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Tom Arnold
University of Richmond, tarnold@richmond.edu

Terry D. Nixon

Richard L. Shockley Jr.

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Intuitive Black-Scholes Option Pricing with a Simple Table

Tom Arnold, Terry D. Nixon, and Richard L. Shockley, Jr.

The Black-Scholes option pricing model (1973) can be intimidating for the novice. By rearranging and combining some of the variables, one can reduce the number of parameters in the valuation problem from five to two: 1) the option's moneyness ratio and 2) its time-adjusted volatility. This allows the computationally complex Black-Scholes formula to be collapsed into an easy-to-use table similar to those in some popular textbooks. The tabular approach provides an excellent tool for building intuition about the comparative statics in the Black-Scholes equation. Further, the pricing table can be used to price options on dividend-paying stocks, commodities, foreign exchange contracts, futures contracts, and exchanges of assets, and can be inverted to generate implied volatility. Formulas for reproducing the tables in Excel are included. [JEL: G10, G12, G13]

■Although three decades old, the option pricing model of Black and Scholes (1973) defines many aspects of option pricing today. With the explosion in financial derivatives and risk management contracts, the development of real option analysis, and the growing popularity of options in compensation packages, many more people (and a more diverse population) need to understand the Black-Scholes model.

Cox and Rubinstein (1985) show how to create a simple two-dimensional table for easy calculation of Black-Scholes European call values.[1] Armed with this table and a basic calculator, we need only three simple calculations to price any European call option. The pricing tables that were once quite popular have been superceded by inexpensive personal computers that can use spreadsheet programs to evaluate the necessary components of the Black-Scholes model (e.g., the cumulative normal distribution function).[2]

Our purpose is to show that the option pricing tables described in Cox and Rubinstein (1985) still have tremendous pedagogical value. First, we show how we can use the tables to help students build intuition about the comparative statics of the Black-Scholes model (i.e., the "greeks") without reference to partial derivatives. Second, we demonstrate that the tables can be used to easily price a wide variety of options beyond plain vanilla calls: options on dividend-paying stocks, commodities, foreign exchange, futures, and asset exchanges; put options can also be priced either by put-call parity or by a similar table. Third, we present a way to invert the tables so that implied volatilities can be quickly extracted from option prices. We provide the Excel spreadsheet code to make reproduction of the tables easy.

We first describe the method for construction of the tables. Through simple manipulation, the Black-Scholes model is reduced to two inputs (the option's *moneyness ratio* and its *time-adjusted volatility*), allowing creation of a two-dimensional pricing table. We then show how the tables can illustrate the sensitivity of an option's price to various model inputs (the "greeks") in an intuitive manner. Next, we invert the pricing table to estimate an option's implied volatility and demonstrate how the tables can be used to price foreign exchange options, futures options, and exchange options (with an application to all-stock tender offers in corporate takeovers).

Tom Arnold is an Assistant Professor of Finance at the University of Richmond in VA, 23173. Terry D. Nixon is an Assistant Professor of Finance at Miami University in Oxford, OH 45056. Richard L. Shockley, Jr. is a Ford Motor Company Teaching Fellow and Assistant Professor of Finance at Indiana University in Bloomington, IN 47405.

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[1]Leurhman (1998) presents a similar table without documentation.

[2]The tables appeared without documentation in early editions of Brealey and Myers, such as the 1996 fifth edition, but are omitted from the current edition.

I. A Tabular Presentation of the Black-Scholes Option Pricing Model

Black and Scholes (1973) developed the now-famous model for valuation of a European call option:

$$C = SN(d_1) - Xe^{-RT}N(d_2)$$

where:

$$d_1 = \frac{\ln\left(S/X\right) + \left(R + 0.5\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and $N(\bullet)$ is the cumulative normal distribution function. C is the value of the call, S is the current value of the stock, X is the strike price, R is the risk-free rate of return per year, T is the number of years (or fraction of a year) until the option expires, and σ is the standard deviation of returns on the stock per year.

Four steps are required to arrive at the tabular form of the Black-Scholes European call pricing model.[3]

Step 1: Define the option's *moneyness ratio* (MR):

$$MR = \frac{S}{Xe^{-RT}}$$

The moneyness ratio is simply the current value of the stock divided by the present value of the strike price (using continuous compounding). When MR > 1, the option is said to be in the money; when MR = 1, the option is at the money; when MR < 1, the option is out of the money.

Step 2: Define the option's *time-adjusted volatility* (TAV):

$$TAV = \sigma\sqrt{T}$$

Multiplying the annual standard deviation of returns by the *square root* of T converts the annual standard deviation of returns to a standard deviation appropriate for the time horizon of the option. For example, suppose the option has six months to maturity – or T = 0.50. If the annual *variance* in returns is σ^2, then the six-month *variance* in returns (which is appropriate for the analysis) is $0.5\sigma^2$, so the six-month *standard deviation* of returns is $\sqrt{0.5\sigma^2} = \sigma\sqrt{0.5} = \sigma\sqrt{T}$.

[3]A less extensive derivation appears in Cox and Rubinstein (1985).

