameter duration is calculated, assume a project has a finite life of "N" years. The project value is defined as the present value of the free cash flows (free cash flow = (revenue – operating expenses –depreciation expense)*(1 - tax rate) + depreciation expense – change in net working capital – change in fixed assets; this definition is similar to that of Brigham and Houston (2004)). The revenue grows at a rate of "g" and many of the parameters are measured as a proportion of revenue: operating expenses (%OP), current liabilities (%CL), current assets (%CA), and fixed assets

(%FA). Depreciation expense is indirectly connected to the revenue through fixed assets because it is measured as a percentage based on straight-line depreciation (%DEP; e.g. 5% = 1 ÷ 20 years) multiplied by the average of the fixed assets.

Following the work of Arnold and James (2000), let revenue have an initial value of "REV_0" that is not part of the valuation (only for convenience) and evaluate the revenue stream portion of the project (call it "PV(REV)"):

$$PV(REV) = \frac{REV_0 * (1+g)}{(k-g)} * \left[1 - \frac{(1+g)^N}{(1+k)^N}\right] \tag{19}$$

The operating expenses are evaluated simply as a proportion of the revenue cash stream: %OP*PV(REV). The evaluation of the change in net working capital (ΔNWC; net working capital defined as current assets less current liabilities) is an adjustment on equation (17) based on Arnold and James (call it "PV(ΔNWC)"):

$$PV(\Delta NWC) = PV(REV) * (\%CA - \%CL) * \left[1 - \frac{1}{(1+g)}\right] \tag{20}$$

The evaluation of the change in fixed assets is similar to equation (20) (call it "PV(ΔFA)"):

$$PV(\Delta FA) = PV(REV) * (\%FA) * \left[1 - \frac{1}{(1+g)}\right] \tag{21}$$

Depreciation expense is based on the average of fixed assets, again following Arnold and James, an adjustment is made to equation (19) to evaluate depreciation expense (call it "PV(DEP)"):

$$PV(DEP) = PV(REV) * (\%FA) * (\%DEP) * \left[1 + \frac{1}{(1+g)}\right] \div 2 \tag{22}$$

Taking advantage of the common factor for "PV(REV_0)" and applying some algebra, the value of the project, "V" (i.e. the discounted value of the free cash flows), becomes:

$$V = \left\{ \begin{array}{l} \dfrac{REV_0 * (1+g)}{(k-g)} \\ * \left[1 - \dfrac{(1+g)^N}{(1+k)^N} \right] \end{array} \right\} * \left\{ \begin{array}{l} (1-\%OP)*(1-T) \\ + \left(\%FA * \%DEP * \left[1 + \dfrac{1}{(1+g)} \right] \div 2 \right) * T \\ - \left((\%CA - \%CL + \%FA) * \left[1 - \dfrac{1}{(1+g)} \right] \right) \end{array} \right\} \tag{23}$$

where "T" is the tax rate.

The goal is to calculate the project parameter duration relative to the revenue growth rate "g". Consequently, the partial derivative of equation (23) needs to be taken with respect to "g". The negative of the derivative is then multiplied by (1 + g) for convenience and divided by equation (23) to find the project parameter duration. Let "A" equal "PV(REV)" and perform the rest of the calculation in pieces:

$$B = -\frac{\Delta A}{\Delta g} * (1+g) = A * \left[\frac{(1+g)}{(1+k)} - 1 \right] + \frac{REV_0 * (1+g)}{(k-g)} * \left[\frac{N * (1+g)^N}{(1+k)^N} \right] \tag{24}$$

$$C = \left\{ \begin{array}{l} (1-\%OP)*(1-T) + \left(\%FA * \%DEP * \left[1 + \dfrac{1}{(1+g)} \right] \div 2 \right) * T \\ - \left((\%CA - \%CL + \%FA) * \left[1 - \dfrac{1}{(1+g)} \right] \right) \end{array} \right\} \tag{25}$$

$$D = -\frac{\Delta C}{\Delta g} * (1+g) = \frac{[(\%FA * \%DEP * T \div 2) + \%CA - \%CL + \%FA]}{(1+g)} \tag{26}$$

The project parameter duration (D_{PRM}) becomes:

$$D_{PRM} = \frac{B * C + A * D}{A * C} = -\left[\frac{\Delta V}{\Delta g} \right] * \left[\frac{(1+g)}{V} \right] \tag{27}$$

A terminal value equation can be added if the project has an infinite life. Further, an initial cost can also be added as a negative value in the denominator of equation (27). This particular example is rather complex because the revenue growth factor is pervasive throughout the sales-driven pro forma analysis of Arnold and James. However, less complex valuation models can be incorporated by following one of the definitions for project parameter duration in equations (14), (17), or (27) depending on the model. Equation (28) completes the above analysis by computing the percentage change in project value based on the project parameter duration from equation (27).

$$\frac{\Delta V}{V} = -D_{PRM} * \left[\frac{\Delta g}{(1+g)} \right] \qquad (28)$$

SECTION 3: Conclusion

Duration and convexity measures are used frequently in the assessment of interest rate risk in regard to bond pricing. This same technique is easily adapted to assess project valuation in regard to a cash flow model parameter or the project discount rate. The benefit is that a single number (or numbers if convexity is considered) provides the same intuition as sensitivity analysis without the complexity or the difficulty in interpreting the results. This investigation can also be used to take duration analysis to the equity level using the cash flows to equity holders and subsequently to the duration of a portfolio of equities.

