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The Influence of Word Problem Structures on Algebraic Expression Construction

Taylyn Hulse
University of Richmond

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by

Taylyn Hulse

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Advisor: David Landy

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Certain learning domains come naturally to humans. Evidence supports that core knowledge systems of objects, number, action and space are innate for infants (Spelke, 2007). These core domains remain throughout development and they also give rise to more complex cognitive skills (Spelke, 2000). As we develop, we form new concepts that transcend the core learning domains (Carey, 2009). These new concepts, unlike core knowledge, are not innate and are learned under social and cultural pressures (Carey, 2009). This means that there is a transition from practicing core knowledge that is learned naturally and higher-functioning cognitive skills that must be specifically taught. In math, this would look like the transition from learning to count to learning algebra. In algebra, students need to be specifically taught how to manipulate the mathematical language that makes up expressions and equations. Though they are both learned, these abilities are theorized to be functions of separate processing systems.

David Geary calls the foundational systems biologically primary systems and biologically secondary systems (1995). Biologically primary systems are the core abilities that span across different cultures. An example of this is how every human culture has language. Culturally specific skills that are more specialized and have stemmed from their evolutionary foundations are biologically secondary systems (Geary, 1995). Though all cultures have language, not all cultures are literate (Geary, 1995). Language is a biologically primary system and literacy has become a biologically secondary system. Similarly, with math ability, all humans have an innate skill to approximate number. This means that we should not see significant differences in primary math skills across cultures (Geary, 1995). Higher math systems, like algebra, however, are more complex.

New research on the development of mathematics cognition has found that instead of biology as being the major source for differences in primary math skills, these differences tend to appear once students begin formal schooling (Geary, 1995). Even before formal schooling, differences in the development of counting have been found to reflect differences in language (Miller, Smith, Zhu & Zhang, 1995). Though English-speaking students and Chinese-speaking students use the same base-10 number system, the two languages label their numbers differently. While English speakers say “eleven” and “twelve”, Chinese words for those numbers would translate to “ten one” and “ten two” (Geary, 1996). When comparing English-speaking students and Chinese-speaking students, there are differences in abstract counting ability at ages 4 and 5, there are no differences found at age 3, when those numbers aren’t as prevalent (Miller et al., 1995). In addition to this, Chinese-speaking and English-speaking students performed equally well on simple problem solving tasks and counting and producing small numbers of objects (Miller et al., 1995). This suggests that basic mathematical ability is equalized across these two cultures until differences in language affect learning number names. Since language is a factor that is not easily changed, our focus is primarily on the problem solving strategies that can be taught through formalized education.

Part of the difficulty of learning mathematics comes from using multiple symbol systems. In the context of an algebraic word problem, there are at least two symbol systems students need to manipulate: the text and the equation. First, students need to read and make sense of the text in the word problem. Once they understand the text, they need to translate the text of the word problem into a series of mathematical symbols that represent the same information. Both of these symbol systems (text, equation) are human constructs that are abstract and need to be specifically taught. Students have learned to use multiple strategies for translating the text of an algebraic word problem into mathematical language. Some strategies, like direct translation and static comparison, rely heavily on the order of the text (Hegarty, Mayer, & Monk, 1995). Other strategies, like using a problem-solving model, are able to extract and manipulate the mathematical relationship outside of the confines of the text (Clement, 1982).

When using direct translation, students use the text to copy down important numbers and words. Students usually write these important pieces in the same left-to-right order as they were presented in the text (Hegarty et al., 1995). This strategy puts little strain on working memory because students are simply copying key elements and not performing any other sort of integration process. The direct translation process, relative to other strategies, is not as cognitively straining because it follows the syntax of the text (Clement, 1982). Following the text, however, makes students vulnerable to influences such as order effects.

Static comparison strategies link variables and coefficients based on how they are presented in the text (Clement, 1982). These strategies rely on structural cues within the text. Students intuitively combine variables and coefficients, assign them a side of the equation and compare them as a static relationship. This is detrimental to problems like "there are six times as many students as professors" (Clement, 1982). Instead of writing the correct answer of 6P=S, students often write 6S=P because the six is linked with students in the text of the problem. Static comparison strategies do not reflect semantic understanding. Rather than thinking of variables as representations for some unknown number, students may think of variables as objects and therefore assign the closest number to that single object and link that number as the object's coefficient. This negates the function of the coefficient, revealing the student's misunderstanding of the mathematical relationship in the problem.