Step 3: Divide both sides of the Black-Scholes formula by S, and substitute the definition of MR in the second term:

$$\frac{C}{S} = \frac{SN(d_1)}{S} - \frac{Xe^{-RT}N(d_2)}{S}$$

$$= N(d_1) - \frac{Xe^{-RT}}{S}N(d_2)$$

$$= N(d_1) - \frac{N(d_2)}{MR}$$

This gives us the value *C/S*, which is the ratio of the value of the call to the value of the stock, or the *call-stock multiplier* (CSM). The CSM will be the final output in our two-dimensional table of option prices. Using the five standard option pricing input variables, one will be able to quickly find the CSM in the table and easily price the option by multiplying the CSM by the stock price.

Step 4: Reduce d_1 and d_2 using the definitions of MR and TAV, so that the Black-Scholes model requires only MR and TAV. To do this, recall the definition of d_1:

$$d_1 = \frac{\ln\left(S/X\right) + \left(R + 0.5\sigma^2\right)T}{\sigma\sqrt{T}}$$

Since $MR = \frac{S}{Xe^{-RT}}$, $\frac{S}{X} = MR \times e^{-RT}$.

Making this substitution along with the definition of TAV into d_1,

$$d_1 = \frac{\ln\left(MR \times e^{-RT}\right) + \left(R + 0.5\sigma^2\right)T}{TAV}$$

$$= \frac{\ln(MR)\ln\left(e^{-RT}\right) + \left(R + 0.5\sigma^2\right)T}{TAV}$$

$$= \frac{\ln(MR) - RT + \left(R + 0.5\sigma^2\right)T}{TAV}$$

$$= \frac{\ln(MR) + 0.5\sigma^2 T}{TAV}$$

$$= \frac{\ln(MR) + 0.5TAV^2}{TAV}$$

Similarly, d_2 can be written:

$$d_2 = \frac{\ln(MR) - 0.5TAV^2}{TAV}$$

The equation for the call-stock multiplier becomes:

$$CSM = \frac{C}{S} = N(d_1) - \frac{N(d_2)}{MR}$$

$$= N\left(\frac{\ln MR + 0.5TAV^2}{TAV}\right) - \frac{1}{MR}N\left(\frac{\ln MR - 0.5TAV^2}{TAV}\right)$$

which requires only two inputs—the moneyness ratio MR and the time-adjusted volatility TAV. The resulting calculation gives us the call-stock multiplier (CSM), which easily prices the option.

All options with equivalent MR and TAV will have the same CSM, so we can form a two-dimensional table of MR and TAV combinations that presents the resulting CSM. All anyone needs to do to calculate an option's price is calculate its MR and TAV, look up the CSM in the table, and then multiply the CSM by the price of the stock.

Exhibit 1 demonstrates how to create the table in Excel. The moneyness ratio proceeds across the columns, increasing to the right, and the time-adjusted volatility is in the first column of the table, increasing down the rows. The increments in step sizes between consecutive MRs and TAVs can be as small as desired, and the table can be of any size.

In Exhibit 2, we show the calculations for moneyness ratios ranging from 0.90 to 1.10 in increments of 0.02, and for time-adjusted volatilities ranging from 0.05 to 1.00 in increments of 0.05.

A. Option Pricing Example

Here is how to use the table. Suppose Home Depot's stock price is currently \$48 per share, and you want to price a nine-month call on the stock with a strike price of \$50. Suppose further that the volatility of returns on Home Depot is 52% per year, and the risk-free rate is 8%. First, calculate the moneyness ratio:

$$MR = \frac{S}{Xe^{-RT}} = \frac{48}{50e^{-(0.08)0.75}} = \frac{48}{47.08} = 1.02$$

Next, calculate the time-adjusted volatility:

$$TAV = \sigma\sqrt{T} = 0.52\sqrt{0.75} = 0.4503 \approx 0.45$$

Now find the column for MR = 1.02 in Exhibit 2, and read down until you reach the intersection with the row for TAV = 0.45. The CSM for this option is 0.1862, so the price of the call is:

$$CSM \times S = 0.1862 \times \$48 = \$8.94$$

B. An Extension—Options on Stocks That Pay Dividends

Suppose the Home Depot stock pays a continuous 5% annual dividend (see Merton, 1973). Let q represent the annual dividend yield and redefine the moneyness ratio as:

$$MR_{Continuous\ Dividend} = \left(\frac{Se^{-qT}}{Xe^{-RT}}\right)$$

The time-adjusted volatility stays the same (0.45), but the moneyness ratio changes:

$$MR_{Continuous\ Dividend} = \left(\frac{Se^{-qT}}{Xe^{-RT}}\right) = \frac{48e^{-(0.05)0.75}}{50e^{-(0.08)0.75}} = 0.98$$

According to Exhibit 2, the associated CSM is now only 0.1698. To get the value of the call when the underlying stock pays a continuous dividend, multiply the CSM times the present value of the stock price discounted at the dividend yield (the numerator of the MR):

$$CSM\ x\ Se^{-qT} = 0.1698 \times \$48e^{-0.05(0.75)} = \$7.85$$

Consider what happened here. If a stock pays a dividend, the MR is *reduced*. Reading across any row in Exhibit 2 from right to left, we see that a decrease in the MR causes a reduction in the option price. In other words, a dividend yield on a stock reduces the value of a call on that stock.[4]

This intuitive approach to the comparative statics of option pricing is the real benefit of the tabular form.