An added dimension for project assessment is the ability of the decision maker to consider the vulnerability of project valuation estimates to shifts in the model parameters or shifts in the discount rate in a very convenient manner by considering a number rather than an entire output of scenarios. Although a given project may have a better expected value, the vulnerability of the project valuation to parameter/discount rate changes may make it less appealing when compared to another project of lesser expected value with more robust parameter/discount rate estimates. More simply, project duration (convexity) and project parameter duration (convexity) provide metrics to assess project forecasting error without additional complex scenario analysis that can be difficult to interpret.

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Appendix: Excel VBA Code

Excel VBA code for project duration when the project has an infinite horizon:

```
Function ProjectDuration_wCG(Inital_CF, Future_CFs, Discount_Rate, Constant_Growth)

CFcount = Application.Count(Future_CFs)
CFcount_m1 = CFcount - 1

Sum1 = 0
Sum2 = 0
Period = 1
Do While Period <= CFcount_m1
Sum1 = Sum1 + Future_CFs(Period) / (1 + Discount_Rate) ^ (Period)
Sum2 = Sum2 + Period * Future_CFs(Period) / (1 + Discount_Rate) ^ (Period)
Period = Period + 1
Loop

PVCG = ((Future_CFs(CFcount) / (1 + Discount_Rate) ^ (CFcount))) * (1 + ((1 + Constant_Growth) /
(Discount_Rate - Constant_Growth)))
ProjectValue = Inital_CF + Sum1 + PVCG

NumeratorCG1 = ((Future_CFs(CFcount) * CFcount) / (1 + Discount_Rate) ^ (CFcount))
NumeratorCG2 = (1 + Constant_Growth) / (Discount_Rate - Constant_Growth)
NumeratorCG3 = ((1 + Discount_Rate) * (1 + Constant_Growth)) / (CFcount * (Discount_Rate -
Constant_Growth) ^ (2))
Numerator_CG = NumeratorCG1 * (1 + NumeratorCG2 + NumeratorCG3)
Numerator = Sum2 + Numerator_CG

ProjectDuration_wCG = Numerator / ProjectValue

End Function
```

Excel VBA code for project duration when the project has a finite horizon:

```
Function ProjectDuration_woCG(Inital_CF, Future_CFs, Discount_Rate)

CFcount = Application.Count(Future_CFs)

Sum1 = 0
Sum2 = 0
Period = 1
Do While Period <= CFcount
Sum1 = Sum1 + Future_CFs(Period) / (1 + Discount_Rate) ^ (Period)
Sum2 = Sum2 + Period * Future_CFs(Period) / (1 + Discount_Rate) ^ (Period)
Period = Period + 1
Loop

ProjectDuration_woCG = Sum2 / (Sum1 + Inital_CF)

End Function
```

Excel VBA code for project convexity when the project has an infinite horizon:

```
Function ProjectConvexity_wCG(Inital_CF, Future_CFs, Discount_Rate, Constant_Growth)

CFcount = Application.Count(Future_CFs)
CFcount_m1 = CFcount - 1

Sum1 = 0
Sum2 = 0
Period = 1
Do While Period <= CFcount_m1
Sum1 = Sum1 + Future_CFs(Period) / (1 + Discount_Rate) ^ (Period)
Sum2 = Sum2 + (Period * (1 + Period) * Future_CFs(Period)) / ((1 + Discount_Rate) ^ (Period + 2))
Period = Period + 1
Loop

PVCG = ((Future_CFs(CFcount) / (1 + Discount_Rate) ^ (CFcount))) * (1 + ((1 + Constant_Growth) /
(Discount_Rate - Constant_Growth)))
ProjectValue = Inital_CF + Sum1 + PVCG

NumeratorCG1 = (CFcount * (CFcount + 1) * Future_CFs(CFcount)) / ((1 + Discount_Rate) ^ (CFcount +
2))
NumeratorCG2 = (1 + Constant_Growth) / (Discount_Rate - Constant_Growth)
NumeratorCG3 = (2 * (1 + Discount_Rate) * (1 + Constant_Growth)) / ((CFcount + 1) * (Discount_Rate -
Constant_Growth) ^ (2))
NumeratorCG4 = (2 * (1 + Discount_Rate) ^ (2) * (1 + Constant_Growth)) / (CFcount * (CFcount + 1) *
(Discount_Rate - Constant_Growth) ^ (3))

Numerator_CG = NumeratorCG1 * (1 + NumeratorCG2 + NumeratorCG3 + NumeratorCG4)
Numerator = Sum2 + Numerator_CG

ProjectConvexity_wCG = Numerator / ProjectValue

End Function
```

Excel VBA code for project convexity when the project has a finite horizon:

```
Function ProjectConvexity_woCG(Inital_CF, Future_CFs, Discount_Rate)

CFcount = Application.Count(Future_CFs)

Sum1 = 0
Sum2 = 0
Period = 1
Do While Period <= CFcount
Sum1 = Sum1 + Future_CFs(Period) / (1 + Discount_Rate) ^ (Period)
Sum2 = Sum2 + (Period * (Period + 1) * Future_CFs(Period)) / ((1 + Discount_Rate) ^ (Period + 2))

Period = Period + 1
Loop

ProjectConvexity_woCG = Sum2 / (Sum1 + Inital_CF)

End Function
```

Author Biography

Tom Arnold:

Tom Arnold is an Assistant Professor of Finance at the Robins School of Business, University of Richmond. He received his PhD from the University of Georgia and holds the designation of Chartered Financial Analyst. His areas of research include derivatives, investments, corporate valuation and decision metrics, real options, and finance pedagogy.

David S. North:

David S. North is an Associate Professor of Finance at the Robins School of Business, University of Richmond. He received his PhD and BA degrees in finance from Michigan State University as well as his MBA in finance from University of Notre Dame. He worked at Ford Motor Company for five years in various corporate finance positions. His research interests include corporate finance, management turnover, and discretionary accruals.