Other strategies, however, seem to withstand order effects because students can extract the relationship between the variables outside of their presentation in the text. A problem solving model strategy builds a model of the relationship between variables and abstractly comprehends how the numbers in the text can influence these variables. Rather than relying on the syntactical order of the text, students will semantically understand the relationship between numbers (Clement, 1982). When testing how a problem model strategy is used, researchers found that students tend to focus on the variable names within the text, while students who use other strategies tend to focus on the numbers within the text (Hegarty et al., 1995). Knowing that students are focusing on the names instead of the numbers supports the idea that strategies influenced by order effects may directly copy numbers in the order they are presented. This also suggests that students using problem model strategies may be learning the mathematical relationship by understanding the connection between variables through their names. This may mean that variable names are more closely tied with successful math strategies than other variable representations.

There are other symbol systems in mathematics education materials that affect student understanding of the mathematical relationship in problem solving. Some algebraic word problems include visual instruction like pictures, in addition to the text.

In some cases, visual representations increase cognitive load and decrease learning (Bobis, Sweller & Cooper, 1993). Our working memory is limited, so we need information to be presented in a way that is efficient for our cognitive systems to process information. Cognitive load theory states that if there is extraneous information in instructional materials, then cognitive load will be unnecessarily increased and make the instructional material ineffective (Bobis et al., 1993). The content of the material may be difficult enough on its own, but increases in extraneous information can make the problem too complex for the limited availability of working memory. Even when redundant information from the text was inserted in a diagram, cognitive load increased (Bobis et al., 1993). If used correctly, however, visual instructions in conjunction with verbal information can help students not only learn the material, but also produce more creative solutions (Mayer, 1997).

When students receive a word problem that has both text and a picture, they need to integrate information from both materials in order to construct a representation that reflects them. There is a theory, called Dual Coding Theory, which states that verbal and nonverbal stimuli are processed in separate systems (Paivio, 1991). This means that the verbal text and visual pictures in an algebraic word problem may be processed separately and may affect cognitively load separately as well. In order for students to engage in meaningful learning, they need to be active participants in selecting important information from these multiple systems, organizing it into an understandable construct, and integrating that information with previous knowledge (Mayer, 1997). Meaningful learning means that students effectively use cognitive processes to build an understanding of the content and can then transfer that knowledge in future problem solving (Mayer,

1997) This concept originally comes from Generative Theory, which is the idea that students learn through a process of discovery and generating a relationship between new information and previous knowledge (Wittrock, 1974). Generative theory in multimedia learning works in the same manner; by generating their own representations from multiple processing systems, students choose which information they need and that information becomes the foundation of their model of the problem (Mayer, 1997).

In this study, we want to understand how students integrate multiple symbol systems that represent the same information. We will give students algebraic word problems with both a textual narrative and a picture. In both the picture and the text, we will manipulate the order in which variables are introduced. Based on the generative theory of multimedia learning, students should select information from the verbal and visual representations and integrate that with information that was previously learned. We are most interested in understanding what information in the verbal and visual structure is most influential in students' algebraic equation construction. By comparing the manipulations in the text and picture to the equations students write on the page, we can determine if students' written equations replicate the spatial structure of the information presented in the word problem and what system had the most influence. We hypothesize that the structure of the text and picture in algebraic word problems will influence students' equation construction. Through this study, we aim to determine if and when the textual structure or pictorial structure has a stronger influence over the other.

Method

Participants

Eighty students from the University of Richmond participated. Students' ages ranged from 18 to 28. Seventy-nine of the participants were compensated with five dollars and one student was compensated with academic credit. All were recruited through the University daily email announcement system.

Materials

Participants took this study in small groups. They were taken into a classroom by a researcher who was blind to their condition and they received a packet with two types of algebraic word problems. The even numbered problems were designed to test how participants thought about know and unknown variables in the problem. There were two known variables and one unknown variable in each, but their values varied from problem to problem. In some problems the unknown variable was the total of two known variables (A=B+C, where B and C are known values, but A is an unknown value). In other problems, the total and one value was known, but the other additive value was unknown (A=B+C, where A and B are known values, but C is an unknown value). In addition to varying the value of the variable, the introduction of the known and unknown values varied to account for ordering effects.