C. Advantages

The tabular form of the call-stock multiplier is simple and flexible. The user does not need a cumbersome cumulative normal distribution table and can focus on the many other aspects of option pricing by determining the related effects of a change in the two relevant parameters (MR and TAV).

Further, all European call options can be priced using this table. The uniqueness of an individual option is captured in its associated MR and TAV calculations; all options with identical MR and TAV parameters will

[4]The convenience yield or rate of return shortfall in a commodity is similar to the dividend yield on a stock, and the implication for option valuation is the same: An increase in the convenience yield on a commodity reduces the value of a call written on that commodity. On the other hand, costly storage of a commodity can be considered a *negative* convenience yield. In this case, increasing the cost of storage increases the MR, and thus *raises* the value of a call on the commodity.

Exhibit 1. Excel Formula for Call-Stock Multiplier

Cell Formula: =NORMSDIST((LN(B$2)+0.5*$A3^2)/$A3)-NORMSDIST((LN(B$2)-0.5*$A3^2)/$A3)/B$2

By freezing column and row references using $, the initial cell formula is copied throughout the rest of the spreadsheet, assuming one has already created a column of time-adjusted volatility values and a row of moneyness ratio values. The Excel setup can be applied to finer gradations of the time-adjusted volatility and the moneyness ratio simply by changing the initial value and/or the incremental increase in value. A table similar to Brealey and Myers' (1996) hedge ratio table can be created by changing the cell formula: =NORMSDIST((LN(B$2)+0.5*$A3^2)/$A3)

To create a put-stock multiplier table, change the cell formula.

Cell Formula: =NORMSDIST((-LN(B$2)+0.5*$A3^2)/$A3)/B$2 - NORMSDIST((-LN(B$2)-0.5*$A3^2)/$A3)

	A	B	C	D
1	**Time-Adjusted Volatility**	**Moneyness Ratio**		
2		*0.90*	*0.92*	*0.94*
3	*0.05*	Cell Formula	Copy Formula	Copy Formula
4	*0.10*	Copy Formula	Copy Formula	Copy Formula

have identical CSMs. Consequently, one does not need to create different tables for options of differing maturities or exercise prices. The option pricing aspect of this tabular approach is not our main contribution, however, as a spreadsheet can easily be created to use Black-Scholes to price options without the table. The primary contribution of this approach is the intuition behind the "greeks".

II. The Greeks

The "greeks" emerge from the call-stock multiplier table readily. The intuition behind the results of taking partial derivatives of the call option price relative to individual option parameters is greatly simplified using a tabular presentation.

First, consider how changes in the underlying security price affect the MR in the CSM table. Because the current underlying security price is in the numerator of the MR, the MR increases as the underlying stock price rises. Consequently, this reflects rightward movement across Exhibit 2, implying that the price of the call option increases. The reader can confirm that an increase in the MR does in fact reflect an increase in the CSM, and, hence, an increase in the price of the call option. Someone with little knowledge of derivatives is still able to see how a change in the underlying asset's price affects the option's price.

This effect represents the option's *delta*—the sensitivity of the option's price to the price of the underlying asset. Delta is found mathematically by taking the first derivative of the Black-Scholes model with respect to the stock price. This is not a simple derivative, since S appears in both integrals which represent the cumulative normal distribution. After much cancellation, the delta of the call turns out to be $N(d_1)$.

Second, Exhibit 2 makes it clear that the delta changes with the stock price. For example, consider a call with a TAV of 20% and an MR of 1.0; the CSM is 0.0797. If the stock price falls, and the MR drops by 0.02 to 0.98, the CSM is reduced by 0.0797 - 0.0707 = 0.0090. But if the stock price rises so that the MR increases by 0.02 (i.e., moves from 1.00 to 1.02), the CSM rises by more: 0.0891 - 0.0797 = 0.0094.

In other words, for TAV of 20%, as the MR rises from 0.98 to 1.00 to 1.02, the CSM changes at a non-constant rate (first by 0.0090, and then by 0.0094). The call value is *convex* in the stock price.[5]

This demonstrates what is known as the *gamma* of a call. Gamma is the second derivative of the call option price relative to the current underlying security price. Generally, delta and gamma are used together to hedge against disadvantageous price movements. The tabular presentation provides a simple way to understand the intuition of these two option sensitivities.

The intuition of other option terminology emerges just as easily. The *vega* of a call measures sensitivity of the call price to an increase in the volatility. The TAV parameter increases as volatility increases; this

[5]We have to be a bit careful here. We are trying to demonstrate that *for a fixed percentage change in the value of the underlying security relative to the present value of the strike price*, the value of the call changes more quickly as the underlying price approaches and goes beyond the present value of the strike price. The movements between moneyness ratios in the table are *not* constant percentage changes in the value of the underlying asset—they are decreasing percentage changes.

Exhibit 2. Call-Stock Multiplier Table

To find the call option price, find the table value in Panel A (i.e., call-stock multiplier, CSM) that corresponds to the appropriate time-adjusted volatility (TAV) and moneyness ratio (MR). Panel B provides a guide as to the appropriate TAV and MR. Panel B also provides a guide for converting the CSM into a call option price.

Panel A. Call-Stock Multiplier (CSM) Based on TAV and MR

Time-Adjusted Volatility	Moneyness Ratio										
	0.90	**0.92**	**0.94**	**0.96**	**0.98**	**1.00**	**1.02**	**1.04**	**1.06**	**1.08**	**1.10**
0.05	0.0003	0.0010	0.0027	0.0060	0.0116	0.0199	0.0311	0.0445	0.0595	0.0754	0.0914
0.10	0.0079	0.0118	0.0169	0.0232	0.0309	0.0399	0.0501	0.0613	0.0734	0.0863	0.0996
0.15	0.0225	0.0283	0.0349	0.0424	0.0507	0.0598	0.0695	0.0799	0.0907	0.1020	0.1136
0.20	0.0399	0.0467	0.0542	0.0622	0.0707	0.0797	0.0891	0.0988	0.1090	0.1193	0.1299
0.25	0.0586	0.0660	0.0738	0.0821	0.0906	0.0995	0.1086	0.1180	0.1276	0.1373	0.1472
0.30	0.0779	0.0857	0.0937	0.1020	0.1105	0.1192	0.1281	0.1372	0.1463	0.1556	0.1649
0.35	0.0976	0.1055	0.1136	0.1219	0.1304	0.1389	0.1476	0.1563	0.1651	0.1740	0.1829
0.40	0.1175	0.1255	0.1336	0.1418	0.1501	0.1585	0.1670	0.1754	0.1839	0.1925	0.2010
0.45	0.1374	0.1454	0.1535	0.1616	0.1698	0.1780	0.1862	0.1945	0.2027	0.2109	0.2191
0.50	0.1573	0.1653	0.1733	0.1813	0.1894	0.1974	0.2054	0.2134	0.2214	0.2293	0.2372
0.55	0.1772	0.1852	0.1931	0.2010	0.2088	0.2167	0.2245	0.2323	0.2400	0.2477	0.2553
0.60	0.1971	0.2049	0.2127	0.2204	0.2282	0.2358	0.2434	0.2510	0.2585	0.2659	0.2733
0.65	0.2168	0.2245	0.2322	0.2398	0.2473	0.2548	0.2622	0.2696	0.2769	0.2841	0.2912
0.70	0.2364	0.2440	0.2515	0.2590	0.2664	0.2737	0.2809	0.2880	0.2951	0.3021	0.3090
0.75	0.2559	0.2634	0.2707	0.2780	0.2852	0.2923	0.2994	0.3063	0.3132	0.3200	0.3267
0.80	0.2752	0.2825	0.2898	0.2969	0.3039	0.3108	0.3177	0.3244	0.3311	0.3377	0.3442
0.85	0.2944	0.3015	0.3086	0.3156	0.3224	0.3292	0.3358	0.3424	0.3489	0.3552	0.3615
0.90	0.3134	0.3204	0.3272	0.3340	0.3407	0.3473	0.3538	0.3601	0.3664	0.3726	0.3787
0.95	0.3321	0.3390	0.3457	0.3523	0.3588	0.3652	0.3715	0.3777	0.3838	0.3898	0.3957
1.00	0.3507	0.3574	0.3639	0.3704	0.3767	0.3829	0.3890	0.3951	0.4010	0.4068	0.4125

Panel B. TAV, MR, and CSM Specifications for European Style Call Options

Option Type	Moneyness Ratio (MR)	Time-Adjusted Volatility (TAV)	Call Price
Non-dividend paying stock (S); Black-Scholes (1973)	$S \div Xe^{-R(T)}$	$\sigma(T)^{0.5}$	$S \times CSM$
Continuous dividend yield stock ($Se^{-q(T)}$); Merton (1973)	$Se^{-q(T)} \div Xe^{-R(T)}$	$\sigma(T)^{0.5}$	$Se^{-q(T)} \times CSM$
Currency option; Garman and Kohlhagen (1983)*	$Se^{-Rf(T)} \div Xe^{-Rd(T)}$	$\sigma(T)^{0.5}$	$Se^{-Rf(T)} \times CSM$
Option on futures (F); Black (1973)	$Fe^{-R(T)} \div Xe^{-R(T)} = F \div X$	$\sigma(T)^{0.5}$	$Fe^{-R(T)} \times CSM$
Option to exchange one asset for another asset (i.e. surrender S_2 for S_1); Margrabe (1978)	$Q_1S_1exp(-q_1(T)) \div Q_2S_2exp(-q_2(T))$	$[(\sigma_1)^2 + (\sigma_2)^2 - 2\rho(\sigma_1)(\sigma_2)]^{0.5}(T)^{0.5}$	$Q_1S_1exp(-q_1(T)) \times CSM$

Note: X is strike price, T is time to maturity measured in years, R is annual risk-free rate, q and q_i are continuous annual dividend yields, σ and σ_i denote annual volatility, ρ denotes correlation, Rf is foreign country annual risk-free rate, Rd is domestic country annual risk-free rate, and Q_i is quantity of S_i.
*In this case, S and X are direct quote exchange rates (i.e., the domestic cost of one unit of foreign currency).

is equivalent to moving down the rows in Exhibit 2 and noticing that the CSM increases. Consequently, an increase in volatility leads to an increase in the call option price.