The odd numbered problems were intended to focus on the introduction of variables in the text and in the picture (Refer to Figure 1). Each problem had a short paragraph and a corresponding picture that described two variables. In some problems, variable A was introduced first in both the text and the picture. In other problems, variable B was introduced first in the text and picture. Other problems had a mixture where A was introduced first in the text and B was introduced first in the picture, or B was introduced first in the text and A was introduced first in the picture. This design allowed researchers to test for participants' reliance on variable placements when writing their expressions.

Procedure

Students were tested in small groups. They sat at a desk with a packet and a pen. The directions on the packet told participants to write an algebraic expression that represented the information presented in the algebraic word problem. The directions also specified that participants should use information from both the text and the picture from the problem to write their expressions. After completing all algebraic word problems, participants filled out a general information questionnaire. Some of the questions included information about academic majors, math courses taken, and math enjoyment.

Data were coded to test the structural mapping between students' written equations and the instructional materials in the algebraic word problems. The structure of the text and picture were marked as to whether or not the target variable was represented on the left side of the text or the left side of the picture. If the target variable was coded as on the left, that meant that it was the first variable to be introduced in that medium. Some problems had the target variable on the left side of both the text and the picture. Some problems had the target variable on the right side of both the text and the picture. Other problems had a mixture of the two: some with the target variable on the left side of the text and right side of the picture and some with the target variable on the right side of the text and left side of the picture. Student responses were also coded for target variable positioning on the left side of the written equation. Data were then analyzed to test the influence of target variable positioning in the instructional materials on the target variable positioning in the written equations.

Results

We varied the structure of verbal (text) and visual (pictures) stimuli in word problems and tested its effect on student-written algebraic expressions. A within-participants logistic regression revealed significant differences between word problem types, $\mathbf{x}^2(3)= 27.6$, $p<.01$ (Refer to Table 1). When the target variable was located on the left of the text and the picture, students wrote the variable on the left of their equation 61% of the time (Refer to Figure 2). When the target variable was located on the right of the text and the picture, students wrote the variable on the left of their equation 22% of the time. By writing the target variable on the right side of their equation instead of the left side in this condition, we can see that students followed the structure of the picture and the text. A significant difference was also found between the conditions where variable location did not match in the text and picture. Students wrote the target variable on the left side of their equation 48% of the time when the variable was left in the text and right in the picture and 25% of the time when the variable was right in the text and left in the picture. Here, students were more influenced by the location of the variable in the text, suggesting that students relied more heavily on the structure of the text than the structure of the picture when writing their equations, $p=.011$.

Discussion

The results supported our prediction that students' written equations would match the physical characteristics of the information given. Students were given algebraic word problems with a textual and pictorial representation of the same information. Researchers recorded what problems had the smaller variable on the left side of the text, picture and the students' responses. When the indicated variable was on the 'left', it was introduced first in the text and positioned first in the picture. When the variable was on the 'right', it was introduced second in the text and positioned second in the picture. Overall, students mirrored these spatial cues.

The biggest difference was between problems with the variable on the left side of both the text and the picture and problems with the variable on the right side of the text and the picture. When the variable was on the left for both the text and picture, students wrote their expressions with the variable on the left more often than the right. Oppositely, when the variable was on the right side of the text and the picture, students wrote their expressions with the variable on the right more often than the left. The other problems had a combination left and right in the picture and text. There was a significant difference between these problems as well. We found that students' expressions followed the positioning of the variable in the text more than the positioning in the picture. This could be due to many factors. Students may have had a preference to using the text. When looking at both pieces of information, students may see the text as clearer and more easily represented than the picture. It is also possible that students did not have a textual preference, but were instead influenced by the order of presentation. Since the text was introduced first, students may have been more influenced by this structure and had less use for the information provided by the picture. Other factors that were not measured by this study could also have major effects on the structure of students' written equations.

The current research supports previous theories on the relationship between spatial properties and information processing. One major field that supports this is in the study of different languages. Some languages are read left to right; others are read right to left or top to bottom. The directionality of language makes a series impact on how speakers process spatial information (Maas & Russo, 2003). Speakers develop a particular motor and perceptual system based on what language they speak. When performing spatial tasks, like recalling pictures that were flashed on a screen, those who spoke languages of different directionality remembered pictures from different sides of the screen (Maas & Russo, 2003). This shows that a developed system in one area, like language, can elicit a strong effect in a nonlinguistic system, like spatial cognition.