Theta measures call option sensitivity to lengthening of the maturity of an option. A change in call option maturity increases both the MR (by reducing its denominator) and the TAV. This dual effect is equivalent to moving to the right and downward in Exhibit 2. Both effects raise the price of the call option.

Rho measures call option sensitivity to an increase in the risk-free rate. In the Black-Scholes model, the risk-free rate is similar to the current underlying security price in that it appears outside *and* inside two integrals. The tabular presentation makes rho much easier to understand.

Raising the risk-free rate reduces the denominator of the MR parameter, but does not change the TAV. Consequently, the MR parameter increases with an increase in the risk free rate. The effect is reflected by left to right movement in Exhibit 2—that is, the value of the call increases.

This also makes the intuition behind the rho clear; a rise in the risk-free rate reduces the present value of the strike price to be paid by the holder of the call, and, hence, increases the value of the call.

Although it is not identified with a Greek letter, the sensitivity of the call option price to an increase in the exercise price can also be evaluated. Raising the exercise price reduces the MR parameter, because the exercise price is in the denominator of the MR. This is reflected by right to left movement in Exhibit 2. Increasing the exercise price reduces the call option price.

III. Implied Volatility

Many option traders assert that they actually trade *volatility*. This interpretation arises because of the five inputs into the Black-Scholes model, four are observable (time to maturity, the strike price, the risk-free rate, and the current price of the underlying asset), while volatility is not. Hence, market participants will agree on everything *but* the volatility parameter, and a good trader can buy calls from participants with lower volatility estimates and sell to others with higher estimates. By viewing a price quote in the market, the volatility trader determines the implied volatility by solving for the volatility level, which along with the four observable parameters gives the market price.

Backing out an option's implied volatility is not a simple task. Given the importance of volatility to option traders, we present two ways to arrive at an option's time-adjusted volatility through the tabular format.

The first way is to use the call-stock multiplier table in Exhibit 2 just as before. For example, suppose Home Depot's stock is still trading at \$48 per share (with no dividends) but you observe that the market price for a nine-month \$50 call on the stock has increased to \$9.86. The MR of the call is $\$48 \div \left(\$50e^{-0.08(0.75)}\right) = 1.02$, and the CSM is $\$9.86/\$48.00 = 0.2054$.

In Exhibit 2, read down the column for MR = 1.02 until you find a CSM of 0.2054. Then read left along the row to determine the TAV. The TAV for this call is 50%, which is the nine-month volatility. To convert this to annual terms, simply multiply 50% by the square root of 4/3 to get an annual implied volatility of 57.7%.

Even though the stock price has not changed, the call has increased in value because volatility has increased. An option trader who is long Home Depot volatility would have profited.

The second approach is to construct a time-adjusted volatility table, similar in layout to the call-stock multiplier table. The primary difference is that instead of using the option's MR and TAV to arrive at its CSM, the option's MR and CSM are used to calculate its TAV (i.e., the TAV table is just the reverse of the CSM table).

The TAV table can be generated through two steps. First, we need a call-stock multiplier table that relocates the TAV parameters to the rightmost vertical axis (Exhibit 3). Second, we use the Excel VLOOKUP command to locate the TAV appropriate for an option with a given MR and CSM (see Appendix for a brief description of VLOOKUP). Thus, the time-adjusted volatility table is based on two familiar parameters: MR and CSM.

The Excel commands for developing the time-adjusted volatility table, given a call-stock multiplier table (slightly adjusted in format), are displayed in Exhibit 3.

Using Home Depot again, the CSM is 0.2054 (\$9.86/\$50.00), or approximately 0.205, and the MR is 1.02. Cross-matching the two values in the time-adjusted volatility table in Exhibit 4 produces a TAV of 0.4985, which is approximately the same that we found before. Again, finer gradations of the input parameters will improve accuracy. That is, a table value of 0.2054 for the CSM, if it were in Exhibit 4, would produce a TAV of 0.50.

IV. Extensions

In this section, we show how options on foreign exchange contracts, options on futures contracts, and options to exchange assets can all be priced with the simple two-dimensional table.

A. Options on Foreign Exchange

By changing the underlying security price S to a spot exchange rate (direct quote) and the annual dividend yield q to the risk-free rate of the foreign country, and by making the exercise price X an exchange rate (direct quote), we can price currency options the same way we price an option on a stock with a continuous dividend (see Garman and Kohlhagen, 1983).

For example, suppose that the current yen-dollar exchange rate is 123.5 (or in *direct* terms \$0.0081 per yen), and you want to hedge against a strengthening yen using a one-year call option. Let the strike on the call be \$0.0086, and assume that the yen risk-free rate is 1% per year while the US dollar risk-free rate is 5% per year, and that the volatility of changes in the foreign exchange rate is 40% per year. The moneyness ratio will be 0.98:

Exhibit 3. Finding Time-Adjusted Implied Volatility Using Excel

The first five rows of the spreadsheet are used to create the call-stock multiplier table, except the column for time-adjusted volatility is made to be the final column instead of the initial column. It is this altered table that the VLOOKUP() command references to create the time-adjusted implied volatility table in rows seven and beyond. Further, notice a finer gradation for time-adjusted volatility is used in Column D.

Cell Formula 1: =VLOOKUP($A9, $A$3:$D$5,4, true)
Cell Formula 2: =VLOOKUP($A9, $B$3:$D$5,3, true)
Cell Formula 3: =VLOOKUP($A9, $C$3:$D$5,2, true)

The same logic applies for put options except that one uses the put-stock multiplier table.

	A	B	C	D
1	Moneyness Ratio			Time-Adjusted Volatility
2	*1.02*	*1.04*	*1.06*	
3	0.1000	0.1096	0.1193	*02280*
4	0.1002	0.1097	0.1195	*0.2285*
5	0.1004	0.1099	0.1197	*0.2290*
6				
7	Call-Stock Multiplier	Moneyness Ratio		
8		*1.02*	*1.04*	*1.06*
9	*0.100*	Cell Formula 1	Cell Formula 2	Cell Formula 3
10	*0.105*	Copy Formula 1	Copy Formula 2	Copy Formula 3

$$MR_{FX} = \left(\frac{Se^{-qT}}{Xe^{-RT}}\right) = \frac{0.0081e^{-(0.01)1}}{0.0086e^{-(0.05)1}} = 0.98$$

and the time-adjusted volatility will be 0.40:

$$TAV = \sigma\sqrt{T} = 0.40\sqrt{1} = 0.40$$

So, from Exhibit 2, the call-stock multiplier will be 0.1501. The value of the option is then determined by multiplying the CSM by the numerator of the MR (the direct quote foreign exchange discounted at the foreign risk-free rate):

$$CSM \times Se^{-qT} = 0.1501\left(\$0.0081 \times e^{-(0.01)1}\right) = \$0.0012$$

so the call costs \$0.0012 per yen, or \$1.20 per 1,000 yen contract.

The comparative statics in this case are informative. As the foreign country risk-free rate rises, the moneyness ratio falls, so the value of the call on the foreign exchange also falls. In other words, the effect of the foreign interest rate on the foreign exchange option is the same as the effect of a dividend on a stock option. On the other hand, an increase in the local risk-free rate raises the MR, and hence increases the value of the call on the foreign exchange. In other words, the effect of the local interest rate on the foreign exchange option is the same as the effect of the risk-free rate on a stock option.

B. Options on Futures

Changing the underlying security to a futures price and changing the annual dividend yield on the underlying security to the risk-free rate lets us price options on futures contracts, again the way we price an option on a stock with a continuous dividend (see Black, 1976).

For futures options, the moneyness ratio is:

$$MR_{Futures\ Option} = \frac{Fe^{-RT}}{Xe^{-RT}} = \frac{F}{X}$$

where F is the currently prevailing futures price. The value of the futures option will be:

$$CSM \times Fe^{-RT}$$

For example, suppose the date is March 1, and the current price of a June futures contract on crude oil is \$21.59 per barrel, the volatility of the June crude futures is 40% per year, and the risk-free rate is 4% per year. Futures options typically expire a few days before the first delivery date of the same month's futures contract, so let's price a three-month call on the June oil futures

Exhibit 4. Time-Adjusted Implied Volatility Table

To find a given call option's implied volatility (annualized), take the table value in Panel A, and divide it by the square root of the time to maturity, where time to maturity is measured as a fraction of a year. The table value is found by applying the appropriate moneyness ratio (MR) and call-stock multiplier (CSM). Panel B supplies the appropriate adjustments for the MR and CSM.