The effect of language directionality also plays a major role in mathematics cognition, especially in regards to the number line. Some educational systems teach students to construct number lines to help form an understanding of how different values relate to each other. Those who read from left to right have shown to have a mental number line construct that has a left to right directionality (Dehaene, Bossini & Giraux, 1993). When testing the Spatial-Numerical Association of Response Codes (SNARC) on parity, Dehaene et al. found that participants responded more quickly to smaller numbers with their left hand and larger numbers with their right hand (1993). This is called the SNARC effect and is strong supporting evidence that smaller numbers are spatially represented on the left side of the mental number line and larger numbers are spatially represented on the right side. This also shows that mental representations that are spatially structured have an influence on physical systems that engage with the world outside of the mind.

Spatial structures in the text can even affect perceptual grouping, which can lead to inaccurate mathematics problem solving strategies. In addition to changes in the physical structures of text and picture, changes in the perceptual grouping of algebraic equations can influence problem-solving techniques (Landy & Goldstone, 2007). The order of operations is an abstract concept that needs to be specifically taught. Usually when students apply the rules of the order of operations, the equations are evenly spaced on the page. A study that manipulated the physical spacing of equations in order of operations problems found that when terms are grouped to contradict the grouping rules in the order of operations, students' accuracy dropped (Landy & Goldstone, 2007). When spacing reinforces the rules of the order of operations or is evenly spaced, however, students perform much better (Landy & Goldstone, 2007). The spatial structure on the page shaped students' visual grouping, which in turn changed their use of the order of operations. All of the evidence presented here, in addition to the results of our study, add to the growing area of research that aims to build a better understanding of mathematics cognition and how it relates to the other processing systems that make up cognition as a whole.

Though our study has strong evidence that the structure of the text influenced algebraic expression construction, there are limitations to what can be explained by our measurements. The data show that students mirrored the physical positioning of the text significantly more than the physical positioning of the picture, but this does not necessarily mean that students prefer textual information than pictorial information. Just as in most mathematics textbooks, the text in our problems were always presented before the pictures. This gives the pictures the context necessary to be understood. This ordering, however, could have influenced the students' reliance on the text. Despite this text and picture distinction, this study shows clear evidence that students are influenced by the physical characteristics of the presented information.

Since it is difficult to measure problem solving strategies before an answer or partial answer is written on the page, it is possible that students were not affected by the structure of information when they were in the process of solving the problem. Instead, the physical mirroring could have been a product after problem solving. Once students read the problem and went through their usual problem-solving strategy, they could have then looked back and decided to mimic the text. Without step-by-step, written work or without talkback problem solving, it is difficult to know exactly what factors influenced students' problem solving techniques.

The current study used simple algebraic problems and compared variable positioning across the text, picture and written expression, but it did not evaluate the accuracy of the answers. This means that we did not assess the success of students' problem solving strategies. If students are mirroring the information from the text and picture during problem solving, they may be successful in simple algebraic word problems with addition and subtraction, but they may find more difficulty in higher levels of math. When problems become more complex, this mirroring strategy may disrupt students' accuracy. This mirroring process could be a version of direct translation or the static comparison approach. In more complex problems, the relationship between variables is less clear. Those who are more successful in solving complex algebraic word problems do not use the direct translation or static comparison and instead create a problem-solving model of the information that is more flexible (Hegarty et al, 1995).

The next steps in this study would be to further test the influence of the text on algebraic expression construction. With our design, students were more influenced by the structure of the text than the structure of the picture, but this result is not likely to always be true. Structural influence may also depend on the context of the problem. The pictures that we used represented a particular mathematical relationship. Equation construction may differ when the picture represents something with a more dynamic relationship. Diagrams that depict a system of movement, like simple machines in physics, may also hold more influence than pictures from algebra. Students may prefer to rely on a visual representation of how the system changes over a textual representation. This research should continue in this direction to determine how different types of diagrams in coordination of text affect equation construction.