Panel A. The Time-Adjusted Implied Volatility Based on the MR and CSM

Call-Stock Multiplier	Moneyness Ratio										
	0.90	*0.92*	*0.94*	*0.96*	*0.98*	*1.00*	*1.02*	*1.04*	*1.06*	*1.08*	*1.10*
0.100	0.3560	0.3360	0.3155	0.2945	0.2735	0.2510	0.2275	0.2030	0.1755	0.1435	0.1015
0.105	0.3685	0.3485	0.3280	0.3075	0.2860	0.2635	0.2405	0.2160	0.1890	0.1585	0.1205
0.110	0.3810	0.3610	0.3405	0.3200	0.2985	0.2765	0.2535	0.2290	0.2025	0.1730	0.1380
0.115	0.3935	0.3735	0.3530	0.3325	0.3110	0.2890	0.2660	0.2420	0.2160	0.1875	0.1540
0.120	0.4060	0.3860	0.3655	0.3450	0.3235	0.3015	0.2790	0.2550	0.2295	0.2015	0.1700
0.125	0.4185	0.3985	0.3780	0.3575	0.3360	0.3145	0.2915	0.2680	0.2430	0.2155	0.1850
0.130	0.4310	0.4110	0.3905	0.3700	0.3490	0.3270	0.3045	0.2810	0.2565	0.2295	0.2000
0.135	0.4440	0.4235	0.4035	0.3825	0.3615	0.3400	0.3175	0.2940	0.2695	0.2435	0.2145
0.140	0.4565	0.4360	0.4160	0.3950	0.3740	0.3525	0.3300	0.3070	0.2830	0.2570	0.2290
0.145	0.4690	0.4485	0.4285	0.4080	0.3865	0.3650	0.3430	0.3200	0.2960	0.2710	0.2435
0.150	0.4815	0.4615	0.4410	0.4205	0.3995	0.3780	0.3560	0.3330	0.3095	0.2845	0.2580
0.155	0.4940	0.4740	0.4535	0.4330	0.4120	0.3910	0.3690	0.3465	0.3230	0.2980	0.2720
0.160	0.5065	0.4865	0.4660	0.4455	0.4250	0.4035	0.3820	0.3595	0.3360	0.3120	0.2860
0.165	0.5190	0.4990	0.4785	0.4585	0.4375	0.4165	0.3945	0.3725	0.3495	0.3255	0.3000
0.170	0.5315	0.5115	0.4915	0.4710	0.4500	0.4290	0.4075	0.3855	0.3625	0.3390	0.3140
0.175	0.5440	0.5240	0.5040	0.4835	0.4630	0.4420	0.4205	0.3985	0.3760	0.3525	0.3280
0.180	0.5565	0.5365	0.5165	0.4965	0.4755	0.4550	0.4335	0.4115	0.3895	0.3660	0.3415
0.185	0.5695	0.5495	0.5295	0.5090	0.4885	0.4675	0.4465	0.4250	0.4025	0.3795	0.3555
0.190	0.5820	0.5620	0.5420	0.5220	0.5015	0.4805	0.4595	0.4380	0.4160	0.3930	0.3695
0.195	0.5945	0.5745	0.5545	0.5345	0.5140	0.4935	0.4725	0.4510	0.4290	0.4065	0.3830
0.200	0.6070	0.5875	0.5675	0.5475	0.5270	0.5065	0.4855	0.4645	0.4425	0.4200	0.3970
0.205	0.6200	0.6000	0.5800	0.5600	0.5400	0.5195	0.4985	0.4775	0.4560	0.4335	0.4110

Panel B. MR and CSM Specifications for European Style Call Options

Option Type	Moneyness Ratio (MR)	Call-Stock Multiplier (CSM)
Non-dividend paying stock (S); Black-Scholes (1973)	$S \div Xe^{-R(T)}$	$Call / S$
Continuous dividend yield stock ($Se^{-q(T)}$); Merton (1973)	$Se^{-q(T)} \div Xe^{-R(T)}$	$Call / (Se^{-q(T)})$
Currency option; Garman and Kohlhagen (1983)*	$Se^{-Rf(T)} \div Xe^{-Rd(T)}$	$Call / (Se^{-Rf(T)})$
Option on futures (F); Black (1973)	$Fe^{-R(T)} \div Xe^{-R(T)} = F \div X$	$Call / (Fe^{-R(T)})$
Option to exchange one asset for another asset (i.e. surrender S_2 for S_1); Margrabe (1978)**	$Q_1S_1exp(-q_1(T)) \div Q_2S_2exp(-q_2(T))$	$Call / (Q_1S_1exp(-q_1(T)))$

Note: X is strike price, T is time to maturity measured in years, R is annual risk-free rate, q and q_i are continuous annual dividend yields, σ and σ_i denote annual volatility, ρ denotes correlation, Rf is foreign country annual risk-free rate, Rd is domestic country annual risk-free rate, and Q_i is quantity of S_i.

*In this case, S and X are direct quote exchange rates (i.e., the domestic cost of one unit of foreign currency).

**The implied volatility is a combination of the two assets and not the volatility for the individual assets.

with a strike of $22.50 per barrel.

The moneyness ratio will be:

$$MR_{Futures\ Option} = \frac{F}{X} = \frac{\$21.59}{\$22.50} = 0.959 \approx 0.96$$

and the time-adjusted volatility will be:

$$TAV = \sigma\sqrt{T} = 0.40\sqrt{0.25} = 0.20$$

From Exhibit 2, we can easily determine that the CSM of this futures option is 0.0622, so the current price of the three-month futures option is:

$$CSM \times Fe^{-RT} = 0.0622 \times \$21.59e^{-0.04(0.25)}$$

$= 0.0622 \times \$21.375 = \$1.3295$ per barrel

or $1,329.50 per 1,000-barrel contract.

It is worth noting that even though there may be a convenience yield or storage costs associated with the asset that underlies the futures contract, convenience yield and costly storage do not enter the calculation for the value of an option on a futures on that asset. The reason is that the futures price already reflects convenience yields and storage costs.

C. Exchange Options

The option to exchange one asset for another asset (developed in Margrabe, 1978) can also by priced using Exhibit 2, although this requires slightly more advanced adjustments to both the MR and TAV. For the exchange option, the MR is:

$$MR_{Exchange\ Option} = \frac{Q_1 S_1 e^{-q_1 T}}{Q_2 S_2 e^{-q_2 T}}$$

where Q_1, S_1, and q_1 are the quantity, current price, and dividend yield on the asset to be received in the exchange, respectively, while Q_2, S_2, and q_2 are the quantity, current price, and dividend yield on the asset to be given up in the exchange.