Structural influence may depend on introduction of instruction. The current study measured the effect of the introduction of variables, but it did not vary the introduction of instructional information. The text was always introduced before the picture, which may have been a major factor in its influence. Students read the text first and may have used that information to make sense of the pictures, therefore understanding the problem with the textual frame in mind. If the pictures were presented before the text, however, it is possible that students may use the structure of the picture as their primary source of information. Then they may use the text as a secondary source to fill in the missing pieces from the information presented in the picture. The next study could consist of varying the placement of instruction on the page. In one condition, the picture could be presented on top of the text. This is does not match typical textbook instruction, but may reveal something important about using diagrams to construct new representations. It would also be useful to vary the left to right orientation. Students may write equations differently if the picture was on the left of the text compared to when it was on the right of the text. Students of different native languages may also differ in this design. By manipulating the introduction of the instructional materials could strengthen our results on the influence of the text, or it could provide more insight as to what informational structures are most influential and in what context.

If successful problem solvers construct models of information instead of using direct translation, then this study has influential implications. Since information presentation significantly changed students' written answers, the students in this study most likely engaged in a strategy that would not be successful in higher-level problem solving. Educators should be conscious of this effect when teaching. Knowing how students naturally think about and solve word problems can help educators facilitate this learning, but it is also important for educators to know when a natural strategy is unsuccessful. Students may have successful learning if lessons would first cater to this natural mirroring of information, but then openly move away from this approach and progress towards a model-based strategy. Something as simple as giving students sets of problems that vary structure may help control for problem solving techniques that are influenced by order effects. This study found that different structures of instructional information have different degrees of influence. Being able to identify what structures students use when problem solving can help isolate the elements of successful strategies that ultimately foster a better understanding of the content.

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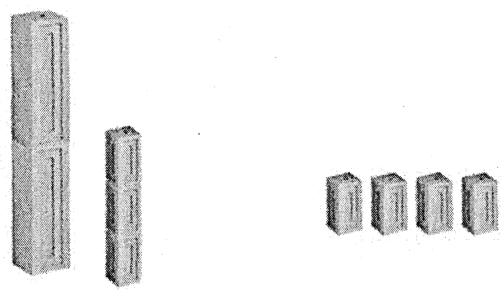
Figure 1.

Sample problem demonstrating the target variable (Tim's blocks) on the left side of the text and the left side of the picture.

Tim has a set of blocks where some blocks are tall and some blocks are small. Jenny decides to play and only has small blocks. The picture below represents how tall the tower would be if we stacked Tim and Jenny's blocks together.



Now write an equation that represents the tallest tower they could make.

Figure 2.

Structural Influence of Text and Picture on Written Equation (95% confidence interval bars shown)

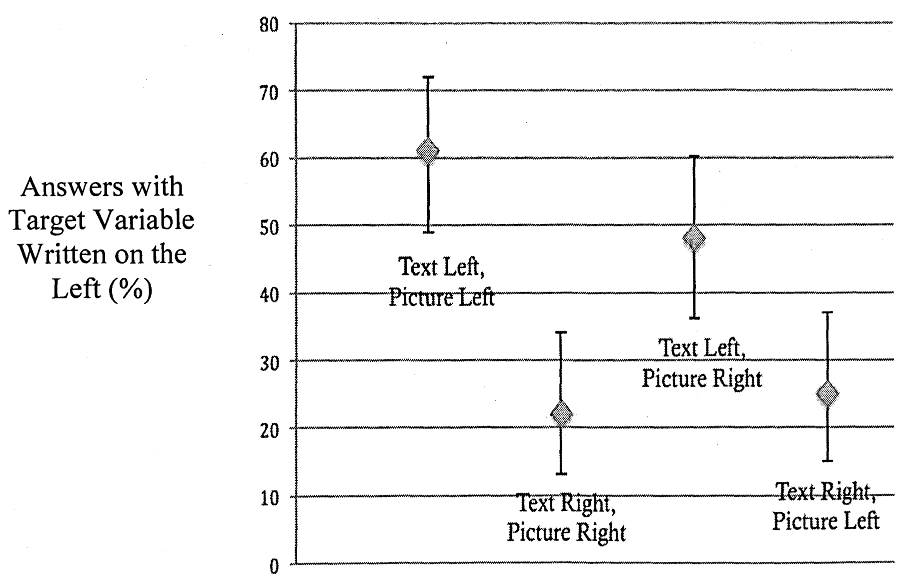


Table 1

Logistic Regression Results

variable	*B*	*SE*
intercept	**-1.29**	0.3
text left	1.2**	0.39
picture left	0.17	0.42
text left: picture left	0.4	0.54