The TAV of the exchange option uses the volatility of both assets:

$$TAV_{Exchange\ Option} = \sqrt{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)T}$$

where σ_1 is the volatility of the asset to be received, σ_2 is the volatility of the asset to be surrendered, and ρ is the correlation of returns between the two assets. The term $\sqrt{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)}$ is the volatility of the ratio S_1/S_2. The value of the exchange option is then $CSM \times (Q_1 S_1 e^{-q_1 T})$.

Firms that initiate all-share tender offers are giving away an option to exchange the target firm's shares for their own. For a completely hypothetical example, suppose that Coca-Cola (with stock price of $49.15) were to launch an all-share tender offer for Disney (whose shares are worth $24.00), where each shareholder of Disney would receive one-half share of Coca-Cola stock for every one share of Disney stock tendered. The tender period would last six months. Coke's dividend yield is 1.7% per year, while Disney's is 0.9% per year.

The MR for this exchange would be:

$$MR_{Exchange\ Option} = \frac{Q_1 S_1 e^{-q_1 T}}{Q_2 S_2 e^{-q_2 T}} = \frac{0.5(\$49.15)e^{-0.017(0.5)}}{1(\$24.00)e^{-0.009(0.5)}}$$

$$= \frac{\$24.37}{\$23.89} = 1.02$$

which, by virtue of being greater than one, implies a very small control premium.

If the volatilities of Coke and Disney are 33% and 39%, respectively, and the return correlation between the two is 0.31, then the TAV for this exchange option is:

$$TAV_{Exchange\ Option} = \sqrt{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)T}$$

$$= \sqrt{\left[(0.33)^2 + (0.39)^2 - 2(0.31)(0.33)(0.39)\right](0.5)} = 0.301 \approx 0.30$$

From Exhibit 2, we can see that the CSM of this exchange option is 0.1281, so the value of the exchange option to the Disney shareholders is:

$$CSM \times (Q_1 S_1 e^{-q_1 T}) = 0.1281(0.5)(\$49.15)e^{-0.017(0.5)} = \$3.12$$

In other words, if a Disney shareholder tenders her shares immediately, her wealth is $0.5(\$49.15) = \$24.575$ per share, but if she holds onto her Disney share and the option to exchange in the tender, her wealth is $\$24.00 + \$3.12 = \$27.12$ per share. The option is worth more alive than dead.

The exchange option can be used in several ways. First of all, it reduces to the Black-Scholes model when the asset to be surrendered in the exchange is a fixed sum of money (the strike price). Second, one can price the option to exchange one commodity futures contract for another. In this case, the continuous dividend yields are zero, and the quantities are the dollars per contract. Third, an index futures exchange can be priced; the quantities will be the dollars paid per index point in the contract.

Note that the value of an exchange option is invariant to the risk-free rate. Neither MR nor TAV changes if R is changed. This is because a change in the risk-free rate of return has an equal effect on each asset in the exchange, and thus washes out.

V. Conclusion

A tabular representation of the Black-Scholes model greatly simplifies calculations and makes it easy to grasp the model's sensitivity to given parameter changes. The table is simple to generate in Excel and can be inverted to produce implied volatility. Although the tabular presentation is not new, the inversion of the table to produce implied volatility and extension of the pricing technique to options other than simple European call options for non-dividend paying stocks are new contributions. Further, the exposition on the option greeks is much more thorough than what has previously existed.

Aside from these benefits, if one simply wants to educate an audience on the application of the Black-Scholes model without going into great detail about the model, the tabular presentation is a very workable alternative. Calculations are simplified and the intuition behind the greeks is easily demonstrated. For individuals who need to understand attributes about options, but not necessarily understand option pricing (say in an introductory finance class or a non-finance class), the tabular representation is a good solution. One can provide the tables and then ask questions or lecture using the table.

The ultimate goal is not to subvert the mathematics of the Black-Scholes model. Rather, it is to provide a low-tech complement.■

Appendix. VLOOKUP Command

In the sample Excel sheet, Column A values can be attributed to the index in Column C. If you want information associated with the index value from Columns D through F, you use the VLOOKUP command, and index a Column A value by Column C to provide the information in Columns D through F.

The syntax for VLOOKUP is:

= VLOOKUP(Cell containing value to be cross-matched with index,
Range of indexed information,
Column number within indexed information for desired information,
True or False condition)

The indexed information must be sorted by the first column for that column to be the appropriate variable for cross-matching information. By entering "true," the user is willing to accept information that is not necessarily in the index but is close to a value within the index. By entering "false," the user demands that the cross-matched variable actually be in the index.

For example, =VLOOKUP(A1, C1:F5, 3, false) returns 25, which corresponds to 4 within Column C (the index column) and the third column within the indexed information. The "false" condition is not necessary, however. Changing the command to =VLOOKUP(7, C1:F5, 3, false) returns an error message, because 7 is not in the index. For the command =VLOOKUP(7, C1:F5, 2, true), a value of 81 is generated, because 7 is closest to the index value of 5.

	A	B	C	D	E	F
1	4		1	Ted	35	$25,000.00
2	4		2	Judy	29	$30,000.00
3	2		3	Andrew	18	$15,000.00
4	3		4	Terry	25	$65,000.00
5	1		5	Alfred	81	$45,000.00

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